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Reliability and Cost trade-off in Multi-Area Power System Generation Expansion Using Dynamic Programming and Global Decomposition

Panida Jirutitijaroen, *Student Member, IEEE*, and Chanan Singh, *Fellow, IEEE*

***Abstract*--This paper proposes an optimization procedure for generation expansion and placement considering generation reliability in multi-area power systems. The objective is to obtain a suitable trade off between system reliability and cost. In this process, reliability evaluation is performed using the concept of global decomposition. The prospective generation locations are pre-selected but the magnitude of generation located in each area is determined by an optimization procedure. The structure of global decomposition is exploited to simplify the problem and solve it using dynamic programming. The method is illustrated by application to a 3-area power system and also implemented for an actual 12-area power system.**

***Index Terms*--Multi-area Power System, Reliability, Power System Optimization, Global Decomposition, Generation Adequacy, Dynamic Programming.**

I. NOMENCLATURE

A. *Indices*

s	Source node
t	Sink node
i,j	Network nodes
I	*{1,2,…,n}* Set of network nodes

B. *Parameters*

$\overline{G}_i$	Capacity of existing generation arc *i* (MW)
a_i^G	Cost of an additional unit at node *i* ($/MW)
C^G	Capacity of an additional generator (MW)
N^G	Total number of additional generators
$\overline{L}_i$	Capacity of load arc *i* (MW)
$\overline{T}_{ij}$	Capacity of existing tie line arc *ij* (MW)
a_{ij}^T	Cost of a tie line between nodes *i* and *j* ($/MW)
C^T	Capacity of an additional tie line (MW)
N^T	Total number of additional tie lines
R	Total available budget ($)
n	Number of areas in the network

C. *Decision variables*

X_{ij}	Flow from node *i* to *j*
X_i^G	Number of additional generators at node *i*, integer
X_{ij}^T	Number of additional tie lines between nodes *i* and *j* where *(i,j) = (j,i)*, integer

II. INTRODUCTION

In the deregulated environment, ISOs need to guide the development of additional generation capacity that is optimal with respect to cost and reliability. In the absence of such a guidance, generation growth may lead to congestion and reliability problems. At present, however, the generation requirement is calculated by simulation and ad hoc methods. A recent study of long term generation adequacy in a multi-area power system is thoroughly analyzed and reported by Rau and Zeng [2]. An optimization procedure along with MARS (Multi Area Reliability Simulation – a Monte Carlo Simulation Program) is proposed to determine an excess or deficient amount of generation in each area. One of the contributions of [2] is to show the relationship of area risk level to load changes [14]. However, the method requires iterations between optimization and risk calculations which are obtained from many runs of MARS.

This paper proposes dynamic programming to determine the location of generators in multi-area power systems with global decomposition as a reliability evaluation tool. Original generation probability distribution in each area is modified to incorporate additional generators. An equation relating the number of additional units in each area to generation probability distribution is developed in this paper. After global decomposition, an equation for reliability is derived and approximated. The problem structure is transformed and solved by Dynamic programming.

There are two ways to formulate the problem. One is to minimize the cost subject to loss of load probability constraint and the other is minimizing the loss of load probability subject to cost constraint. Either objective can be achieved by the method described in this paper. The basic formulation is based on minimizing the loss of load probability with budget as a

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P. Jirutitijaroen and C. Singh are with the Department of Electrical and Computer Engineering, Texas A&M University, College Station, TX 77843 USA (e-mail: pjirut@ece.tamu.edu; singh@ece.tamu.edu).

constraint but the same formulation can be used for the alternate objective. If the minimized loss of load probability is not acceptable, the budget can be changed and reliability re-optimized. Through such an iterative procedure, a suitable solution that minimizes the cost subject to a reliability constraint can be found. Since decomposition needs to be performed only once in the proposed method, such an iterative procedure is quite efficient. The proposed method is illustrated by application to a 3-area system and also applied to a 12-area system that represents an actual system.

III. BACKGROUND

Power system reliability analysis is done at various levels depending upon the objectives of the study. In single area analysis, all generators and loads are assumed to be connected to a single bus as the objective is to find whether the total installed generation is adequate to satisfy the load. In composite system reliability, all the major nodes are preserved as the transmission capability is regarded quite important. In the middle of this is the multi-area reliability analysis where the generators in a given area are assumed to be connected to a single bus but the capacities of tie lines between areas are considered. It should be noted, however, that limitations of intra-area transmission are included indirectly in determining the capacities of inter-area tie lines. It should be noted that this paper is focused on multi-area formulation of reliability. Once the decision to locate the additional generation in a particular area has been made, more detailed analysis within that area can be pursued using methods investigated in [1], [4], [5], [6]. It should be noted that in the multi-area formulation, network flow method is generally accepted to be adequate [2], [3]. However, if needed, DC load flow method can also be used [7].

Multi-area reliability analysis has two major approaches; Monte-Carlo simulation and state-space decomposition. In Monte-Carlo simulation, failure and repair history of components are created using their probability distributions. Reliability indices are estimated by statistical inferences. The basic concept of state-space decomposition, originally proposed in [13], is to classify the system state space into three sets; acceptable sets (*A* sets), unacceptable sets (*L* sets), and unclassified sets (*U* sets) while the reliability indices are calculated concurrently. Advanced versions of decomposition such as simultaneous-decomposition for including load and planned outages in a computationally efficient manner are described in [8], [10], [11], [12].

Global decomposition is an efficient reliability evaluation technique for this type of analysis. With global decomposition, the additional generators in prospective areas are included in the system. This global state space is valid for all generation combinations. The concept is based on the fact that decomposition depends on state capacities and not state probabilities. The unavailability (forced outage rate) of additional generators is also considered in the formulation. The major advantage of this technique is that decomposition is performed only once. Reliability indices of each combination can be evaluated by allowing zero probability to the omitted states. One of the contributions of this paper is to derive the relationship of reliability index to the additional generation in each area.

