Security-Constrained Adequacy Evaluation of Bulk Power System Reliability

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***Abstract* -- A framework of security-constrained adequacy evaluation (SCAE) based on analytical techniques is proposed to assess the ability of a bulk power system to supply electric load while satisfying security constraints. It encompasses three main steps: (a) critical contingency selection, (b) effects analysis, and (c) reliability index computation. Effects analysis is the most essential but computationally demanding procedure. It is important that effects analysis simulate contingencies in a realistic manner efficiently. However, effects analysis based on the traditional power flow technology often lacks the realistic system model and diverges when the system is severely stressed. In this paper, a non-divergent optimal quadratized power flow (NDOQPF) algorithm is proposed for effects analysis. It is implemented based on a single phase quadratized power flow (SPQPF) model that can improve solution efficiency, and a constrained optimization problem, which incorporates operational practices, security constraints, and remedial actions, is formulated to simulate contingencies realistically. The non-divergence of power flow is achieved by introducing fictitious bus injections that are driven to zero as the solution progresses. It guarantees convergence if a solution exists; if a solution does not exist, it provides a suboptimal solution that may include load shedding. The NDOQPF algorithm is capable of efficiently solving the RTO/ISO operational model as well. Such operational procedure is formulated as an optimization problem with the objective function being the bid cost function and congestion constraints. The proposed SCAE framework also includes an improved contingency selection/enumeration scheme based on SPQPF and reliability index computations. The methodology is demonstrated with the IEEE reliability test system.**

***Index Terms*—Bulk system reliability evaluation, contingency selection/enumeration, effects analysis, remedial actions, non-divergent optimal quadratized power flow, security constraint.**

I. Introduction

TRADITIONALLY, reliability evaluation for the bulk power system has formed an important part in power system planning and operating procedures. The long-term reliability evaluation is usually executed to assist the long-range system planning while the short-term reliability prediction may be sought to assist operators in day-to-day operating decisions [1, 2]. Nowadays, the electric power industry restructuring introduces some new issues, such as energy market and transmission system open access, which inevitably force system operation points very close to their physical limits. This scenario requires the development of reliability assessment techniques that allow more rigor in system modeling and higher computational efficiency in the evaluation procedure [3, 4].

Bulk power system reliability analysis is generally performed using analytical or Monte Carlo simulation techniques. Three aspects of system reliability can be evaluated [5]: (a) adequacy evaluation that estimates the system ability to supply aggregate electric energy demands from customers, (b) security-constrained adequacy evaluation (SCAE) that estimates the ability of the system reserve to avoid load curtailment under contingencies, and (c) security evaluation that estimates the system ability to operate in stable conditions when disturbances occur. In past several decades, adequacy evaluation has been a topic of major interest [6-8]. Recently, more research effort focuses on SCAE and security evaluation techniques that study system reliability from different facets [5, 9-11].

SCAE based on analytical techniques includes three main steps: (a) critical contingency selection, (b) effects analysis, and (c) reliability index computation. Effects analysis is the most important but computationally demanding procedure. It is important that contingencies be simulated in a realistic manner that captures the system response, including major controls and operational practices. Effects analysis must also be efficient so that the overall computational effort of SCAE is reasonable. Effects analysis based on the traditional power flow technology often uses the simplified system model and breaks down (non-convergence) when multi-level contingencies are considered and the system is severely stressed.

In this paper, a non-divergent optimal quadratized power flow (NDOQPF) algorithm is proposed for effects analysis. It is implemented based on a single phase quadratized power flow (SPQPF) model to improve efficiency. A constrained optimization problem, integrating operational practices, security constraints, and remedial actions, is formulated to simulate contingencies realistically. The non-divergence is achieved by the introduction of fictitious bus injections that are driven to zero as the solution progresses. The methodology guarantees convergence if a solution exists; in case a solution does not exist, it provides a non-optimal solution that may include load shedding. The proposed NDOQPF methodology is capable of efficiently solving the ISO/RTO operational

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model as well. Specifically, in the deregulated system, the ISO/RTO determines the least expensive dispatch of generation based on energy bids while satisfying system constraints. Remedial actions are applied if system congestions occur. Such operational procedure is mathematically formulated as a constrained optimization problem with the objective function being the bid cost function and congestion constraints. This operational model is embedded in the proposed NDOQPF. The proposed SCAE framework also includes an improved contingency selection/enumeration scheme based on SPQPF and reliability index computation. The overall methodology is demonstrated with the IEEE reliability test system (RTS).

II. Methodology

This section describes the proposed SCAE methodology for bulk power system reliability assessment. Figure 1 shows the overall developed computational algorithm of the methodology, which is implemented based on the SPQPF model and mainly includes contingency selection/enumeration, effects analysis, and reliability index computation. The reliability evaluation procedure is illustrated below.

