

# The Impact of Uncertainty on Incentives to Collude in Electricity Markets

Mary B. Cain and Fernando L. Alvarado

*Abstract*-- Market power in any market can result in higher prices than in a truly competitive market. Electricity markets pose unique problems to detection of market power due to the properties of electricity: it cannot be stored, which implies that the amount of energy supplied must always equal the amount demanded, and it must obey the laws of physics. A group of generators may exercise market power through tacit collusion, all raising prices together without communicating their intentions. Generators will use their expectations of market conditions to choose a competitive (low) bid, or a non-competitive (high) bid. We use expected residual demand to describe market conditions, and calculate threshold levels where a generator would decide to change from low to high bid. We examine the effects of uncertainty in load and competitor behavior on collusion decisions in both uniform price and discriminatory price auctions.

*Index Terms*-- market power, power system economics

## Nomenclature and representative values

$B_L$	Low bid price (competitive bid)	(\$20.00)
$B_H$	High bid price (non-competitive bid)	(\$50.00)
$M$	Generator capacity	(60 MW)
$L$	Load	
$p_{LH}$	Probability that load is high	
$p_{GjH}$	Probability that generator $j$ bids high	
$T_L$	Load threshold for increasing bid	

## I. INTRODUCTION

**M**ARKET power in any market can result in higher prices than in a truly competitive market. Electricity markets pose unique problems to detection of market power. These problems arise from electricity's properties: it cannot be stored, which implies that the amount of energy supplied must match the amount demanded at every instant, and its transmission across the country must obey the laws of physics (Kirchoff's Laws).

Many factors determine whether or not firms that know they have market power will actually exercise it to increase their profits. These include the penalties imposed when

market power exercise is detected, the rules of the market, the willingness of other players to collude, the potential increase in profits, and uncertainty in future market conditions.

Alvarado and Rajaraman discuss market power detection through observing whether an individual market participant's behavior is compatible with the behavior of price takers [1]. They also study the effect on bidding of a non price-taker generator with some degree of market power.

The output of a "price maker" (a marginal producer) has, by definition, some impact on the market price. Alvarado and Rajaraman [2] and de la Torre, Arroyo, Conejo and Contreras [3] use residual demand curves<sup>1</sup> as exogenous inputs to determine the optimal strategy for price makers in electricity markets. Thus a residual demand curve for a price maker reflects its impact on price, and also includes the expected behavior of other participants and the expected total demand, reducing the data needed for analysis.

Green and Porter find that firms will compete at Cournot<sup>2</sup> levels when market prices are below a certain trigger level, but that when prices rise above that level, firms have individual monopolies and will produce at a monopoly level until market prices drop [4]. They point out that firms will not remain at Cournot levels forever; after some amount of time, they will again produce at monopoly levels. They apply dynamic programming to determine the trigger level and the number of periods after which firms will revert to Cournot output levels when the market prices fall below the trigger level.

Guan, Ho and Pepyne analyze price spikes in electricity markets, and find that generators will withhold capacity from the market when market conditions are above a certain threshold [5]. They demonstrate that there are two Nash equilibria in their model, and withholding drives the market to the higher-priced equilibrium. When enough suppliers withhold, the supply curve shifts and a price spike occurs. The authors call this condition "opportunistic tacit collusion." They compare historical prices from the California market and find that this model accounts for

---

This work was supported in part by the CERTS consortium under DOE Interagency Agreement DE-AI-99EE35075 and by the Power Systems Engineering Research Center (PSERC).

M. B. Cain is a graduate student, and F. L. Alvarado is a Professor Emeritus in the Department of Electrical and Computer Engineering, University of Wisconsin, Madison, WI 53706 USA.

---

<sup>1</sup> Residual demand is the market demand minus the output of all other market participants.

<sup>2</sup> In Cournot competition, firms simultaneously choose the quantity they will produce, and sell this quantity at the market clearing price. Cournot equilibria occur when and all firms choose quantities that are the best response to the expected actions of their competitors.

many of the actual price spikes. They do not investigate the time variation of the price spikes.

Correia, Overbye and Hiskens study automated agents using stochastic bidding strategies, and find that without explicit collusion, the agents find a region where both have higher earnings than at the competitive equilibrium [6]. Lesieutre, Thomas and Mount also discuss this “win-win” region, and study a method using sensitivity of revenue and dispatch to bid to locate generators who might benefit from exercise of market power as a group [7].