IV. PROBLEM FORMULATION

Multi-area power systems can be formulated as a network flow problem where each node in the network represents an area in the system and each arc represents tie line connection between areas. Source and sink nodes are introduced to represent generation capacity and load as shown in Fig. 1. The capacity of every arc in the network is a random variable because generation, tie line and load capacity are random with discrete probability distributions. For computational efficiency, all arc capacities are rounded off to a fixed increment so that only the minimum capacity state and number of states in each arc need to be known. States with very small probability are ignored.

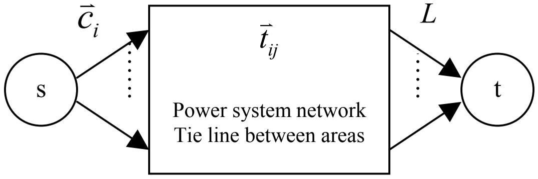

Fig. 1. Power System Network Capacity Flow Model

The decision variables of network flow problem are integer as the number of additional generators is an integer value. This network flow problem is called stochastic integer programming problem due to randomness in capacity arcs, reliability constraints, and integer decision variable. The standard formulation is derived in the following with the objective of minimizing loss of load probability subject to cost and network capacity constraints. The additional generators have capacity of C^G MW each and additional tie lines have capacity of C^T MW each. The objective function to minimize loss of load probability is given below.

$$\text{Min } \Pr\{X_{1t}, X_{2t}, \ldots, X_{nt} : X_{1t} < \overline{L}_1 \cup X_{2t} < \overline{L}_2 \ldots \cup X_{nt} < \overline{L}_n\} \quad (1)$$

The problem has the following constraints;

Capacity constraints

- Flow in generation arc

$$X_{si} \le \overline{G}_i + C^G X_i^G \qquad \forall i \in I \quad (2)$$

- Flow in tie line

$$\left| X_{ji} - X_{ij} \right| \le \overline{T}_{ij} + C^T X_{ij}^T \qquad \forall i, j \in I, i \ne j \quad (3)$$

- Flow in load arc

$$X_{it} \le \overline{L}_i \qquad \forall i \in I \quad (4)$$

Conservation of flow at node *i* in the network

$$X_{si} + \sum_{\substack{j \in I \\ j \ne i}} X_{ji} = \sum_{\substack{j \in I \\ j \ne i}} X_{ij} + X_{it} \qquad \forall i \in I \quad (5)$$

Maximum number of additional generators

$$\sum_{i \in I} X_i^G = N^G \quad (6)$$

Maximum number of additional transmission line

$$\sum_{\substack{i \in I \\ i \ne j}} \sum_{j \in J} X_{ij}^T = N^T \quad (7)$$

Budget constraint

$$\sum_{i\in I} a_i^G X_i^G + \sum_{\substack{i\in I\\ i\neq j}}\sum_{j\in J} a_{ij}^T X_{ij}^T \le R \tag{8}$$

Non negativity

$$X_{ij}, X_i^G, X_{ij}^T \ge 0 \qquad \forall i,j \in I \tag{9}$$

The expression within the parenthesis in equation 1 represents the system loss of load event. The problem is thus formulated so as to minimize the loss of load probability index subject to cost constraint. If the optimal system reliability obtained through this process does not satisfy the requirements, the cost constraint can be relaxed to allow more additional generators in the system and the LOLP can be re-optimized

All possible additional generation units are included in each prospective area of the system before performing global decomposition. In the global decomposition process, constraints (2) to (7) and (9) have already been included. The problem has only one additional constraint which is the budget constraint (8). However, the objective function has no available explicit formulation in terms of the decision variables. In order to express the objective function in terms of number of additional generators in each area, generation probability distribution is modified to incorporate additional units as described in the following.

A. Generation Probability Distribution Equation Incorporating the Additional Units

The generation probability distribution of each area is modified as additional units are added to the area. The expression for the modified probability distribution is developed as a function of the number of additional units. For the sake of simplicity, this expression (11) is presented assuming that the capacity of additional units in a given area is the same. This, however, is not an inherent limitation of the method but if the units are non-identical then (11) becomes more complex. The equation in the case of non-identical units is given in Appendix A.

The capacity, C_j, of an additional unit j is assumed as multiple, μ_j, of the fixed increment, η, used in the discrete probability distribution, i.e.

$$\mu_j = \frac{C_j}{\eta} \tag{10}$$

The following equation describes generation probability incorporating the additional units, y_j in area j.

$$P_{G_i^j}^{y_j} = \sum_{k=0}^{y_j} P^0_{G^j_{i-\mu_j k}} \binom{y_j}{k} \left(\mathrm{FOR}_j\right)^{y_j-k} \left(1-\mathrm{FOR}_j\right)^k \tag{11}$$

where

$P^0_{G^j_{i-\mu_j k}}$	Probability of original generation at level i- $\mu_j k$ before additional units in area j, 0 if $i \le \mu_j k$
FOR_j	Forced outage rate of additional units in area j

Equation (11) describes the probability of a given generation level in each area in terms of number of additional units in this area.

B. Concept of Global Decomposition

The system state space consists of generation states in each area and inter-area tie line states. It is defined as (12).

$$\Omega = \begin{bmatrix} M_1 & M_2 & \dots & M_N \\ m_1 & m_2 & \dots & m_N \end{bmatrix} \tag{12}$$

where

M_k	Maximum state of arc k
m_k	Minimum state of arc k
N	Number of arcs in the network

A system state, x, can assume any value between its minimum and maximum state as shown in (13).

$$x = \begin{bmatrix} x_1 & x_2 & \dots & x_N \end{bmatrix} \tag{13}$$

where $m_k \le x_k \le M_k$

x_k	State of arc k

The maximum possible number of additional generators in each area is included in the state space before performing global decomposition. The additional generation capacity in each area is rounded off to the closest integral multiple of the fixed increment, η, used when constructing its distribution.

Let the budget constraint be of the following form,

$$a_1 y_1 + a_2 y_2 + \dots + a_N y_N \le R \tag{14}$$

where

a_j	Cost per additional generator to area j
R	Total budget available

The maximum number of additional generation levels in area j is calculated from (15)

$$\gamma_j^{\max} = \mu_j \left\lfloor \frac{R}{a_j} \right\rfloor \tag{15}$$

where

$\gamma_j^{\max}$	Maximum number of additional generation levels in area j
μ_j	Multiple of additional capacity to a fixed increment, as defined in (10)

Global decomposition approach analytically partitions the state space into the following three different sets of states.

