

# PLANNING OF RECONFIGURABLE POWER SYSTEMS

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## ABSTRACT

The objectives of this project are to develop (a) theory and method for planning hybrid (discrete, continuous) power system controllers, and (b) economic systems, applicable to power system energy markets, for incentivizing, recovering, and allocating costs of design and installation of such controllers. The paper reports on first year efforts which include (a) application of discrete-event system theory to the reactive power planning problem, illustrated on a nine bus test system, and (b) development of cost allocation rules based on analysis of a cooperative game formulation using locational marginal pricing, illustrated on a 3 bus test system. In addition, we report on our efforts to develop a course for seniors in electrical engineering and in economics on electricity markets, system control, and planning.

## KEY WORDS

Power system planning, reconfiguration, hybrid control, discrete event system, cooperative games, locational marginal prices.

## 1. INTRODUCTION

Future reliability levels of the electric transmission system require proper long-term planning to strengthen and expand transmission capability to accommodate expected transmission usage from normal load growth and increased long-distance power transactions. There are three basic options for strengthening and expanding transmission: (1) build new transmission lines, (2) build new generation at strategic locations, and (3) introduce additional control capability. Although all of these will continue to exist as options, options (1) and (2) have and will continue to become less and less viable. As a result, there is significantly increased potential for application of additional power system control in order to strengthen and expand transmission in the face of growing transmission usage. Incentives for doing so are: there is little or no right-of-way, and relative to building new transmission or generation facilities, capital investment is less.

There are 3 types of control technologies that exist today and will continue to be available to power system control engineers: generation controls, power-electronic based transmission controllers, and system protection schemes (SPS). Of these, the first two exert continuous feedback control action; the third exerts discrete open-loop control action. Thus, power system control is hybrid [1, 2] in that it consists of continuous and discrete control. Since power systems are already hybrid, and since good solutions may also be hybrid, assessment of control alternatives for expanding transmission must include procedures for gauging cost and effectiveness of hybrid control schemes.

Design of power system control today is done using a combination of linear system theory and time domain simulation. Continuous controllers are located and tuned using linearized methods and checked for robustness using time domain simulation of the nonlinear model. Discrete controllers are designed based on repeated time domain simulation. The only coordination between continuous and

discrete control design occurs via time domain simulation as a check on the proposed design. Such procedures require tedious trial and error analysis of many different operating conditions, and they result in acceptable control design for the conditions and disturbances simulated; but engineers do not know whether they are optimal or even among the better designs, and whether they are robust with respect to conditions and disturbances not simulated.

Our objective is to develop a unified approach to planning of hybrid controls for reconfiguring power systems. (a) We assume no new transmission equipment (lines, transformers) is installed and generation expansion occurs only at existing generation facilities. This assumption represents the extreme form of the industry trend of relying on control to strengthen/expand transmission capability without building new transmission or strategically siting new generation. (b) We consider design solutions *for each disturbance* based on control, where options are limited to SPS (discrete control) or FACTS-type devices (continuous control), but not both. This identifies problems for which SPS and FACTS are competing alternatives, providing the ability to compare the approaches for effectiveness and cost. (c) The final control design coordinates use of both continuous and discrete controllers. (d) The control design is driven by controller effectiveness, controller cost, the contingency set, and uncertainty in loading conditions/generation availability. (e) The design is performed relative to 2 sets of severe, but credible contingencies. One set is comprised of NERC class-C contingencies; the other set of NERC class-D contingencies. This enables comparison of control cost for maintaining today's system strength against control costs for increasing system strength to withstand the next higher event class. This difference, then, can be considered as a rough cost-estimate associated with a first-level strengthening of power systems to withstand class-D contingencies. (f) We identify organizational issues within the industry associated with proper and adequate *cost recovery* for control design,

installation, operation, and maintenance through design of appropriate procedures, an issue that is complicated by the difficulty in identifying the extent that different entities benefit from the control. (g) We are designing a course for undergraduate seniors that will impart fundamental skills and knowledge related to planning, control, and markets.

## 2. RESEARCH SUMMARY

Research during the past year has involved control design development, as described in Section 2.1, and economic analysis, as described in Section 2.2.