First of all, feasible contingencies up to a certain level are defined. Then the contingency enumeration scheme provides an enumeration for each level of contingencies and the selection method is applied to rank contingencies according to their impact on system operation. This procedure can be repeated for a number of electric load levels. Ranked contingencies are then subjected to effects analysis.

Three options for effects analysis are available in branch A, B, and C: (a) network solution with remedial actions, (b) system simulation, and (c) market simulation. These options provide the means to perform effects analysis in regulated or deregulated environment and evaluate the adequacy or security aspect of system reliability. The network solution with remedial actions applies the proposed NDOQPF algorithm for effects analysis under the regulated environment. Market simulation approach considers effects analysis based on the NDOQPF algorithm in deregulated power systems. The system simulation approach is capable of considering the full security evaluation in effects analysis. This security assessment scheme is not addressed in this paper.

The results of effects analysis for all evaluated contingencies are stored and processed to provide reliability indices, which provide system planners and operators a quantitative way to indicate system reliability levels.

The implementation of SPQPF and the three major constituent parts of the SCAE methodology are described in detail in the following sections.

A. Single Phase Quadratized Power Flow

Traditional power flow (TPF) models usually suffer from lack of ability to model complex component characteristics and slow convergence. To improve the accuracy and efficiency in contingency selection and effects analysis, an advanced power flow model, i.e., the single phase quadratized power flow (SPQPF) model [12], is applied in the proposed SCAE methodology. A comparison between the formulation of TPF and SPQPF models is provided below to illustrate the advantages of SPQPF.

The TPF model consists of power balance equations at each bus in the system. These equations are expressed in Polar coordinates in terms of system states (bus voltage magnitudes and angles), and nonlinear trigonometric terms appear in equations inevitably. In addition, most system loads are induction machines and their models contain relatively high order nonlinear terms. Consequently, the TPF formulation results in a set of very complex and highly nonlinear equations. When iterative method, such as Newton-Raphson method, is used to solve such equations, many iterations may be required and in some cases, the solution it is hard to find [13].

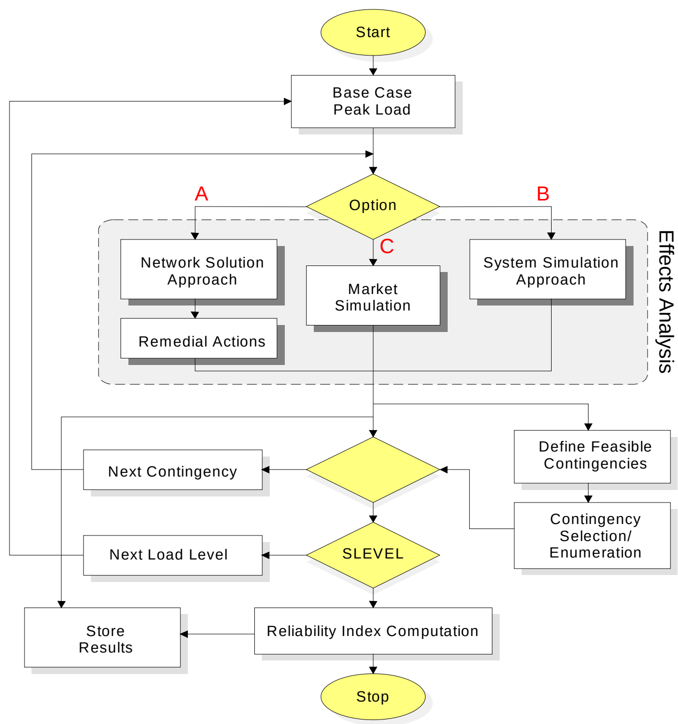

Figure 1. Overall computational algorithm of SCAE methodology for bulk power system reliability assessment.

The SPQPF model, however, is set up based on the application of the Kirchhoff's current law at each bus, with the intention that most of power flow equations are linear in the large-scale system. Also, system states are expressed in Cartesian coordinates (bus voltage real and imaginary parts) that can avoid trigonometric terms. As trigonometric functions are absent, power flow equations become less complex. Moreover, since Newton's method is ideally suitable to solve quadratic equations, all power flow equations are quadratized, which are achieved by introducing additional state variables. As a result, all power flow equations are linear or quadratic. The formulation of the quadratized power flow model provides superior performance in two aspects: (a) faster convergence and (b) ability to model complex load characteristics in the quadratized form.

B. Contingency Selection/Enumeration

Contingency selection/enumeration is to identify the contingencies that may lead to system unreliability, such as system loss of load, which is achieved by an advanced performance index (PI) based contingency ranking approach and an improved contingency enumeration scheme.