1. Sets of acceptable states (*A* sets): The success states that all area loads are satisfied.
2. Sets of unacceptable states (*L* sets): The failure states or Loss of load states that some area loads are not satisfied.
3. Set of unclassified states (*U* sets): The states that have not been classified into *A* or *L* sets.

The process of partitioning the state space into *A* and *L* sets involves determining maximum flow in the network. Ford-Fulkerson algorithm is implemented with breadth-first search for finding existing flow in the system. At the beginning of the decomposition, state space (Ω, as in (12)) is the first *U* set, unclassified set. At every step of decomposition, one *A* set, N *L* sets and N *U* sets are generated from one *U* set. The *A* sets will be deleted to minimize memory usage since the goal of this evaluation is to extract all *L* sets for system loss of load probability calculation. The *U* sets have to be kept and partitioned further to *A*, *U* and *L* sets.

The concept of global decomposition is based on the fact that decomposition depends only on state capacities and not the state probabilities. This allows us to include the maximum possible number of generators in each area in the decomposition. The sets obtained from this state space are valid for all scenarios of distribution of additional generators.

Probability of each scenario can then be evaluated by allowing zero probability for the excluded states because of the omission of corresponding additional generators included in the original decomposition.

C. Reliability Equation

After global decomposition is performed, the state space is completely partitioned into a number of L sets. Probability of a set is calculated from (16).

$$\Pr(\omega) = \prod_{k=1}^{N} \sum_{m_k^{\omega} \le x_k \le M_k^{\omega}} p_{x_k} \tag{16}$$

where

ω	A given set
x_k	State of arc k
p_{x_k}	Probability of state x_k of arc k
M_k^{ω}	Maximum state of arc k in set ω
m_k^{ω}	Minimum state of arc k in set ω

Now area generation probability can be written as a function of additional units in the area as given in (11). Therefore, equation (16) can be written in terms of number of additional units in each area as in (17) and (18). The equation in the case of non-identical units is given in Appendix B.

$$\Pr(\omega) = \prod_{j=1}^{N} h_j(y_j) \tag{17}$$

and

$$\begin{aligned} h_j(y_j) &= \sum_{k=0}^{y_j} \left[\left(\sum_{i=m_j^{\omega}}^{M_j^{\omega}} P^0_{G^j_{i-\mu_j k}} \right) \binom{y_j}{k} (\mathrm{FOR}_j)^{y_j-k} (1-\mathrm{FOR}_j)^k \right] \\ &= \sum_{k=0}^{y_j} \left[\left(\overline{P}^0_{G^j_{M_j^{\omega}-\mu_j k}} - \overline{P}^0_{G^j_{m_j^{\omega}-\mu_j k-1}} \right) \binom{y_j}{k} (\mathrm{FOR}_j)^{y_j-k} (1-\mathrm{FOR}_j)^k \right] \end{aligned} \tag{18}$$

where

y_j	Number of additional units in area j
$\overline{P}^0_{G^j_{i-\mu_j k}}$	Cumulative probability of original generation at level i- $\mu_j k$ before additional units in area j

Probability of an A set and U set can also be expressed in terms of number of additional units in all areas as (17) and (18). Loss of load probability is calculated from summation of probability of loss of load sets. The objective function for the optimization problem can be expressed as (19).

$$LOLP = \sum_{i=1}^{n_d} \sum_{k=1}^{d_i} \Pr(L_k^i) \tag{19}$$

where

n_d	Total number of decompositions
d_i	Number of L sets generated at i^{th} decomposition

D. Approximation after Global Decomposition

The problem is transformed into a single cost constraint (8) with an objective function to minimize loss of load probability (19). In order to apply dynamic programming, separable functions in both objective function and constraints are favorable structures. However, equation (19) is a very complex function with no specific pattern and has a nonlinear relationship between the decision variables which are the number of additional generators in each area. Because of the restriction of the function structure, it is more efficient to do the optimization with L sets only from the first stage of global decomposition process.

This introduces an approximation to the problem. The budget function is assumed to be a linear summation of the decision variables. The analysis will later reveal that the probability equation structures of the first L sets are in separable form that can be simply solved by dynamic programming.

Most of the time, the first L sets generate higher probability than all other L sets generated in the decomposition process. The L sets of the first stage global decomposition are then chosen based on this advantage for optimization procedure in this paper. The structure of the first L sets is shown in (20).

$$L_i = \begin{bmatrix} M_1 & M_2 & \cdots & v_i - 1 & \cdots & M_N \\ v_1 & v_2 & \cdots & 1 & 1 & 1 \end{bmatrix} \tag{20}$$

where

v_i	Minimum capacity of arc i that the system remains in success state given all other arcs remain at their maximum states

Loss of load probability equation from the first L sets can be written as (21) and (22). The equation in the case of non-identical units is given in Appendix B.

$$\Pr(L) = \sum_{i=1}^{N} \Pr(L_i) = \sum_{i=1}^{N} \prod_{j=1}^{N} g_j^i(y_j) \tag{21}$$

and

$$g_j^i(y_j) = \sum_{k=0}^{y_j} \left[\left(\overline{P}^0_{G^j_{v_i-\mu_j k-1}} \right) \binom{y_j}{k} (\mathrm{FOR}_j)^{y_j-k} (1-\mathrm{FOR}_j)^k \right] \tag{22}$$

where

$\overline{P}^0_{G^j_{v_i-\mu_j k-1}}$	Cumulative probability of capacity arc v_i-$\mu_j k$-1 generation area j before unit addition

It can be observed that for set L_i the state of arc i+1 takes all value from its minimum, 1, to its maximum, M_N states. This means that the probability of these arcs from i+1 to N is one. Therefore, equation (21) can be rewritten as (23). The development is derived in Appendix C. Equation (23) also has a separable structure; this will be clear when dynamic programming is applied to solve this problem.