The structure of this paper is as follows: section II discusses the factors affecting collusion, section III focuses on a two-generator system with changing load and competitor behavior, section IV discusses extension of these results to larger systems, and section V concludes the paper.

## II. FACTORS AFFECTING A GENERATOR’S DECISION TO COLLUDE

In this paper we focus on market power exercise by a group of generators raising bids or reducing output together in order to increase the price they are paid for producing electricity. The collusion is implicit, meaning that the generators do not communicate their intended bids, and each generator keeps its revenues (there are no side payments). A generator’s decision to collude is based on its expectations of load level, behavior of other market participants, and likelihood of market regulators punishing collusion.

### A. Load Level

For a given load level, a generator expects some known amount of its power to be dispatched in the system. When a generator knows that a portion of its power must be dispatched to meet load, it has an incentive to raise its bid, or reduce its output to increase price, for this portion of its output power. Actual system load depends on several factors, including weather, hours of daylight and economic conditions. Electricity companies use complex models to forecast load, but these forecasts always have some uncertainty. In this paper we use residual demand; this is the system demand minus the supply of all other generators. Residual demand incorporates all of the factors in load forecasts, and does not require large amounts of data. We model uncertainty in load forecasts as uncertainty in residual demand. We analyze both certain and uncertain residual demand.

### B. Expected Behavior of Other Generators

We assume that in order for the strategy of raising bids to increase prices to work, two or more generators must participate. If a generator knows that other market participants will also submit high bids in a given load condition, it has more incentive to do likewise. Conversely, if a generator knows that its competitors never submit high bids, it also knows that submitting a high bid leaves it in danger of not being dispatched and thus not maximizing its revenue. We model generators bidding

above the competitive price as ‘bad citizens’ and generators that bid at the competitive price as ‘good citizens’. This situation is similar to the prisoner’s dilemma, common in game theory texts.

Another factor to consider is how different market participants’ actions affect the price at a generator’s bus. If generators that raise their bids have no effect on our price, we have little incentive to join them in high bidding.

### C. Penalties for and Likelihood of Market Power Detection

Market monitoring and oversight may impact generators’ decisions to raise bids. If generators are likely to be accused of exercising market power and there is a significant penalty, they are less likely to collude. However, if the potential profits of colluding are higher than the expected penalty, the penalty may not deter the generators from exercising market power. Likewise, if market power often goes undetected, generators may not expect to be penalized.

## III. TWO GENERATOR EXAMPLE

The following example is for a two generator system shown in Fig. 1. To simplify the analysis, we ignore transmission constraints, transmission losses, load price responsiveness, and we assume only one electricity market. Withholding can be economic (offering power at a high price), or physical (making some generator capacity unavailable). We investigate two types of auctions: a uniform price auction, where all generators are paid the highest clearing price, and a discriminatory price auction, where generators are paid what they bid. Both are simultaneous auctions – all generators submit bids at the same time without knowledge of each other’s bids. We assume that if bids are equal, the load will be equally split among generators. In the following sections we calculate the expected dispatch and revenue for generator 1 under various load conditions and expectations about generator 2, using analysis similar to [2]. We assume that the generators have zero cost (such as hydro generators), so that revenue and profit are equal.

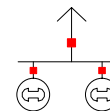


Fig. 1. Two generator system

### A. Certain Load and Competitor Behavior

When generators can exactly predict the load level and competitor bids, they will know when they can and cannot benefit from withholding. Suppose that the two generators in Fig. 1 have the same capacity of  $M$  MW, and that they can make an offer at high price  $B_H$  or low price  $B_L$ .  $B_L$  is the competitive price, and  $B_H$  is the non-competitive price. The actual load is denoted  $L$ ; high load means  $L > M$ , and low  $L < M$ .

### 1) Generator 2 Always Bids the Competitive Price:

First, assume that generator 2 is a ‘good citizen’, meaning it always bids  $B_L$ . The first four rows of Table I show the dispatch and revenue for generator 1 for different combinations of load level and bids. Both types of auction give the same results in this case. When generator 2 always bids low, generator 1 will only receive the high price when it bids high and is dispatched.