Since traditional PI linearization based contingency ranking methods are prone to misranking due to the highly nonlinear nature of the TPF model, in this paper, an advanced PI method, i.e., the state linearization approach [13] is applied to reduce the error introduced by the PI linearization. In this approach, the contingency ranking problem is reformulated using the SPQPF model that has milder nonlinearities (by construction), and an indirect differentiation procedure is used to yield higher order sensitivity terms in calculating PI changes before and after contingencies. Results of the state linearization approach has shown a better performance in improving contingency ranking accuracy compared to PI linearization methods based on TPF models [13-15].

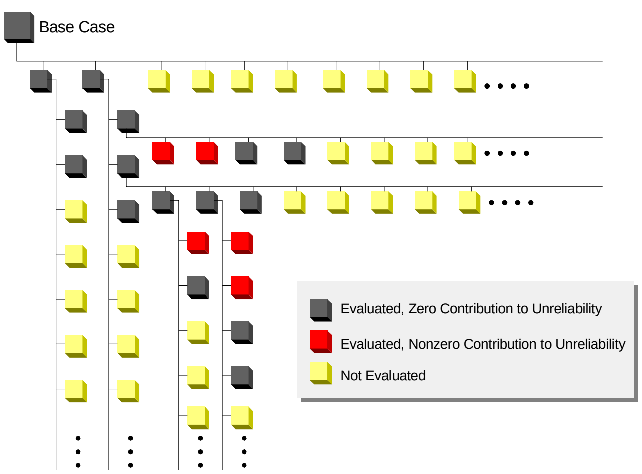

Figure 2. A Wind-Chime enumeration scheme.

Figure 2 shows a Wind-Chime contingency enumeration scheme [14], which can systematically enumerate system contingencies according to their outage levels and rank each level contingencies in terms of severity based on the results of PI changes obtained from the state linearization approach. In each outage level, the reliability evaluation starts from the highest ranked contingencies. If there are several successive contingencies that are evaluated but have zero contribution to system unreliability, the rest of the contingencies need not be investigated since they are considered to have less severe impact on system reliability. Therefore, computational effort can be saved by performing evaluation only on the most severe contingencies. Figure 2 shows the resulting three types of system contingencies: (1) evaluated contingencies with nonzero contribution to unreliability, (2) evaluated contingencies with zero contribution to unreliability, and (3) not evaluated contingencies.

C. Effects Analysis Based on NDOQPF Algorithm

A non-divergent optimal quadratized power flow algorithm is presented for effects analysis to simulate the system response to a critical contingency, including major controls and operational practices. Consider a system and let vectors $\mathbf{x}$ and $\mathbf{u}$ represent the system state variables and control variables of remedial actions (RAs) such as unit real/reactive generation adjustments, switched capacitors/reactors, transformer tap/phase shift adjustments, and the last RA would be used is load shedding. Assuming a given operating state vector $\mathbf{x}^0$ and control variable vector $\mathbf{u}^0$ and considering a general system bus k as shown in Figure 3, unless $\mathbf{x}^0$ and $\mathbf{u}^0$ represent a power flow solution, a mismatch value exists at bus k equal to $I^0_{mk\_r} + jI^0_{mk\_i}$. Assume that a fictitious current source ($I_{mk\_r} + jI_{mk\_i}$) is placed at bus k and let the output of the current source be equal to the mismatch value $I^0_{mk\_r} + jI^0_{mk\_i}$, the Kirchhoff's current law is then satisfied at bus k . Similar fictitious sources/mismatches can be assumed for other SPQPF equations such that $\mathbf{x}^0$ and $\mathbf{u}^0$ represent the current operating condition of the system. The actual operating condition can be obtained by gradually driving the output of the fictitious sources/mismatches to zero. This transition can be achieved along a trajectory that maintains feasibility and optimality. Mathematically, above procedure is formulated as an optimization problem:

$$
\begin{aligned}
&\text{Minimize} \quad J = \mu \sum_{i=1}^{n} |m_i| + \sum_{j=1}^{r} \lambda_j |\Delta u_j| \\
&\text{Subject to} \quad G(\mathbf{x},\mathbf{u},\mathbf{m}) = 0 \\
&\quad V_j^{\min} \le |\tilde{V}_j| \le V_j^{\max} \qquad j = 1,\cdots,b \\
&\quad |\tilde{I}_j| \le I_j^{\max} \qquad j = 1,\cdots,l \\
&\quad P_{gj}^{\min} \le P_{gj} \le P_{gj}^{\max} \qquad j = 1,\cdots,g_{ed} \\
&\quad Q_{gj}^{\min} \le Q_{gj} \le Q_{gj}^{\max} \qquad j = 1,\cdots,g_{pv} \\
&\quad \Delta u_j^{\min} \le \Delta u_j \le \Delta u_j^{\max} \qquad j = 1,\cdots,r \\
&\quad 0 \le |m_j| \le |m_j|^{\max} \qquad j = 1,\cdots,n\,, \qquad (1)
\end{aligned}
$$