$$\begin{aligned} \Pr(L) &= \sum_{i=1}^{N} \Pr(L_i) \\ &= g_1^1(y_1) + \left(1 - g_1^1(y_1)\right) g_2^2(y_2) + \ldots + \\ &\left(1 - g_1^1(y_1)\right)\left(1 - g_2^2(y_2)\right) \cdots \left(1 - g_{N-1}^{N-1}(y_{N-1})\right) g_N^N(y_N) \end{aligned} \tag{23}$$

It should be observed that if any arc has 'v' value equal to one, the effect of unit addition in that area to system reliability cannot be calculated from (23). In the process of obtaining L sets, 'v' values can be expressed as the smallest capacity of an area generation before the system enters loss of load state. In other words, the capacity below the 'v' values will result in system failure (or loss of load in any area). The 'v' values obtained from the decomposition are very important variables since they are the preliminary indications of area generation deficiency. It can also be implied from the 'v' value that the components with 'v' value equal to one do not contribute as much to the system reliability. In other words, without the capacity in that area ('v' value equal to one), the system can still remain in success state. Therefore, in this application, the

'v' value will indicate the prospective area for the optimization process.

If a solution from optimization does not satisfy system reliability criteria, a budget constraint can then be relaxed to incorporate more generation units. Global decomposition is not required to be performed once again if the maximum number of additional units in each area remains the same. The problem can then be re-optimized until the reliability criterion is satisfied. The next section will discuss the application of dynamic programming in solving the problem.

E. Dynamic Programming Application to the Problem

Dynamic programming is an optimization procedure that can be applied to a problem with discrete decision variables which are the number of additional units in each area in this application. The problem has the following formulation.

$$\text{Min } \Pr(L)$$

$$\text{s.t. } a_1y_1 + a_2y_2 + \ldots + a_Ny_N \le R$$

The first step in solving a problem with dynamic programming is to define stages and states of the problem. In this application, stages represent area of interest and states are the available budget at each stage as shown in Fig. 2. At each stage, a decision is made on how much should be spent on each area generation.

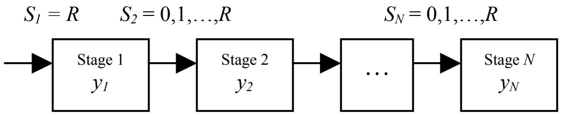

Fig. 2. State Diagram of the Problem

where

S_j Available budget at stage j

Dynamic programming initially solves the smallest sub problem, which contains the smallest number of variables. The optimal solution to the next sub problem (next stage) is calculated using the solution from the previously computed smaller sub problem (previous stage). A recursive function can be derived to describe this relationship. In this problem, the last stage is the smallest sub problem; therefore, the solution obtained from the last stage is the starting point of a recursive function. The following analysis covers the recursive function derivation for the first stage L set optimization.

Let $f_j(S_j)$ be the optimal objective function at stage j with available budget S_j. The recursive function for this application is written as (24).

$$f_j(S_j)=\begin{cases} g_j^j\left(\left\lfloor \dfrac{S_j}{a_j}\right\rfloor\right) & ,j=N \\ \min\limits_{0\le y_j\le \left\lfloor \frac{S_j}{a_j}\right\rfloor} \left\{g_j^j(y_j)+\left(1-g_j^j(y_j)\right)f_{j+1}(S_j-a_jy_j)\right\} & ,j=1,\ldots,N-1\end{cases} \tag{24}$$

where $g_j^j(y_j)$ is as defined in (22).

With this recursive function, solutions from each stage can be evaluated. The recursive function describes a relationship between (already computed) solutions from a previous stage to a current stage. The optimal solution can then be traced back when the solution to the first stage is found.

V. ILLUSTRATION OF THE METHOD

A three area test system, its parameters, and the probability table for each area before additional units, are shown in Fig. 3, TABLE I and TABLE II

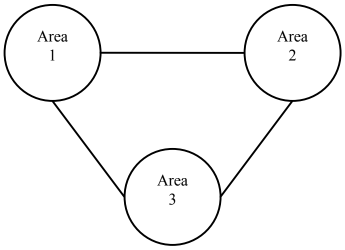

Fig. 3. Three Area Test System

TABLE I
THREE AREA GENERATION PARAMETERS

State of Cap. arc	Area 1		Area 2		Area 3	
	Cap (MW)	Cum. Prob.	Cap (MW)	Cum. Prob.	Cap (MW)	Cum. Prob.
7			600	1.000000		
6	500	1.000000	500	0.737856	500	1.000000
5	400	0.672320	400	0.344640	400	0.672320
4	300	0.262720	300	0.098880	300	0.262720
3	200	0.057920	200	0.016960	200	0.057920
2	100	0.006720	100	0.001600	100	0.006720
1	0	0.000320	0	0.000064	0	0.000320

TABLE II
THREE AREA TIE-LINE PARAMETERS

State of Cap. arc	Tie-line					
	1-1		1-2		1-3	
	Cap (MW)	Cum. Prob.	Cap (MW)	Cum. Prob.	Cap (MW)	Cum. Prob.
2	100	1	100	1	100	1
1	0	0.1	0	0.1	0	0.1

The additional unit parameters of three area system are shown in TABLE III. It is assumed that the budget is $200 million and the additional generators have capacity of 100 MW each. The problem is to locate the best generation combination in the three areas.

TABLE III
THREE AREA ADDITIONAL UNIT PARAMETERS

Area j	a_j ($m)	Load (MW)	FOR of additional units
1	60	300	0.15
2	100	400	0.05
3	80	300	0.10

From the budget constraint, the maximum possible number of additional units is 3 units in area 1, 2 units in area 2, and 2 units in area 3. These units are included in the system before performing global decomposition. Probability distribution tables for generations and tie-lines are developed with a capacity increment of 100 MW. In the following, the solution steps from the first *L* set optimization are presented.