Clearly, when load is low, the best strategy for generator 1 is to bid  $B_L$ . When load is high, the best strategy depends on the load level. Assuming that  $B_H$ ,  $B_L$ , and  $M$  are fixed, the criteria that generator 1 will use to decide whether to bid  $B_H$  or  $B_L$  under high load conditions is based on the load  $L$ . We calculate this load threshold by equating the revenues of generator 1 at high load for the two possible bids. Generator 1 will bid  $B_H$  when the load is above the threshold in (1).

$$T_L = B_H M / (B_H - 0.5B_L) \quad (1)$$

Table I  
Generator 1 outcomes for certain load, certain generator 2 bids

Load	Gen.2 Bid	Gen.1 Bid	Gen.1 Dispatch	Gen.1 Revenue Discriminatory	Gen.1 Revenue Uniform
Low	$B_L$	$B_L$	$\frac{1}{2} * L$	$\frac{1}{2} * L * B_L$	$\frac{1}{2} * L * B_L$
Low	$B_L$	$B_H$	0	0	0
High	$B_L$	$B_L$	$\frac{1}{2} * L$	$\frac{1}{2} * L * B_L$	$\frac{1}{2} * L * B_L$
High	$B_L$	$B_H$	$L - M$	$(L - M) * B_H$	$(L - M) * B_H$
Low	$B_H$	$B_L$	$L$	$L * B_L$	$L * B_L$
Low	$B_H$	$B_H$	$\frac{1}{2} * L$	$\frac{1}{2} * L * B_H$	$\frac{1}{2} * L * B_H$
High	$B_H$	$B_L$	$M$	$M * B_L$	$M * B_H$
High	$B_H$	$B_H$	$\frac{1}{2} * L$	$\frac{1}{2} * L * B_H$	$\frac{1}{2} * L * B_H$

### 2) Generator 2 Always Bids the Non-Competitive Price:

Now, consider the case when generator 2 is always a ‘bad citizen’ (bids  $B_H$ ), shown in the lower half of Table I. When load is low, the bidding decision for generator 1 depends on the prices. Generator 1 will bid  $B_H$  when the (2) is true. Generator 1 increases its revenue by bidding  $B_L$  and serving more of the load. This holds for both auction types.

$$B_H \geq 2 B_L \quad (2)$$

The threshold load level for generator 1 to increase its bid from  $B_L$  to  $B_H$  when the load is high and generator 2 bids  $B_H$  for a discriminatory price auction is shown in (3).

$$T_L = 2 B_L M / B_H \quad (3)$$

Equation (4) shows the threshold for a uniform price auction. The load threshold is the combined capacity of the 2 generators, implying that generator 1 should always bid  $B_L$ . Generator 1 will never bid above the competitive price when it knows that generator 2 will bid high, and that load is greater than the capacity of generator 1.

$$T_L = 2 M \quad (4)$$

### 3) Effect of Competitor Bid on Load Threshold Level:

In a discriminatory price auction, the load threshold is higher when generator 2 bids low than when generator 2 bids high. This can be demonstrated by calculating numerical values for the load thresholds, or by setting

equation (1)  $>$  (3) and rearranging to get  $B_H > B_L$ , which is always true, so the assumption that (1)  $>$  (3) is also true. Generator 2 bidding high allows generator 1 to benefit from bidding high at lower load levels than when generator 2 bids low.

In a uniform price auction, generator 1 will never bid high when load is high and generator 2 bids high, since the threshold in (4) occurs when load is greater than the system capacity.

The difference in the two auctions is due to the different payment systems. In a discriminatory price auction, when one generator bids high and one low, the low bidder is fully dispatched and receives the low price, while the high bidder is the marginal unit and is only partially dispatched. When both bid high, in our model, they split the load and both receive the high price. In a uniform price auction, when one generator bids high and the other low, the low bidder is fully dispatched and receives the high price. When both bid high, they split the load. Thus it is only beneficial to the 2<sup>nd</sup> bidder to bid high when load is greater than the generators’ combined capacities.

### 4) Generator 2 Bids Correlated With Load:

Table II shows the outcomes for generator 1 when generator 1 knows that generator 2 bids are directly correlated with load level (generator 2 bids the competitive price  $B_L$  when load is low, and bids the non-competitive price  $B_H$  when load is high). For low load, the revenues are the same as for low load when generator 2 always bid low in section A-1. For high load, the results are the same as in section A-2 for high load and generator 2 always bidding high. For low load, generator 1 should always bid  $B_L$ , since bidding  $B_H$  results in zero revenue. For high load it can use the threshold in (3) or in (4) depending on the type of auction.