where

μ penalty coefficient for mismatches

m_i mismatch value of the i^{th} SPQPF equation

λ_j weight coefficient of the j^{th} remedial action

Δu_j amount of the j^{th} remedial action adjustment

n number of SPQPF equations

r number of total control variables

$G(\mathbf{x},\mathbf{u},\mathbf{m}) = 0$ SPQPF equations

$\mathbf{x}$, $\mathbf{u}$, $\mathbf{m}$ state, control, mismatch variable vectors respectively

b number of buses

l number of circuit branches

g_{ed} number of generators participating economic dispatch

g_{pv} number of PV generators

The first term in the objective function is a penalty function that forces mismatches to zero and thus reaching feasibility. The second term is a pre-selected function to be optimized. In (1), this term expresses the weighted amount of RAs. However, it can be substituted with any other functions of interest.

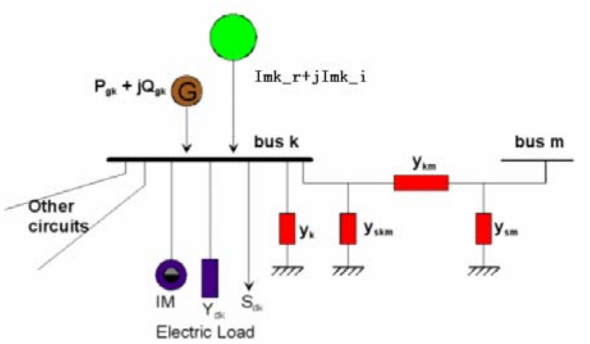

Figure 3. A general bus k in a power system with a fictitious current source.

The defined optimization problem is a large-scale problem. The size of this problem can be drastically reduced with simple transformations. Specifically, mismatch values can be substituted with one artificial control variable v as follows:

$$m_i = m_i^0 v, \quad 0 \le v \le 1, \tag{2}$$

where variable v represents the normalized change of mismatches. Note that this transformation replaces all mismatch values (a total of n) with a single variable v.

Variables P_{gj} (a total of g_{ed}) can also be replaced with only two variables that implicitly incorporate the economic dispatch process. Assume that the initial generation P_{gj}^0 ($j = 1, \cdots, g_{ed}$) is an economic dispatch schedule with respect to initial electric load P_L and Lagrange multiplier λ^0. Assume now that load has increased from P_L to $P_L + dP_L$ and consider the economic dispatch problem:

$$\text{Min} \quad \sum_{j=1}^{g_{ed}} f_j(P_{gj}) = \sum_{j=1}^{g_{ed}} (a_j + b_j P_{gj} + c_j P_{gj}^2)$$

$$\text{S.t.} \quad \sum_{j=1}^{g_{ed}} P_{gj} - (P_L + dP_L) - q = 0, \tag{3}$$

where

q system transmission loss

a_j, b_j, c_j generation cost coefficients for j^{th} generating unit

If $dP_L = 0$, then the solution for the above problem is P_{gj}^0. When $dP_L \ne 0$, according to Lagrange theory, the solution must satisfy following necessary conditions:

$$\frac{\partial f_j(P_{gj})}{\partial P_{gj}} - \lambda (1 - \frac{\partial q}{\partial P_{gj}}) = 0, \quad j = 1, \cdots, g_{ed}$$

$$\sum_{j=1}^{g_{ed}} P_{gj} - (P_L + dP_L) - q = 0 \tag{4}$$

Upon linearization of above equations around operating point P_{gj}^0 and λ^0,

$$\begin{bmatrix} 2c_1 & 0 & 0 & \cdots & -(1-\frac{dq}{dP_{g1}}) \\ 0 & 2c_2 & 0 & \cdots & -(1-\frac{dq}{dP_{g2}}) \\ \vdots & & & & \\ (1-\frac{dq}{dP_{g1}}) & (1-\frac{dq}{dP_{g2}}) & & \cdots & 0 \end{bmatrix} \begin{bmatrix} dP_{g1} \\ dP_{g2} \\ \vdots \\ d\lambda \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} dP_L \tag{5}$$

where $q = \sum_{j=1}^{g_{ed}} P_{gj} - (P_L + dP_L)$,

and $$\frac{dq}{dP_{gj}} = \frac{\partial q}{\partial P_{gj}} - \frac{\partial q}{\partial x} J^{-1} \frac{\partial G}{\partial P_{gj}}. \tag{6}$$

Solution of above equations yields:

$$\begin{bmatrix} dP_{g1} \\ dP_{g2} \\ dP_{g3} \\ \vdots \\ d\lambda \end{bmatrix} = \begin{bmatrix} 2c_1 & 0 & 0 & \cdots & -(1-\frac{dq}{dP_{g1}}) \\ 0 & 2c_2 & 0 & \cdots & -(1-\frac{dq}{dP_{g2}}) \\ \vdots & & & & \\ (1-\frac{dq}{dP_{g1}}) & (1-\frac{dq}{dP_{g2}}) & & \cdots & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} dP_L \tag{7}$$

or $dP_{gj} = p_{fj} dP_L$,

where p_{fj} is the economic participation factor for generator j.