After the first decomposition, loss of load sets are shown below.

$$L_1 = \begin{bmatrix} 1 & 9 & 8 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 9 & 2 & 8 & 2 & 2 & 2 \\ 2 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} 9 & 9 & 1 & 2 & 2 & 2 \\ 2 & 3 & 1 & 1 & 1 & 1 \end{bmatrix}$$

The problem can be formulated as (25)

$$\begin{aligned} &\min \Pr(L) = g_1^1(y_1) + \left(1 - g_1^1(y_1)\right)\left\{g_2^2(y_2) + \left(1 - g_2^2(y_2)\right)g_3^3(y_3)\right\} \\ &s.t. \quad 60y_1 + 100y_2 + 80y_3 \le 200 \\ &\qquad\qquad y_j \in I^+ \end{aligned} \tag{25}$$

where

$$g_1^1(y_1) = \sum_{k=0}^{y_1}\left[\overline{P}^0_{G^1_{1-k}} \binom{y_1}{k} (0.15)^{y_1-k}(0.85)^k\right] \tag{26}$$

$$g_2^2(y_2) = \sum_{k=0}^{y_2}\left[\overline{P}^0_{G^2_{2-k}} \binom{y_2}{k} (0.05)^{y_2-k}(0.95)^k\right] \tag{27}$$

$$g_3^3(y_3) = \sum_{k=0}^{y_3}\left[\overline{P}^0_{G^3_{1-k}} \binom{y_3}{k} (0.1)^{y_3-k}(0.9)^k\right] \tag{28}$$

TABLE IV relating available budget S_j (from 0 to 10) with the probability of each y_j, $g_j^j(y_j)$ is developed.

TABLE IV
AVAILABLE BUDGET AND MODIFIED PROBABILITY WITH ADDITIONAL UNITS

Budget ($m)	Probability and number of additional unit in each area					
	y_1	$g_1^1(y_1)$	y_2	$g_2^2(y_2)$	y_3	$g_3^3(y_3)$
0-59	0	0.003200	0	0.001600	0	0.003200
60-79	1	0.000480	0	0.001600	0	0.003200
80-99	1	0.000480	0	0.001600	1	0.000320
100-119	1	0.000480	1	0.000141	1	0.000320
120-159	2	0.000072	1	0.000141	1	0.000320
160-179	2	0.000072	1	0.000141	2	0.000032
180-199	3	0.000072	1	0.000141	2	0.000032
200	3	0.000011	2	0.000010	2	0.000032

The procedure yields a solution to locate 1 unit in area 2 and 1 unit in area 3. The total cost is $180 million. This combination yields the minimum probability of the first *L* set which is 0.0005. TABLE V shows all possible combinations of units, their cost, and corresponding system LOLP. It can be seen from this table that the optimal solution is the same as that obtained from the dynamic programming approach.

The proposed method, however; does not guarantee the optimal solution as the optimization is based on partial information. For example, consider the case of area loads to be 400, 500, and 400 in area 1, 2, and 3 respectively. In this case, the dynamic programming approach gives the solution of locating 1 unit in area 2, and 1 unit in area 3. TABLE VI gives all possible combinations with their cost and reliability. It is seen that 2 units in area 1 and 1 unit in area 3 give lower LOLP than found by dynamic programming based on the first *L* sets.

TABLE V
COMPARISON OF SOLUTION FROM ENUMERATION

Unit combination			Cost ($m)	LOLP of the first *L* set obtained from global decomposition	LOLP
γ_1	γ_2	γ_3			
0	**1**	**1**	**180**	**0.000493**	**0.005660**
1	1	0	160	0.000509	0.006059
2	0	1	200	0.001639	0.007002
0	2	0	200	0.000650	0.007079
3	0	0	180	0.001921	0.010472
0	0	2	160	0.001923	0.011531

TABLE VI
COMPARISON OF SOLUTION FROM ENUMERATION WITH LOAD 400, 500, AND 400 IN AREA 1, 2, AND 3

Unit combination			Cost ($m)	LOLP of the first *L* set obtained from global decomposition	LOLP
y_1	y_2	y_3			
2	0	1	200	0.018132	0.083689
0	**1**	**1**	**180**	**0.010023**	**0.115748**
3	0	0	180	0.023606	0.117351
1	1	0	160	0.010340	0.121880
0	2	0	200	0.013644	0.124654
0	0	2	160	0.023688	0.148523

It can be seen that even though the solution is not the optimal, it is less expensive than the optimal one. It is also the second best solution and therefore should be acceptable as a near optimal one. Next, the proposed method is implemented for a 12-area test system.

VI. IMPLEMENTATION OF THE METHOD

A 12-area power system is shown in Fig. 4. The test system is a multi-area representation of an actual power system [9] that has 137 generation units and 169 tie line connections between areas. Transfer capabilities between areas are shown in Appendix D. TABLE VII shows area generations and loads as well as availability and cost per generator in prospective areas which are areas 1 to 5, and 9 to 12.

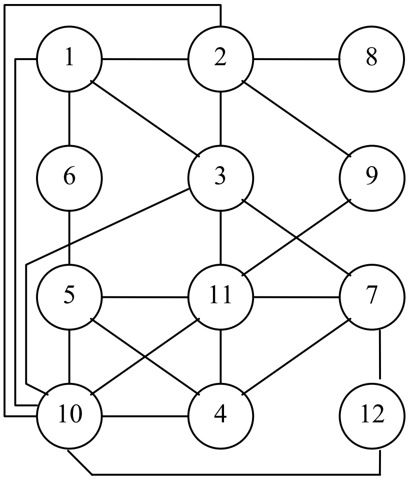

Fig. 4. Twelve Area Test System

It is assumed that the additional generators have capacity of 200 MW each. Probability distribution tables for generation and tie-lines are developed with a capacity increment of 50

MW. System loss of load probability before unit additions is 0.0041 for original load and 0.0153 for 10% increased load. Reliability indices presented in the analysis are calculated from complete decomposition approach.

TABLE VII
GENERATION AND LOAD PARAMETERS OF TWELVE AREA TEST SYSTEM

Area j	Load (MW)	10 % increased Load (MW)	Generation (MW)	FOR of additional units	Cost ($m)
1	1750	1900	2550	0.025	250
2	16650	18300	23600	0.025	250
3	9300	10250	15100	0.025	250
4	2000	2200	3100	0.025	250
5	550	600	900	0.025	250
6	0	0	550	-	-
7	0	0	3500	-	-
8	0	0	400	-	-
9	1100	1200	2100	0.025	250
10	2200	2400	3100	0.025	250
11	2600	2850	4150	0.025	250
12	750	850	900	0.025	250

The analysis is implemented with two load scenarios; original and 10% increased, and repeated with two budgets; $0.5 and $1 billion. Maximum number of additional units allowed in each area is two units for $0.5 billion budget and four units for $1 billion budget, which gives 45 and 495 possible generator combinations respectively. The purpose of having two different budgets is to test the correctness of the proposed method. The proposed method is applied to smaller budget first and then to larger budget. To verify the correctness of the solution from this procedure, an optimal solution for each scenario is also obtained by enumeration.