### 2.1 Control Design

Our proposed hybrid control planning approach requires 4 basic steps: (i) contingency selection, (ii) development of generation/load growth futures, (iii) identification of control policies for discrete-action controller design and continuous controller design, and (iv) development of the control plan. We have articulated the basis for these steps in [3, 4]. We focused here on implementing step (iii).

The fundamental problem to be solved in step (iii) is, for a specified contingency and corresponding set of conditions resulting in violation of class-C performance criteria (no uncontrolled loss of load), identify effective and economic controls such that the only interrupted load resulting from the disturbance is planned and controlled. We assume that uncontrolled load interruption occurs for system out-of-step conditions (brought on by machine groups losing synchronism with one another over a weak tie, often characterized by large power swings and interarea oscillations) or voltage instability (fast or slow, brought on by insufficient reactive resources in one or more network regions). Each disturbance is mitigated using either the minimum cost selection of discrete actions *or* the minimum cost selection of continuous controllers that eliminate uncontrolled load loss. We do not use combinations of discrete and continuous controllers for a single disturbance in order to enable comparing cost and effectiveness of the 2 approaches. This does not preclude having both types of solutions in the final control plan because the final control plan will be developed based on composition of control policies for all disturbances.

We have begun development of the approach by restricting the type of control to switchable shunt or series capacitors, leaving for later the consideration of continuous control design. In addition, we will not consider controlled load interruption in the planning stage since its every actuation incurs a significant cost. The problem then becomes to (a) choose the nodes (buses) or links (branches) of the network in which to place the capacitors, (b) determine amount of capacitance at each selected location, and (c) determine switching sequence. We use a two-part strategy where (a) and (b) are solved first based on steady-state voltage security (with requirements on voltage stability margin and voltage magnitude), and (c) is solved second based on transient analysis. The underlying paradigm for the

design is the hybrid automaton, as illustrated in Fig. 1, where each node represents a discrete configuration of switches, and the recovery node R is reached from origin node O through several paths. The optimal path is O-3-7-8-R. Node R2 represents another possible but higher cost recovery node. This approach has been successfully applied in the design of control strategies for autonomous vehicles [5,6].

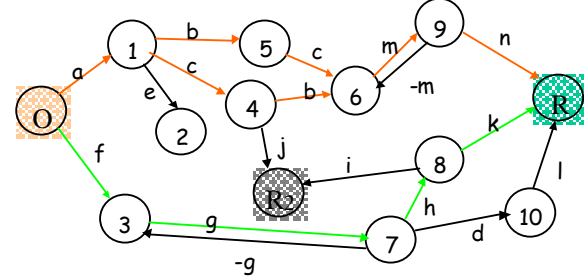


Fig. 1: Hybrid automaton

#### 2.1.1 Location

Here, we assume any equilibrium associated with a node is in the domain of attraction of the equilibrium represented by any other node that is one switch away, that is, the system remains stable for any single switching action. The implication of this assumption is that the location and amount of switches may be determined based only on steady-state (power flow) analysis. The performance requirement under this type of analysis is that the system maximum loading (loadability) in the post-contingency state must exceed system loading by a defined percentage of that loading; 15% is commonly used. An upper bound for loadability is the loading for which the system Jacobian matrix becomes singular, and continuation methods [7, 8] can efficiently locate this point. Loadability may also be defined in terms of circuit loadings, but this is unnecessary given that the only controls considered at this point, capacitive switches, do not significantly influence circuit loadings. The problem that we desire to solve is similar to the reactive power planning problem attended to in the literature [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. We formulate this problem as follows:

$$\min J(\bar{a}) = F_c + F_s - \lambda \sum_k L_k + \mu \sum_k F_k + \gamma F_{operating} \quad (1)$$