In order to allow flexibility in the model, two economic participation factors are defined, one (p_f^+) for generation increase and the other (p_f^-) for generation decrease. Two additional variables w_1 and w_2 are introduced to represent the total system generation increase and decrease, respectively. They satisfy the condition $w_1 w_2 = 0$, which is incorporated in the SPQPF model. Therefore,

$$dP_{gj} = p_{fj}^+ w_1 + p_{fj}^- w_2, \quad j = 1, \cdots, g_{ed}, \tag{8}$$

where

$$p_{fj}^+ = \begin{cases} p_{fj} & P_{gj}^{\min} \le P_{gj}^0 < P_{gj}^{\max} \\ 0 & P_{gj}^0 = P_{gj}^{\max} \end{cases}$$

$$p_{fj}^- = \begin{cases} p_{fj} & P_{gj}^{\min} < P_{gj}^0 \le P_{gj}^{\max} \\ 0 & P_{gj}^0 = P_{gj}^{\min} \end{cases}. \tag{9}$$

This transformation guarantees that any changes in unit output will preserve an optimal economic dispatch. Also, it reduces variables P_{gj} (a total of g_{ed}) to only two variables, w_1 and w_2.

$$P_{gj} = P_{gj}^0 + dP_{gj} = P_{gj}^0 + p_{fj}^{+} w_1 + p_{fj}^{-} w_2 \,,\quad j = 1,\cdots, g_{ed} \tag{10}$$

Another feature that has been incorporated in SPQPF is the ability to allocate reactive power burden among PV generator units connected to the same bus, which is implemented based on the following derivation:

$$\frac{Q_{ji} - Q_{ji}^{\min}}{Q_{ji}^{\max} - Q_{ji}^{\min}} = k_i \,, \qquad i = 1, \cdots, b_{mpv} \,,\; j = 1,\cdots, g_{pvi}$$
$$Q_{ji} = k_i (Q_{ji}^{\max} - Q_{ji}^{\min}) + Q_{ji}^{\min}$$
$$Q_i = \sum_{j=1}^{g_{pvi}} Q_{ji} = k_i \sum_{j=1}^{g_{pvi}} (Q_{ji}^{\max} - Q_{ji}^{\min}) + \sum_{j=1}^{g_{pvi}} Q_{ji}^{\min} \,, \tag{11}$$

where

k_i a proportional constant for bus i

b_{mpv} number of buses connected to multiple PV generators

g_{pvi} number of PV generators connected to bus i

Upon substitution of above transformations in the formulation in (1),

Minimize $J = \mu \sum_{i=1}^{n} | m_i^0 | v + \sum_{j=1}^{r} \lambda_j \, | \Delta u_j |$

Subject to $G'([\mathbf{x}, \mathbf{k}, w_1, w_2], \mathbf{u}, v) = 0$

$$V_j^{\min} \le | \tilde{V}_j | \le V_j^{\max} \qquad j = 1,\cdots, b$$
$$| \tilde{I}_j | \le I_j^{\max} \qquad j = 1,\cdots, l$$
$$P_{gj}^{\min} \le P_{gj} \le P_{gj}^{\max} \qquad j = 1,\cdots, g_{ed}$$
$$\sum_{j=1}^{g_{pvi}} Q_{ji}^{\min} \le \sum_{j=1}^{g_{pvi}} Q_{ji} \le \sum_{j=1}^{g_{pvi}} Q_{ji}^{\max} \qquad i = 1, \cdots, b_{mpv}$$
$$j = 1,\cdots, g_{pvi}$$
$$Q_{gj}^{\min} \le Q_{gj} \le Q_{gj}^{\max} \qquad j = 1,\cdots, g_{spv}$$
$$\Delta u_j^{\min} \le \Delta u_j \le \Delta u_j^{\max} \qquad j = 1,\cdots, r$$
$$0 \le v \le 1 \qquad j = 1,\cdots, n \,, \tag{12}$$

where

g_{spv}: number of buses with single PV generator.

The solution method for the above problem is iterative and includes two steps in each iteration: (a) the linearization of the objective function and operating constraints and (b) the solution of the resulting linear programming (LP) problem. The efficient co-state method [12] is employed to perform the linearization in the first step. The simplex method or interior point method can then be used in the second step to solve the LP problem.