The additional units are included in the state space before performing global decomposition according to the budget. Once global decomposition is performed, candidate areas are determined by the '*v*' value and the optimal solution from the first *L* set optimization is computed. TABLE VIII shows the solution for the system with original load and $0.5 billion budget. TABLE IX shows the solution with 10% increased load and $0.5 billion budget. TABLE X shows the solution with original load and $1 billion budget. TABLE XI shows the solution with 10% increased load and $1 billion budget. TABLE XII shows the comparison between the optimal solution from proposed method and that from enumeration.

TABLE VIII
SOLUTION WITH $0.5 BILLION BUDGET

Area	1	2	3	4	5	9	10	11	12
'*v*' value	1	248	147	27	1	1	16	23	1
Solution from DP	-	0	0	2	-	-	0	0	-
Optimal Solution	0	0	0	2	0	0	0	0	0

TABLE IX
SOLUTION WITH $0.5 BILLION BUDGET AND 10% INCREASED LOAD

Area	1	2	3	4	5	9	10	11	12
'*v*' value	1	284	166	31	1	1	20	28	1
Solution from DP	-	0	0	2	-	-	0	0	-
Optimal Solution	0	0	0	2	0	0	0	0	0

TABLE X
SOLUTION WITH $1 BILLION BUDGET

Area	1	2	3	4	5	9	10	11	12
'*v*' value	1	240	147	27	1	1	16	23	1
Solution from DP	-	0	0	3	-	-	1	0	-
Optimal Solution	0	1	0	3	0	0	0	0	0

TABLE XI
SOLUTION WITH $1 BILLION BUDGET AND 10% INCREASED LOAD

Area	1	2	3	4	5	9	10	11	12
'*v*' value	1	276	166	31	1	1	20	28	1
Solution from DP	-	0	1	3	-	-	0	0	-
Optimal Solution	0	2	0	2	0	0	0	0	0

TABLE XII
COMPARISON BETWEEN THE SOLUTIONS FROM THE PROPOSED METHOD AND THE OPTIMAL SOLUTIONS

Scenarios	LOLP of optimal solution from proposed method	LOLP of optimal solution from enumeration	% Difference
Original Load, $0.5 B	0.001640	0.001640	0
Increased Load, $0.5 B	0.010182	0.010182	0
Original Load, $1 B	0.001320	0.001284	2.80
Increased Load, $1 B	0.008975	0.007834	14.56

Results show that when the maximum number of additional units in each area is small, the proposed method accurately provides optimal solution. However, when the maximum number of additional units in each area is higher, the proposed method provides a solution with LOLP close to the one obtained from the optimal solution. This is due to the fact that all possible additional units are included in the state space before performing global decomposition. Since all the states are assumed to exist in the system, the first *L* sets from global decomposition can underestimate capacity deficiency, and thus, LOLP. Even though the proposed method may not necessarily provide the optimal solution, it still gives a solution close to optimal one that can be adjusted with some other sensitivity techniques to obtain the optimal solution.

Results also show that when system loss of load probability is small, the proposed method produces optimal or close to optimal solution. System with small loss of load probability tends to have much more generation than load. When the system loses its generation in any single area, the surrounding areas can provide assistance allowed by transfer capabilities from its neighborhood areas. This means that there is smaller probability that the system will lose its generations in two areas or more to produce loss of load state.

The first *L* sets from global decomposition are partitioned from the overall state space by lowering generation in one area while keeping generation in other areas at their maximum until the system reaches loss of load state. They provide information about the effect of additional generation in any area to partial system loss of load probability which is produced from loss of generation in a single area i.e. L_i is produced from loss of generation in area *i*. The combined effects of loss of area generation in two (or three or more) areas are evaluated at the second (or third and so on) stage of decomposition.

However, when there are small probabilities that loss of two or more area generators creates loss of load states, the effects of additional generation on loss of load probability are mainly dominated by loss of generation in one area and transfer capability of its surrounding areas, or equivalently, the first L set from global decomposition.

VII. ADVANTAGE OF THE PROPOSED APPROACH

Dynamic programming reduces the number of computations made during the search for the best solution. Number of multiplications depends on available budget, R as seen in the three-area test system. Number of possible combinations is shown in (29). In the worst case, multiplications will be performed to all components in the table of the size $R\times\left(y_j+1\right)$ at each stage (except the last stage). The operations used in exhaustive search and dynamic programming are compared in TABLE XIII. The computations will have a significant advantage for larger systems using dynamic programming.

$$\text{Number of possible combinations} = \prod_{j=1}^{N}\left(y_j+1\right) \tag{29}$$

TABLE XIII
COMPARISON OF NUMBER OF COMPUTATION BETWEEN EXHAUSTIVE SEARCH AND DYNAMIC PROGRAMMING

Operation	Exhaustive Search	Dynamic Programming
Multiplication	$(N-1)\times\prod_{j=1}^{N}\left(y_j+1\right)$	At most $(N-1)\times\sum_{j=1}^{N}\left(R\times\left(y_j+1\right)\right)$
Comparison	$\prod_{j=1}^{N}\left(y_j+1\right)-1$	At most $(N-1)\times\sum_{j=1}^{N}\left(R\times y_j\right)$

Dynamic programming can also be applied to a problem with different cost constraints, as long as the constraint is in separable form. For example, a summation of one variable function as in (30), the same analysis can be applied

$$\sum_{j=1}^{N}h_j\left(y_j\right) \tag{30}$$

As an example, the constraint can also be of the quadratic form as in (31). It is also possible to consider problems with multiple constraints; however, the algorithm may not be as efficient.

$$\sum_{j=1}^{N}\alpha_j y_j^2+\beta_j y_j+\varphi_j \tag{31}$$

Another advantage of the approach is that the proposed technique can be extended to incorporate transfer capability adequacy analysis between areas. Global decomposition allows us to include additional state capacities of any arc in the network which can either be additional generation or additional tie lines. The formulation given in (23) is in terms of network arc. Thus, the same analysis can be applied so that generation and tie line expansion can be analyzed together by the proposed method.