Table II  
Gen. 1 outcomes for certain load, gen. 2 bids correlated to load level

Load	Gen.2 Bid	Gen.1 Bid	Gen.1 Dispatch	Gen.1 Revenue Discriminatory	Gen.1 Revenue Uniform
Low	$B_L$	$B_L$	$\frac{1}{2} * L$	$.5 * L * B_L$	$.5 * L * B_L$
Low	$B_L$	$B_H$	0	0	0
High	$B_H$	$B_L$	$M$	$M * B_L$	$M * B_H$
High	$B_H$	$B_H$	$\frac{1}{2} * L$	$\frac{1}{2} * L * B_H$	$\frac{1}{2} * L * B_H$

### B. Uncertain Load and Certain Competitor Behavior

We look at the three cases for competitor behavior, bidding high, bidding low, and bidding correlated to load. In each case, there is probability  $p_{LH}$  that load is high, and  $(1 - p_{LH})$  that load is low. Table III shows the outcomes for these three cases.

Table III  
Generator 1 outcomes for uncertain load, known generator 2 bids

Gen2 Bid	Gen.1 Bid	Gen. 1 Expected Dispatch	Gen. 1 Expected Revenue
B <sub>L</sub>	B <sub>L</sub>	.5*L	.5*L*B <sub>L</sub>
B <sub>L</sub>	B <sub>H</sub>	p <sub>LH</sub> *(L - M)	p <sub>LH</sub> *(L - M)*B <sub>H</sub>
B <sub>H</sub>	B <sub>L</sub>	p <sub>LH</sub> *M + (1-p <sub>LH</sub> )*L	[p <sub>LH</sub> *M + (1-p <sub>LH</sub> )*L]*B <sub>L</sub> (Discriminatory)
B <sub>H</sub>	B <sub>L</sub>	p <sub>LH</sub> *M + (1-p <sub>LH</sub> )*L	p <sub>LH</sub> *M*B <sub>H</sub> + (1-p <sub>LH</sub> )*L*B <sub>L</sub> (Uniform)
B <sub>H</sub>	B <sub>H</sub>	.5*L	.5*L*B <sub>H</sub>
Load	B <sub>L</sub>	p <sub>LH</sub> *M+(1-p <sub>LH</sub> )*L*1/2	p <sub>LH</sub> *M+(1-p <sub>LH</sub> )*L*1/2*B <sub>L</sub>
Load	B <sub>H</sub>	p <sub>LH</sub> *L*1/2	p <sub>LH</sub> *L*1/2*B <sub>H</sub>

Equation (5) shows the threshold for generator 1 to increase its bid from B<sub>L</sub> to B<sub>H</sub> when generator 2 bids low for either auction type.

$$T_L = p_{LH} B_H M / (p_{LH} B_H - 0.5 B_L) \quad (5)$$

When generator 2 bids high in a discriminatory price auction, the threshold is (6), and for a uniform price auction in (7).

$$T_L = p_{LH} B_L M / [1/2 B_H - (1-p_{LH}) B_L] \quad (6)$$

$$T_L = p_{LH} B_H M / [1/2 B_H - (1-p_{LH}) B_L] \quad (7)$$

When generator 2 bids are correlated with the load, the results are the same for the two auction types. Equation (8) shows the threshold for this situation.

$$T_L = 2 p_{LH} B_L M / [p_{LH} B_H - (1-p_{LH}) B_L] \quad (8)$$

Fig. 2 shows the variation of these thresholds with probability of high load. For this example, M = 60, B<sub>L</sub> = 20, and B<sub>H</sub> = 50. When generator 2 always bids high, the load threshold is low and increases as the probability of high load increases, but when generator 2 bids low the load threshold decreases with increasing probability of high load. When generator 2 bids high and load is low, generator 1 has higher revenue if it bids low and is fully dispatched. When a generator knows its competitors will bid high in a discriminatory price auction, it will also bid high for relatively low loads. However, when competitors are known to bid low, the load must be higher for a generator to benefit from bidding high. When generator 2 bids are correlated to the load, the threshold is the close to that for generator 2 always bidding low for low p<sub>LH</sub>, then decreases to intersect with the generator 2 bids high (discriminatory auction) curve as p<sub>LH</sub> approaches 1.