Subject to:

$$0.95 \leq V_i \leq 1.05 \quad (2)$$

$$L_k \geq 15\% \quad (3)$$

$$P_i - P_{shed\_i} - V_i^2 \sum_{j \in N(i)} (G_{sj} + G_{ij}) + \quad (4)$$

$$V_i \sum_{j \in N(i)} V_j [G_{ij} \cos \theta_{ij} + B_{ij}(a_{ij}, B_{series\_ij}) \sin \theta_{ij}] = 0$$

$$Q_i - Q_{shed\_i} + V_i^2 \sum_{j \in N(i)} [B_{sj} + B_{ij}(a_{ij}, B_{series\_ij})] + \quad (5)$$

$$(B_{si} + a_i B_{shunt\_i}) V_i^2$$

$$-V_i \sum_{j \in N(i)} V_j [B_{ij}(a_{ij}, B_{series\_ij}) \cos \theta_{ij} - G_{ij} \sin \theta_{ij}] = 0$$

$$0 \leq B_{shunt\_i} \leq B_{max\_i} \quad (6)$$

$$0 \leq B_{series\_ij} \leq B_{max\_ij} \quad (7)$$

where  $J$  is the objective function,  $F_C$  is the cost of shunt capacitors given by

$$F_c = \sum_i (c_{fix\_i} + c_{var\_i} B_{max\_i}) a_i \quad (8)$$

$F_s$  is the cost of series capacitors given by

$$F_s = \sum_{ij} (c_{fix\_ij} + c_{var\_ij} B_{max\_ij}) a_{ij} \quad (9)$$

$a_i$  and  $a_{ij}$  are selector functions given by

$$a_i = \begin{cases} 1 & \text{site } i \text{ is selected for shunt capacitors expansion} \\ 0 & \text{otherwise} \end{cases}$$

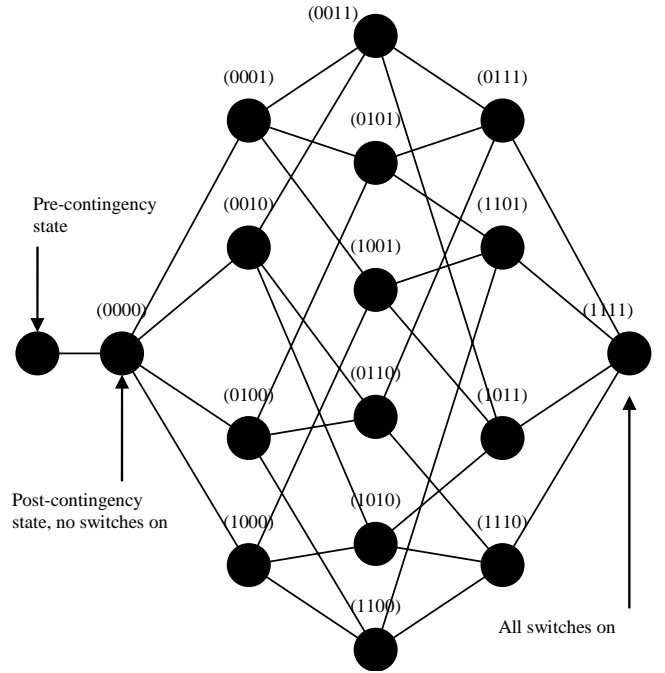
$$a_{ij} = \begin{cases} 1 & \text{site } ij \text{ is selected for series capacitors expansion} \\ 0 & \text{otherwise} \end{cases}$$

$\bar{a}$  is the collection of all  $a_i$ 's and  $a_{ij}$ 's,  $c_{fix\_i}$  is the fixed cost for shunt capacitors,  $c_{var\_i}$  is the variable cost for shunt capacitors,  $B_{max\_i}$  is the maximum capacity for shunt capacitors,  $c_{fix\_ij}$  is the fixed cost for series capacitors,  $c_{var\_ij}$  is the variable cost for series capacitors,  $B_{max\_ij}$  is the maximum capacity for series capacitors,  $\lambda$ ,  $\mu$ ,  $\gamma$  are weighting factors,  $L_k$  is the stability margin for contingency  $k$ ,  $F_k$  is controlled load shedding cost for contingency  $k$ ,  $F_{operating}$  is the operating cost for the base case such as the active power losses in an electric power network,  $V_i$  is the bus  $i$  voltage amplitude,  $\theta_{ij}$  is the voltage angle difference between buses  $i$  and bus  $j$ ,  $V_j$  is the bus  $j$  voltage amplitude,  $N$  is the number of system buses,  $N(i)$  is the set of buses directly connected to bus  $i$  ( $i=1, \dots, N$ ),  $P_i$  is the active power injected by any generator or load connected to bus  $i$ ,  $P_{shed\_i}$  is the active load shedding at bus  $i$ ,  $Q_i$  is the reactive power injected by any generator or load connected to bus  $i$ ,  $Q_{shed\_i}$  is the reactive load shedding at bus  $i$ ,  $G_{sij}$  is circuit shunt conductance,  $B_{sij}$  is circuit shunt susceptance,  $G_{ij}$  is the circuit series conductance,  $B_{ij}$  is circuit series susceptance and is a function of the susceptance of a series capacitor,  $B_{si}$  is susceptance of all shunt elements present at bus  $i$ ,  $B_{shunt\_i}$  is susceptance of shunt capacitors, and  $B_{series\_ij}$  is susceptance of series capacitors.