VIII. LIMITATIONS OF THE PROPOSED APPROACH

The structure of the optimization problem does not allow simple analysis when considering L sets from more than one stage of decomposition. The problem is then simplified by considering sets from the first stage of decomposition only; the first L sets optimization. This introduces an approximation to the problem given that the objective of the problem is to maximize reliability of the overall system. This approximation is reasonable since the first L sets provide preliminary information on the generation deficiency in each area from the overall state space which represents the system characteristic. The optimization process depends significantly on these sets. Thus, it is important that the sets from decomposition phase give substantial information to the optimization phase.

IX. CONCLUSION

An optimization procedure is proposed to find an optimal or near optimal generation location in multi area power systems. The term near optimal is used to indicate that the LOLP of the proposed solution is equal or close to that of the optimal solution. The problem has reliability constraint that does not have an explicit expression and therefore complicates the optimization process. Global decomposition is introduced to effectively evaluate reliability index of different generation combinations. Even though reliability equation can be derived, it is a very complex function. This reliability equation is then approximated by considering only L sets from the first decomposition since they possess separable structure which can be solved by dynamic programming.

The problem is formulated with minimization of LOLP index as an objective function subject to cost constraint. If a solution obtained from the optimization provides unsatisfactory system reliability, budget constraint can be relaxed to include more additional units in each area. The problem can be re-optimized without performing global decomposition again if the maximum number of additional units in each area remains unchanged. This iterative procedure between cost constraint and reliability optimization can be used to get a satisfactory solution. The procedure is quite efficient as decomposition needs to be performed only once.

Generation expansion problem including reliability constraint is a very challenging and complex optimization problem. Due to the problem complexity, certain assumptions and approximations have been made. The paper proposes a method to explicitly incorporate reliability into consideration. The main contribution is to propose an approximation to LOLP equation. The proposed approach simplifies the problem by optimizing over a smaller set of state space and thus the solution cannot guarantee global optimality. However, the solution from this approach is likely to be near optimal, if not optimal, and can provide a starting point to which sensitivity analysis can be applied to locate the optimal solution. Meta-heuristic techniques, which require good starting solution for efficiency, can be applied along with the proposed method to ensure an optimal solution.

X. APPENDIX

A. Generation Probability Distribution Equation Incorporating Non-identical Additional Units

Let the additional units in area *j* have the capacities, $C_j^1, C_j^2, \ldots, C_j^m$ and corresponding number of additional units in area *j* is $y_j^1, y_j^2, \ldots, y_j^m$ respectively.

Each additional unit has the capacity as multiple, μ_j^t, $t = 1,2,\ldots,m$ of the fixed increment, η, used in the discrete probability distribution. The following equation describes generation probability incorporating the additional units, y_j^t, in area *j*.

$$P_{G_i^j}^{y_j^t} = \sum_{k=0}^{y_j^t} P_{G^j_{i-\mu_j^t k}}^{t-1} \binom{y_j^t}{k} \left(\mathrm{FOR}_j^t\right)^{y_j^t - k} \left(1 - \mathrm{FOR}_j^t\right)^k, t = 1,\ldots,m \tag{A.1}$$

where

$$P_{G_i^j}^{t} = \sum_{k=0}^{y_j^{t,\max}} P_{G^j_{i-\mu_j^t k}}^{t-1} \binom{y_j^{t,\max}}{k} \left(\mathrm{FOR}_j^t\right)^{y_j^{t,\max} - k} \left(1 - \mathrm{FOR}_j^t\right)^k, t = 1,\ldots,m \tag{A.2}$$

and

$P_{G^j_{i-\mu_j^t k}}^{t-1}$ Probability of generation at level $i - \mu_j^t k$ with additional units, $y_j^1, y_j^2, \ldots, y_j^{t-1}$ in area *j*, 0 if $i \le \mu_j^t k$

FOR_j^t Forced outage rate of additional unit *t* in area *j*

$y_j^{t,\max}$ Maximum number of additional units of capacity C_j^t

B. Reliability Equation for Non-identical Additional Units

Equation (18) is rewritten as the following.

$$\begin{aligned} h_j\left(y_j^t\right) &= \sum_{k=0}^{y_j^t} \left[\left(\sum_{i=m_j^\omega}^{M_j^\omega} P_{G^j_{i-\mu_j^t k}}^{t-1} \right) \binom{y_j^t}{k} \left(\mathrm{FOR}_j^t\right)^{y_j^t - k} \left(1 - \mathrm{FOR}_j^t\right)^k \right] \\ &= \sum_{k=0}^{y_j^t} \left[\left(\overline{P}_{G^j_{M_j^\omega - \mu_j^t k}}^{t-1} - \overline{P}_{G^j_{m_j^\omega - \mu_j^t k - 1}}^{t-1} \right) \binom{y_j^t}{k} \left(\mathrm{FOR}_j^t\right)^{y_j^t - k} \left(1 - \mathrm{FOR}_j^t\right)^k \right], t = 1,\ldots,m \end{aligned} \tag{B.1}$$

where

y_j^t Number of additional units of capacity C_j^t in area *j*

$\overline{P}_{G^j_{i-\mu_j k}}^{t-1}$ Cumulative probability of generation at level $i - \mu_j^t k$ with additional units, $y_j^1, y_j^2, \ldots, y_j^{t-1}$ in area *j*, 0 if $i \le \mu_j^t k$

Then, equation (22) is modified as follows.

$$g_j^i\left(y_j^t\right) = \sum_{k=0}^{y_j^t} \left[\left(\overline{P}_{G^j_{v_i - \mu_j^t k - 1}}^{t-1} \right) \binom{y_j^t}{k} \left(\mathrm{FOR}_j^t\right)^{y_j^t - k} \left(1 - \mathrm{FOR}_j^t\right)^k \right], t = 1,\ldots,m \tag{B.2}$$