The mismatch variable v is adjusted step by step from 1 to 0 during the solution procedure. Given $v_{\min}$ in each step, the solution of the above optimization problem provides new control settings. These controls are implemented in the system and the power flow solution is updated. Note that the unit outputs will be updated by (10). Any violated operating constraints are added to the set of active constraints. A nonzero value of mismatch variable indicates that the algorithm has not converged yet. The participation factors are updated with the aid of (7) and the procedure is repeated until no mismatch and no constraint violation exist. The flow chart of the NDOQPF algorithm is illustrated in Figure 4.

The proposed algorithm achieves power flow non-divergence by introducing fictitious sources/mismatches that are driven to zero as the solution progresses. Therefore, the NDOQPF algorithm guarantees convergence if a solution exists; if a solution does not exist, the algorithm provides a sub-optimal solution that may include load shedding. Also, the solution moves along the economic dispatch by virtue of (8) in each iteration. The algorithm mimics the real time operation in which any generation-load imbalance is allocated to the individual units according to their economic participation factors. Another improvement is that violated operating constraints appear gradually since the system is gradually loaded. The appearance of an overwhelming number of violated constraints, as can occur to the traditional power flow solution, is improbable with the proposed method.

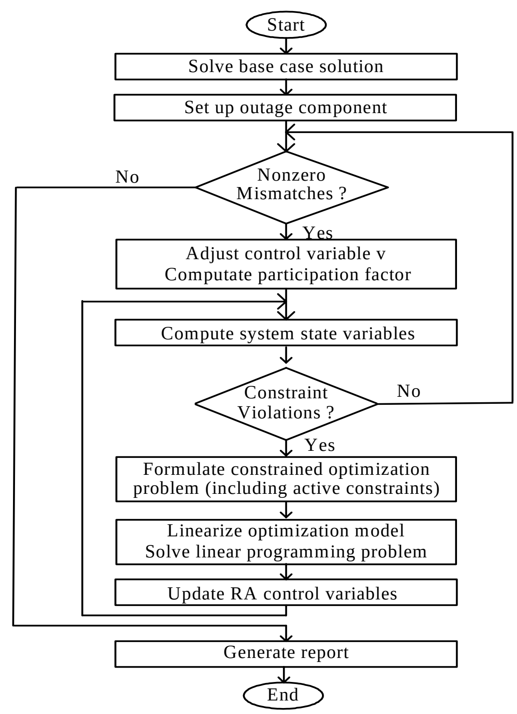

Figure 4. Flow chart of non-divergent optimal power flow.

D. Application of NDOQPF in Deregulated Power Systems

The challenge of power industry restructuring is the creation of competitive markets while at the same time maintaining system reliability [16]. Many of current models for competitive markets employ the ISO/RTO that operate transmission systems, ensure fair access and system reliability, and provide open markets for power transactions. In a power market, energy is the primary commodity, and market participants (power suppliers or consumers) submit their energy bids (an offer to supply or consume energy at a price) to the ISO/RTO. Upon receiving these bids, the ISO/RTO determines the least expensive dispatch of generation while satisfying customers' demand and system reliability requirements. If transmission congestion occurs, energy bids and other remedial actions are appropriately selected to relieve congestion. This process is referred as congestion management and it is no different than the usual remedial action procedures used in a regulated environment. The only difference is that power bids are also used as remedial actions.

Mathematically, such ISO/RTO operational procedure is formulated as a constrained optimization problem. The objective is to minimize the overall system cost (defined with energy bids) while satisfying system constraints. The objective function is usually piecewise linear or staircase, and system constraints are power flow equations, transmission capability limits, bus voltage limits, and so on. This problem is formulated as a NDOQPF as following:

Minimize $J = \sum_{i=1}^{g_p} C_i(P_{gi})$

Subject to $G(\mathbf{x},\mathbf{u}) = 0$

$$\begin{aligned}
&V_i^{\min} \le |\tilde{V}_i| \le V_i^{\max} && i = 1,\cdots,b\\
&|\tilde{T}_i(\mathbf{x},\mathbf{u})| \le \bar{T}_i && i = 1,\cdots,l\\
&P_{gi}^{\min} \le P_{gi} \le P_{gi}^{\max} && i = 1,\cdots,g_p\\
&Q_{gi}^{\min} \le Q_{gi} \le Q_{gi}^{\max} && i = 1,\cdots,g_{pv}\\
&\Delta u_i^{\min} \le \Delta u_i \le \Delta u_i^{\max} && i = 1,\cdots,r,
\end{aligned} \qquad (13)$$