This is a mixed integer nonlinear programming problem, with  $\bar{a}$  being the collection of discrete variables and  $B_{max}$  being the collection of continuous variables. We propose an algorithm for solving this problem under the assumption that  $B_{max}$  is fixed; this converts the problem to an integer programming problem where the decision variables are locations for shunt and series capacitor switches. The algorithm assumes the existence of a list of contingencies and a list of candidate control locations. The former is identified based on fast screening as in [22], and the latter is identified based on the sum of sensitivities of voltage stability margin with respect to the susceptance of shunt or series capacitor evaluated at the voltage collapse point under the specified contingencies. The algorithm begins at a node and searches from that node in a prescribed direction, either

backward from the node corresponding to all switches closed (the strongest node) or forward from the node corresponding to all switches open (the weakest node). We give only the backward algorithm here since the forward algorithm is similar.

**Backward Algorithm:** Consider the graph (automaton) where each node represents a configuration of discrete switches, and two nodes are connected if and only if they are different in one switch configuration. The graph has  $2^N$  nodes where  $N$  is the number of candidate switches. We pictorially conceive of this graph as consisting of layered groups of nodes, where each successive layer has one more switch "on" than the layer before it, and the  $n^{\text{th}}$  layer (where  $n=0, \dots, N$ ) consists of a number of nodes equal to  $N!/n!(N-n)!$ . Fig. 2 illustrates for the case of 4 switches. The algorithm has 4 steps.



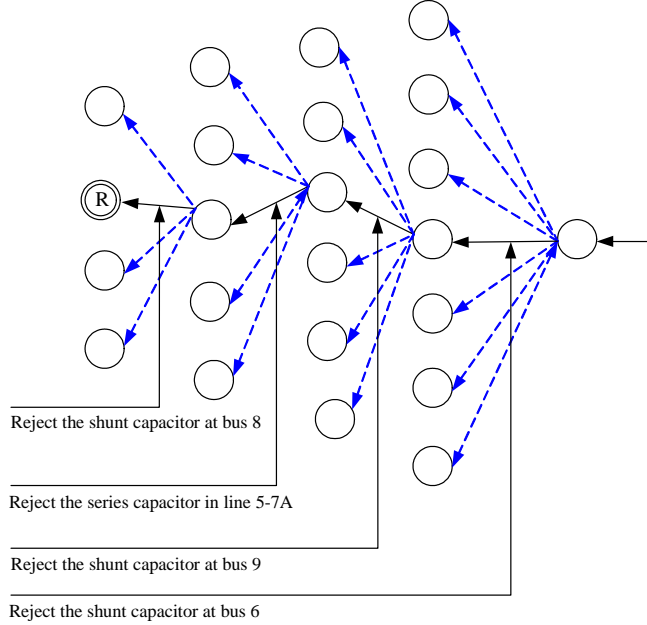
**Fig. 2: Automaton for 4-switch problem**