C. The First L set Equation

Detailed derivation of (23) is given as follow. From (21), probability equation of the L_1 set is (C.1)

$$\Pr\left(L_1\right) = \prod_{j=1}^{N} g_j^1\left(y_j\right) = g_1^1\left(y_1\right) g_2^1\left(y_2\right) \cdots g_N^1\left(y_N\right) \tag{C.1}$$

Due to the structure of the first L sets described by (20), the states of component from 2 to *N* in L_1 set take all possible value from 1 to their maximum states, i.e.,

$$g_j^1\left(y_j\right) = 1,\ j = 2,\ldots,N \tag{C.2}$$

Therefore, (C.1) can be written as (C.3).

$$\Pr\left(L_1\right) = g_1^1\left(y_1\right) \tag{C.3}$$

Consider probability equation of L_2, the same argument can be applied to the states from 3 to *N*, the probability of L_2 set is (C.4).

$$\Pr\left(L_2\right) = g_1^2\left(y_1\right) g_2^2\left(y_2\right) \tag{C.4}$$

Since the capacity arc of area 1 in both L_1 and L_2 takes all possible value from 1 to M_1. This gives (C.5).

$$g_1^1\left(y_1\right) + g_1^2\left(y_1\right) = 1 \tag{C.5}$$

Thus, equation (C.4) can be simplified as (C.6).

$$\Pr\left(L_2\right) = \left(1 - g_1^1\left(y_1\right)\right) g_2^2\left(y_2\right) \tag{C.6}$$

The same analysis is then applied to the all L_i sets, the probability equation of L_i set is (C.7).

$$\Pr\left(L_i\right) = \left(1 - g_1^1\left(y_1\right)\right)\left(1 - g_2^2\left(y_2\right)\right) \cdots \left(1 - g_{i-1}^{i-1}\left(y_{i-1}\right)\right) g_i^i\left(y_i\right) \tag{C.7}$$

This gives first L sets probability as (23).

D. Transfer Capability of a 12-area Power System

Transfer capability of a 12-area power system [9] is shown in TABLE D.I.

TABLE D.I
TRANSFER CAPABILITY

From Area	To Area	Transfer Capability (MW)
1	2	4550
1	3	300
1	6	100
1	10	150
2	3	1050
2	8	150
2	9	900
2	10	450
3	7	400
3	10	200
3	11	50
4	5	50
4	7	300
4	10	200
4	11	150
5	6	400
5	10	50
5	11	650
7	11	350
7	12	950
9	10	150
9	11	150
10	11	150
10	12	100

XI. ACKNOWLEDGMENT

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XII. REFERENCES

[1] S. Kannan, S. M. R. Slochanal and N. P. Padhy, “Application and Comparison of Metaheuristic Techniques to Generation Expansion Planning Problem”, *IEEE Trans. Power Syst.*, vol. 20, no. 1, pp. 466-475, Feb. 2005.

[2] N. S. Rau and F. Zeng, “Adequacy and Responsibility of Locational Generation and Transmission – Optimization Procedure”, *IEEE Trans. Power Syst.*, vol. 19, no. 4, pp. 2093-2101, Nov. 2004.

[3] A. A. Chowdhury et. al., “Assessing Mid-Continent Area Power Pool Capacity Adequacy Including Transmission Limitations”, in *Proc. 8th Int. Conf. Probabilistic Methods Applied to Power Systems*, Iowa, Sept. 2004.

[4] H. M. Khodr et al., “A Linear Programming Methodology for the Optimization of Electric Power–Generation Schemes”, *IEEE Trans. Power Syst.*, vol. 17, no. 3, pp. 864-869, Aug. 2002.

[5] H. T. Firmo and L. F. L. Legey, “Generation Expansion Planning: An Iterative Genetic Algorithm Approach”, *IEEE Trans. Power Syst.*, vol. 17, no. 3, pp. 901-906, Aug. 2002.

[6] J. Zhu and M. Chow, “A Review of Emerging Techniques on Generation Expansion Planning”, *IEEE Trans. Power Syst.*, vol. 12, no. 4, pp. 1722-1728, Nov. 1997.

[7] J. Mitra and C. Singh, “Incorporating the DC load flow model in the decomposition-simulation method of multi-area reliability evaluation”, *IEEE Trans. Power Syst.*, vol. 11, no. 3, pp. 1245-1254, Aug. 1996.

[8] Z. Deng and C. Singh, “A New Approach to Reliability Evaluation of Interconnected Power Systems Including Planned Outages and Frequency Calculations”, *IEEE Trans. Power Syst.*, vol. 7, no. 2, pp. 734-743, May 1992.

[9] S. Sung, “Multi-Area Power System Reliability Modeling”, Ph. D. Dissertation, Dept. Elec. Comp. Eng., Texas A&M Univ., College Station, Texas, 1992.

[10] C. Singh and Z. Deng, “A New Algorithm for Multi-Area Reliability Evaluation–Simultaneous Decomposition-Simulation Approach”, *Electric Power System Research*, vol. 21, pp. 129-136, 1991.

[11] A. Lago-Gonzalez and C. Singh, “The Extended Decomposition Simulation Approach for Multi-Area Reliability Calculations”, *IEEE Trans. Power Syst.*, vol. 5, no. 3, pp. 1024-1031, Aug. 1990.

[12] C. Singh and A. Lago-Gonzalez, “Improved Algorithm for Multi-Area Reliability Evaluation Using the Decomposition-Simulation Approach”, *IEEE Trans. Power Syst.*, vol. 4, no. 1, pp. 321-328, Feb. 1989.

[13] P. Doulliez, E. Jamoulle, “Transportation Networks with Random Arc Capacities”, *Reveu Francaise d’Automatique, Informatique et Recherche Operationelle*, vol. 3, pp 45-66, 1972.

[14] Garver L. L., “Effective Load Carrying Capability of Generating Units”, *IEEE Trans. Power Appar. Syst.*, vol. PAS-85, no. 8, pp. 910-919, Aug. 1966.