where

C_i bidding function of the generator i

n number of generators participating the market

$\tilde{T}_i$ load of transmission line i

$\bar{T}_i$ load limit of transmission line i

Note that congestion management is incorporated in the optimization problem by introducing the remedial action control vector $\mathbf{u}$. The proposed NDOQPF algorithm is capable of efficiently solving the generic ISO/RTO operational model. As a matter of fact, this algorithm is especially effective in the deregulated environment due to its ability in handling the power flow non-convergence problem. In the deregulated environment, the transmission system is more likely to be heavily stressed and the system may be operated in different and unusual power flow patterns from what it was originally designed for [17]. These new scenarios unavoidably cause post contingency power flow analysis running into divergent situations more frequently than that in the regulated system. The ability to provide always a solution (optimal or suboptimal) to the ISO/RTO operator under all conditions is the major advantage of the proposed NDOQPF algorithm.

E. Reliability Index Computation

Reliability indices are computed on the basis of identifying the set of states that satisfy a specific failure criterion and the transition rates from any state inside the set to a state outside the set. For example, for a state space diagram that includes both evaluated and non-evaluated states (contingencies) as shown in Figure 5, a state j at certain load level is characterized with a certain probability p_j and transition rates to and from other system states, such as λ_{jk} and λ_{ij}. Consider an event S_r, which contains a set of evaluated states that satisfy a specific failure criterion. This set is identified by retrieving effects analysis results. For this event, three different classes of reliability indices can be computed:

(1) Probability index $P_r[S_r] = \sum_{j \in S_r} p_j$,

(2) Frequency index $f_{S_r} = \sum_{j \in S_r} p_j \sum_{i \notin S_r} \lambda_{ji}$,

(3) Duration index $T_{S_r} = \dfrac{P_r[S_r]}{f_{S_r}}$,

where

p_j : the occurrence probability of the failure state j

λ_{ji} : the transition rate from state j to state i

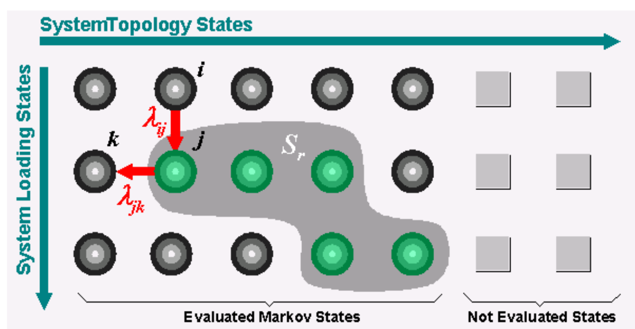

Figure 5. Example state-space diagram.

III. Case Study

The proposed SCAE methodology and its performance are demonstrated with the IEEE 24-bus reliability test system (RTS) [18] as shown in Figure 6. In this system, the peak load level is applied to generate a highly stressed system condition, and available system RAs include active generation re-dispatching, reactive power rescheduling, reactor/capacitor bank switching, and load shedding. In this section, representative case study results are described to illustrate the features of the proposed methodology. As an example, Table 1 lists 10 highest ranked first level independent contingencies that result in operating constraint violations and therefore require RAs. These evaluation results are obtained from the NDOQPF algorithm, which incorporates operational practices, such as real power economic and reactive power proportional dispatch, and applies available RAs when necessary to alleviate any post contingency constraint violations during the solution procedure. Contingencies in Table 1 are listed in the order of their rank determined by the contingency ranking algorithm. The first six contingencies also require load shedding to keep the system operating normally.

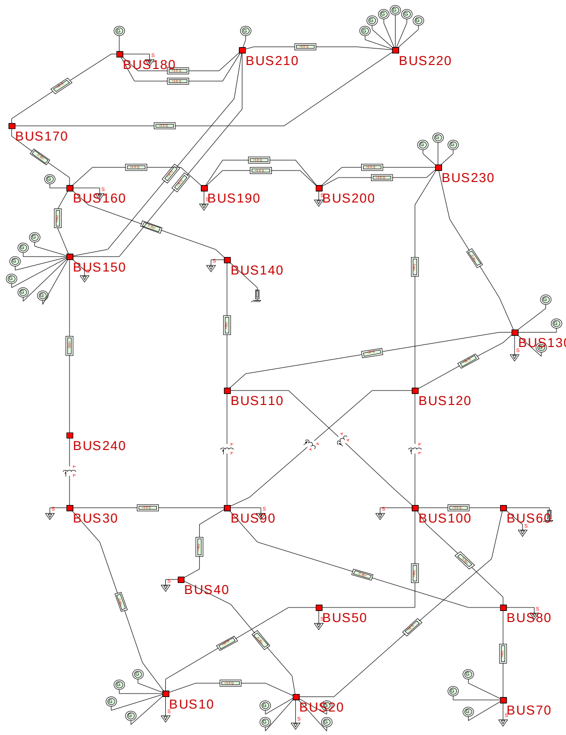

Figure 6. The IEEE 24-bus reliability test system.