- 1) Choose the node with all candidate switches on.
- 2) For the chosen node, check if safety (2)-(3) and stability (4)-(5) are satisfied for all contingencies on the list. If not, then stop, the solution corresponds to the previous node (if there is a previous node, else no solution exists).
- 3) For the chosen node, eliminate (open) the switch that provides the largest (most positive) contribution to objective function  $J$  (therefore it's elimination contributes most to minimizing  $J$ ). We denote this as switch  $i^*$ :

$$i^* = \arg \left\{ \max_i \frac{\partial J}{\partial B_i} \right\} = \arg \left\{ \max_{i \in S} \left( c_{var\_i} - \lambda \sum_k \frac{\partial L_k}{\partial B_i} \right) \right\} \quad (10)$$

where  $S = \{\text{set of closed switch for the chosen node}\}$ , and  $c_{var\_i}$  is the corresponding variable cost of new shunt or series capacitor control. Inspection of (10) indicates that, among all closed switches,  $i^*$  will have large cost and/or small effect on post-contingency loadabilities  $L_k$ .





**Fig. 4: Automaton for the backward algorithm**

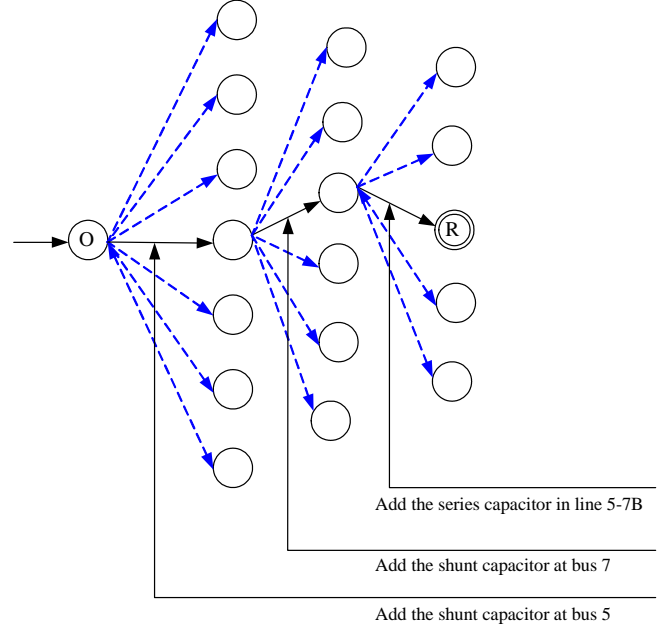
Table II shows that, with only 2 controls, the loading margin is unacceptable at 8.43%. Therefore the final solution must utilize 3 controls. These controls are a shunt capacitor at bus 5, a series capacitor in line 5-7B, and a shunt capacitor at bus 7. The location of these controls are intuitively pleasing given that, under the contingency, Load A, the largest load, must be fed radially by a long transmission line, a typical voltage collapse situation.

**Table III: Steps taken in forward algorithm**

No		no cntrl	1 cntrl add # 1	2 cntrls add # 2	3 cntrls add # 4
1	Sensitivity of shunt capacitor at bus 5	5.5307	3.7038	3.6491	
2	Sensitivity of shunt capacitor at bus 7	0.7123	0.7571	0.754	
3	Sensitivity of series capacitor in line 5-7A	0.6539	0.6728	0.6686	
4	Sensitivity of series capacitor in line 5-7B	0.6539	0.6728	0.6686	
5	Sensitivity of shunt capacitor at bus 8	0.4961	0.524	0.5248	
6	Sensitivity of shunt capacitor at bus 9	0.1557	0.1622	0.1636	
7	Sensitivity of shunt capacitor at bus 6	0.0611	0.0565	0.0588	
	loadability (MW)	435	445	450	480
	stability margin (%)	4.82	7.23	8.43	15.66

The solution obtained from the forward algorithm is the same as that obtained using the backward algorithm: shunt capacitors at buses 5 and 7 and a series capacitor in branch 5-7B. This is guaranteed to occur if switch sensitivities do not change as the switching configuration is changed, i.e., if the system is linear. We know power systems are nonlinear, and the changing sensitivities across the columns for any given row of Tables II or III confirm this. However, we also observe from Tables II and III that the sensitivities do not change much, thus giving rise to the agreement between the algorithms.

Table III summarizes the steps taken by the forward algorithm in terms of switching sensitivities, where we have again assumed  $c_{var,j}=0$ . Here, at each step, the switch with the maximum loadability sensitivity is added (closed), as indicated in each column by the numerical value within the box. Figure 5 shows the corresponding search via the automaton.