### C. Certain Load and Uncertain Competitor Behavior

Next we examine the effects of uncertain competitor behavior on the criteria for generator 1 to submit a high bid. Generator 1 has estimated that generator 2 has probability p<sub>G2H</sub> of being a ‘bad citizen’ (bidding B<sub>H</sub>) and (1 - p<sub>G2H</sub>) of being a ‘good citizen’ (bidding B<sub>L</sub>).

#### 1) Discriminatory Price Auction:

Table IV shows the outcomes for the various bidding possibilities of the two generators for a discriminatory price auction.

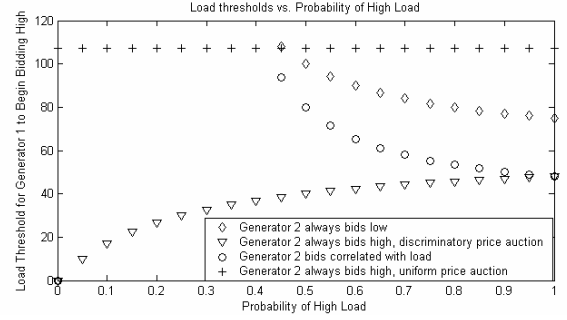


Fig. 2. Load threshold vs. probability of high load for known competitor bids

Table IV  
Generator 1 outcomes for certain load, unknown generator 2 bids, discriminatory price auction

Load	Gen.1 Bid	Gen. 1 Expected Dispatch	Gen. 1 Expected Revenue
Low	B <sub>L</sub>	p <sub>G2H</sub> *L+(1-p <sub>G2H</sub> )*.5*L	[p <sub>G2H</sub> *L + (1-p <sub>G2H</sub> )*.5*L]*B <sub>L</sub>
Low	B <sub>H</sub>	.5*L*p <sub>G2H</sub>	.5*L*p <sub>G2H</sub> *B <sub>H</sub>
High	B <sub>L</sub>	p <sub>G2H</sub> *M+(1-p <sub>G2H</sub> )*.5*L	[p <sub>G2H</sub> *M + (1-p <sub>G2H</sub> )*.5*L]*B <sub>L</sub>
High	B <sub>H</sub>	p <sub>G2H</sub> *.5*L + (1-p <sub>G2H</sub> )*(L-M)	[p <sub>G2H</sub> *.5*L + (1-p <sub>G2H</sub> )*(L-M)]*B <sub>H</sub>

When load is low the decision threshold in (9) is based on the bid prices and the probability of generator 2 bidding high.

$$p_{G2H} \geq B_L / (B_H - B_L) \quad (9)$$

When load is high, the load threshold for generator 1 to change its bid is shown in (10).

$$T_L = \frac{M(p_{G2H}(B_L - B_H) + B_H) - p_{G2H}B_H}{(1 - p_{G2H})(B_H + .5B_L) + .5B_H} \quad (10)$$

#### 2) Uniform Price Auction:

Table V shows the outcomes for the various bidding possibilities of the two generators for a uniform price auction. As in the discriminatory price auction, the low load bidding criteria is independent of load level (11).

$$p_{G2H} \geq B_L / B_H \quad (11)$$

Table V  
Generator 1 outcomes for certain load, unknown generator 2 bids, uniform price auction

Load	Gen.1 Bid	Gen. 1 Expected Dispatch	Gen. 1 Expected Revenue
Low	B <sub>L</sub>	p <sub>G2H</sub> *L+(1-p <sub>G2H</sub> )*.5*L	p <sub>G2H</sub> *L*B <sub>L</sub> + (1-p <sub>G2H</sub> )*.5*L*B <sub>H</sub>
Low	B <sub>H</sub>	.5*L*p <sub>G2H</sub>	.5*L*p <sub>G2H</sub> *B <sub>H</sub>
High	B <sub>L</sub>	p <sub>G2H</sub> *M+(1-p <sub>G2H</sub> )*.5*L	p <sub>G2H</sub> *M*B <sub>H</sub> + (1-p <sub>G2H</sub> )*.5*L*B <sub>L</sub>
High	B <sub>H</sub>	p <sub>G2H</sub> *.5*L + (1-p <sub>G2H</sub> )*(L-M)	[p <sub>G2H</sub> *.5*L + (1-p <sub>G2H</sub> )*(L-M)]*B <sub>H</sub>