Table 1: SCAE of First Level Independent Contingencies for IEEE 24-Bus RTS (C: Circuit G: Generator)

Rank No.	Outage Component	Constraint Violations (Yes/No)	RAs w/o load shedding (Yes/No)	Load Shedding (Yes/No)
1	C60-100	Yes	Yes	Yes
2	C150-240	Yes	Yes	Yes
3	C30-240	Yes	Yes	Yes
4	C160-170	Yes	Yes	Yes
5	C20-60	Yes	Yes	Yes
6	C130-230	Yes	Yes	Yes
7	C120-230	Yes	Yes	No
8	C160-190	Yes	Yes	No
9	C150-210	Yes	Yes	No
10	C150-160	Yes	Yes	No

Some of the second level contingencies under the peak load level may cause power flow divergent situations by applying traditional power flow in effects analysis. This problem can be solved by using the NDOQPF algorithm in effects analysis. For example, consider the second level contingency that involves the outages of circuits C30-90 (first level) and C150-240 (second level), the traditional power flow for this second level contingency diverges while the NDOQPF algorithm converges and provides required RAs. For illustration purposes, the formulation and solution for this second level contingency are described in greater detail. The first level outage of C30-90 does not cause any constraint violations, while the control variable v is reduced from 1.0 directly to 0.0. The solution of the first level outage, represented with $\mathbf{x}^0$ and $\mathbf{u}^0$, is used as the base case for the second level outage of C150-240. Under the operating point ($\mathbf{x}^0, \mathbf{u}^0$) after the first level outage and the additional outage of the circuit C150-240, the mismatch vector $\mathbf{m}^0$ is calculated first. Next the problem (12) is formulated. The artificial control variable v is then gradually reduced as follows:

$$v: 1.0 \rightarrow 0.667 \rightarrow 0.333 \rightarrow 0.0$$

Note that when v is 1.0, the mismatch vector is $\mathbf{m}^0$ and the operating point is $\mathbf{x}^0, \mathbf{u}^0$, the system does not have any constraint violations. In the progress of the solution, the optimization problem in equation (12) is formulated at each step of the control variable v and solved using the linear programming technique. The procedure is repeated until the variable v reaches zero. During the solution procedure, the active constraints are obtained and summarized below:

Active constraints:

$$V_{30} \geq V_{30}^{\min}, V_{240} \geq V_{240}^{\min}, \; Q_{G10-4} \leq Q_{G10-4}^{\max}, \; Q_{G220-4} \leq Q_{G220-4}^{\max},$$
$$P_{G180-1} \leq P_{G180-1}^{\max}, P_{G210-1} \leq P_{G210-1}^{\max} \; P_{G230-1} \leq P_{G230-1}^{\max}.$$

For this contingency the available remedial actions (excluding load shedding) cannot eliminate all violated constraints, and load shedding as a remedial action is allowed. The final solution includes the following remedial actions, which include load shedding at bus 30:

$$\Delta Q_{G220} = 6.63\,MVAR$$
$$Load_{Bus30} = -113.82\,MW - j23.4\,MVAR$$

In addition, the total amounts of changes in system real power and reactive power outputs due to the real power economic dispatch and the reactive power proportional adjustment among PV generator units connected to the same buses are as following:

$$\Delta P = -106.91\,MW \;, \Delta Q = -30.0\,MVAR$$

The solution of this second level contingency illustrates the major advantages of the NDOQPF, i.e., the ability to always provide a solution.

For the IEEE 24-bus RTS, when only first level contingencies are considered under the peak load level, reliability indices in terms of system loss-of-load probability, frequency, and duration are computed as follows: Probability: 0.0012; Frequency: 0.4794 per year; Duration: 21.93 hr/per year.

IV. Conclusions

A comprehensive security-constrained adequacy evaluation (SCAE) methodology based on analytical techniques is proposed for bulk power system reliability assessment. In this methodology, a system simulation method based on the quadratized power flow model is adopted to (a) improve the accuracy of contingency ranking and (b) provide a robust effects analysis approach. Specifically, a non-divergent optimal quadratized power flow (NDOQPF) algorithm is developed to incorporate major operational practices, security constraints, and remedial actions in one unified framework. The NDOQPF algorithm is able to simulate post contingencies in a realistic and efficient manner and provide always a solution (optimal or suboptimal) under all system conditions. Also, the NDOQPF algorithm is applicable to both regulated and deregulated power system environment. The SCAE methodology considers contingency selection/enumeration and reliability index computations as well. The overall SCAE framework is demonstrated with the IEEE reliability test system.

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