Equation (12) shows the threshold at high load.

$$T_L = M B_H / [B_H - 1/2 p_{G2H} B_H - 1/2(1-p_{G2H}) B_L] \quad (12)$$

Fig. 3 illustrates how the load threshold varies with probability of generator 2 bidding high for high and low load and the two auction types. In a discriminatory price auction with high load, the load threshold for generator 1 to bid high is flat for low probability, and increases slightly when generator 2 has a high probability of bidding high. In a uniform price auction, the threshold has a similar shape but is higher. When load is low, the threshold in either auction depends only on the probability of generator 2 bidding high. Below this threshold, generator 1 has higher revenues when it bids low than when it bids high and is not dispatched. Above this probability generator 1 sees higher revenues bidding high and splitting the load with generator 2 than bidding the competitive price. The discriminatory price auction has a higher probability threshold than the uniform price auction.

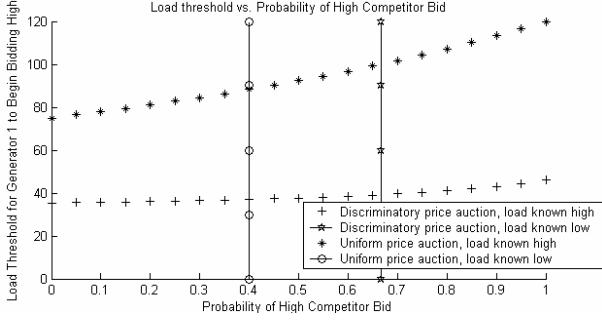


Fig. 3. Load thresholds vs. probability of high competitor bid for certain load level

#### D. Uncertain Load and Unknown Competitor Behavior

Table VI shows the expected dispatch and revenue for generator 1 using the probabilities  $p_{LH}$  and  $p_{G2H}$  for both auction types. The load threshold for generator 1 to increase its bid in a discriminatory auction is shown in (13). The load threshold for generator 1 to increase its bid for a uniform price auction is shown in (14).

Fig. 4 displays the variation of the load threshold with probability of high load for different probabilities of generator 2 bidding high in a discriminatory price auction. When probability of high load is low, the threshold is very dependent on the probability of generator 2 bidding high, and as probability of high load increases the thresholds are close together for the different generator 2 bidding probabilities. As the probability of generator 2 bidding at the non-competitive price increases, the load threshold decreases.

Fig. 5 shows the variation of load threshold with probability of high generator 2 bid for different probabilities of high load in a discriminatory auction. When there is 0 probability of high load the load threshold

$$T_L = \frac{p_{LH} M [p_{G2H} B_L + (1 - p_{G2H}) B_H]}{B_H p_{LH} (1 - p_{G2H}) + \frac{1}{2} B_H p_{G2H} - B_L p_{G2H} (1 - p_{LH}) - \frac{1}{2} B_L (1 - p_{G2H})} \quad (13)$$

$$T_L = \frac{p_{LH} B_H M}{B_H (\frac{1}{2} p_{G2H} + p_{LH} - p_{G2H} p_{LH}) - B_L [p_{G2H} (1 - p_{LH}) + \frac{1}{2} (1 - p_{G2H})]} \quad (14)$$

is zero. For any other probabilities of high load the load threshold decreases with increasing probability of generator 2 bidding high. The load threshold decreases steeply for low probabilities of high load, and decreases more gradually for higher probabilities of high load.

Table VI  
Generator 1 outcomes for uncertain load, uncertain generator 2 bids, discriminatory price auction

Auction	Gen.1 Bid	Gen. 1 Expected Dispatch	Gen. 1 Expected Revenue
Disc.	$B_L$	$p_{LH} * p_{G2H} * M + p_{G2H} (1 - p_{LH}) L + (1 - p_{G2H}) L * .5$	$[p_{LH} * p_{G2H} * M + p_{G2H} (1 - p_{LH}) L + (1 - p_{G2H}) L * .5] * B_L$
Disc.	$B_H$	$p_{LH} (1 - p_{G2H}) (L - M) + p_{G2H} * .5 * L$	$[p_{LH} (1 - p_{G2H}) (L - M) + p_{G2H} * .5 * L] * B_H$
Uniform	$B_L$	$p_{LH} * p_{G2H} * M + p_{G2H} (1 - p_{LH}) L + (1 - p_{G2H}) L * .5$	$p_{LH} * p_{G2H} * M * B_H + [p_{G2H} (1 - p_{LH}) L + (1 - p_{G2H}) L * .5] * B_L$
Uniform	$B_H$	$p_{LH} (1 - p_{G2H}) (L - M) + p_{G2H} * .5 * L$	$[p_{LH} (1 - p_{G2H}) (L - M) + p_{G2H} * .5 * L] * B_H$

Fig. 6 shows the load threshold variation with probability of high load in a uniform price auction for various probabilities of high competitor bid. The curves are similar to those in Fig. 4 for a discriminatory price auction, but intersect further to the left and at a higher load level. The load threshold in a uniform price auction is higher at high load than for a discriminatory price auction, since a generator can benefit from competitors' high bids when it bids low, is fully dispatched, and receives the higher price as payment.

Fig. 7 shows the load threshold varying with probability of high generator 2 bid at different probabilities of high load. These curves are similar to those in Fig. 5, but intersect at a higher load level and lower probability of generator 2 bidding high.

#### IV. EXTENSION TO LARGER SYSTEMS

To apply this analysis to a larger, more realistic power system, we would use these steps: determine a group of generators who are likely to collude, calculate the residual demand for the group, and then determine load thresholds for each generator in the group in relation to the probabilities of load level and of each of the other generators bidding non-competitively. Tacit collusion may occur in larger groups when several generators are owned by one holding company, or in situations where multiple generators withhold output to constrain a line, while others benefit from the ability to be dispatched at higher bids. Large groups of generators are not likely to successfully exercise market power through tacit collusion – the costs of coordinating the collusive group grow with the size of the group [8].

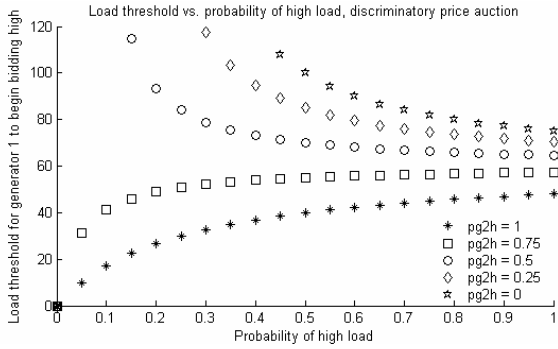


Fig. 4. Discriminatory price auction load threshold vs. probability of high load for different probabilities of high competitor bid

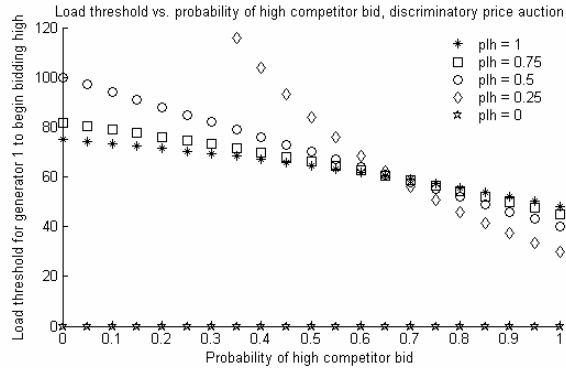


Fig. 5. Discriminatory price auction load threshold vs. probability of high competitor bid for different probabilities of high load

## V. CONCLUSIONS

In this paper, we have studied the criteria a generator in a two generator power system can use to decide to bid the competitive low price or the non-competitive high price. This is effectively that generator's decision whether to implicitly collude with the other generator. We compare a discriminatory price and a uniform price auction. The load threshold for bidding high generally decreases with increasing likelihood of high load and/or of high competitor bid. The threshold in a uniform price auction with high load or high competitor bid is higher than that for a discriminatory auction. Low competitor bid increases the threshold level relative to that for high competitor bid in a simultaneous discriminatory price auction, but decreases it in a uniform price auction. This suggests that uniform price auctions are less susceptible to gaming than discriminatory price auctions – both generators benefit from bidding above the competitive price only at the highest load levels, while in a discriminatory price auction both benefit at lower load levels.

We plan to extend this analysis to multi-period auctions and larger power systems as future work. We also will examine more subtle market power situations, such as when one of two jointly owned generators at different buses lowers its bid in order to create congestion and benefit the other generator.

## VI. ACKNOWLEDGEMENT

The authors thank David Watts for his helpful comments.

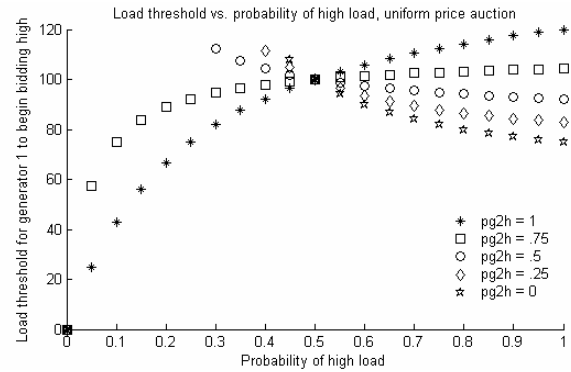


Fig. 6. Uniform price auction load threshold vs. probability of high load for different probabilities of high competitor bid

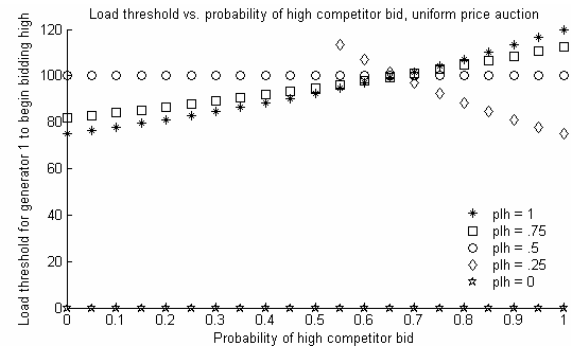


Fig. 7. Uniform price auction load threshold vs. probability of high competitor bid for different probabilities of high load

## VII. REFERENCES

- [1] R. Rajaraman and F. Alvarado, "(Dis)Proving Market Power", Laurits R. Christensen Associates, Inc., September 5, 2003 [Online]. Available: [http://www.ksg.harvard.edu/hepg/Papers/Rajaraman.mkt\\_power.Sept\\_03.pdf](http://www.ksg.harvard.edu/hepg/Papers/Rajaraman.mkt_power.Sept_03.pdf)
- [2] R. Rajaraman and F. Alvarado, "Optimal Bidding Strategy in Electricity Markets Under Uncertain Energy and Reserve Prices", Power Systems Engineering Research Center, PSERC Publication 03-05, April 2003 [Online]. Available: [http://www.pserc.wisc.edu/cgi-pserc/getbig/publicatio/reports/2003report/alvarado\\_bidding\\_report\\_0411.pdf](http://www.pserc.wisc.edu/cgi-pserc/getbig/publicatio/reports/2003report/alvarado_bidding_report_0411.pdf)
- [3] S. de la Torre, J. M. Arroyo, A.J. Conejo, J. Contreras, "Price Maker Self-Scheduling in a Pool-Based Electricity Market: A Mixed-Integer LP Approach", *IEEE Transactions on Power Systems*, Vol. 17 No. 4, November 2002.
- [4] E. J. Green and R. H. Porter. "Noncooperative Collusion Under Imperfect Price Information", *Econometrica*, Vol. 52 No. 1, pp. 87-100, Jan. 1984.
- [5] X. Guan, Y. C. Ho, D. L. Pepyne. "Gaming and Price Spikes in Electric Power Markets", *IEEE Transactions on Power Systems*, Vol. 16, No. 3, pp. 402-408, Aug. 2001.
- [6] P. Correia, T. Overbye, I. Hiskens. "Supergames in Electricity Markets: Beyond the Nash Equilibrium Concept", *Proceedings of the 14th Power Systems Computation Conference*, Sevilla, Spain, 24-28 June 2002, Session 07, Paper 4.
- [7] B. C. Lesieutre, R. J. Thomas, and T. D. Mount. "Identification of Load Pockets and Market Power in Electric Power Systems", *Proceedings of the IEEE-PES Summer Meeting*, July 2003, Toronto, Ontario, Canada.
- [8] R. Selten. "A Simple Model of Imperfect Competition, where 4 are Few and 6 are Many", *International Journal of Game Theory*, Vol. 3, Issue 3, 1973, pp. 141-201.