**Fig. 5: Automaton for the forward algorithm**

For large systems, however, we do not expect the two algorithms to identify the same solution. And of course, neither algorithm is guaranteed to identify the optimal solution. But both algorithms will generate *good* solutions. We may also develop modifications of these algorithms where, for example, we choose the next switch based on assessment of sensitivity sums for the next two switches, or a second-ranked switch may be chosen initially. Such modifications will lead to different, but good solutions. Ultimately, we are interested in filling in the automaton with a significant number of nodes corresponding to good solutions. This will not only facilitate good reactive power planning designs, but it will also lead to a tool that is useful for studying proper sequencing in using the controls to respond to contingencies. We have just begun to consider this issue; further discussion is provided in Section 2.1.3.

The optimization problem of (1)-(9) could be solved by a traditional integer programming method, e.g., the branch-and-bound algorithm. However, our algorithm has complexity linear in the number of switches  $N$ , whereas branch and bound has complexity  $2^N$ . The gain in complexity comes at the expense of optimality: branch-and-bound finds a optimal solution, whereas our algorithm finds a solution that is set-wise minimal (any subset of switches won't work). There can exist more than one minimal set solution, and to compute an optimal solution, one will have

to examine all of them which we avoid for the sake of complexity gain.

### 2.1.2 Amount

In the previously described examples, we allowed only one switch per control location, and therefore each potential control location was represented by a single node in the automaton. However, we may model multiple switches at a single control location in an effort to identify amount of susceptance, and the same backward and forward algorithms apply, although the graph over which we search is enlarged.

Alternatively, we may partition the solution between integer and continuous, solving them separately. For example, we may apply the above algorithms to identify locations, and then develop a continuous-valued optimization to identify amount. Or we could embed a continuous-valued optimization within the backward algorithm, for example, at step 2. We will continue to explore these possibilities in the near future.

### 2.1.3 Switching sequence

A recovery node  $R$  is identified in the above backward/forward search on an automaton representing discrete configuration of switches. Define the discrete operation mode of a power system just previous to a discrete disturbance as the original node of an automaton which is represented by node  $O$  in Figure 4.

An automaton is associated with each contingency  $r$ . The nodes of the automaton correspond to the steady state equilibria,  $Ze(r)$ . The edges, one-step discrete control actions, called maneuvers, consisting of a set of switches occurring simultaneously, are denoted by  $E(r)$ . A subset of these edges, denoted by  $Es(r)$ , is between a pair of nodes that are stable (the parent node of the edge is in the region of stability of the child node of the edge). A further subset of  $Es(r)$  is also safe, meaning the transients in continuous variables caused by the one-step switching associated with such an edge lie within the safety limits of voltage magnitudes and power flows. Denote this set of safe and stable edges by  $Ess(r)$ .

The steady-state conditions of (1)-(5) are necessary but not sufficient for acceptability of the identified controls under the given contingencies and conditions. We must also be able to identify at least one *sequence* of stable and safe switching from the pre-contingency state through the contingency to an acceptable equilibrium. In terms of the automaton, this means we must identify a sequence of stable and safe edges from the original node through the contingency switching to a recovery node. The safety of a stable edge can be checked via time-domain simulation and using formal verification methods for hybrid systems [27, 28] when they apply.

We assume that a path of safe and stable edges from original to recovery nodes exists. If the automaton is rich enough, there may exist several such paths. These will correspond to different feasible switching sequences. It is assumed there is cost associated with any edge. For example,

part of the cost could be given by the time to hop from one node to the next. In searching for these paths, we could first search for all paths connecting the origin and the recovery nodes, and then order them according to their cost. Another approach is to do both operations at the same time using dynamic programming [29]. The search algorithm starts from the recovery node. There may be paths with smaller cost requiring more steps to reach the recovery node. In such cases, the algorithm should continue searching for optimal policies made of more than  $N$  maneuvers. It is possible to compute the length of the longest (non cyclic) path. Such length will cap the number of steps needed for the algorithm to terminate. The result will be a list of optimal paths connecting origin and recovery nodes indexed by the length or the number of maneuvers that they require, from which the absolute optimal path can be easily picked.

The search restricted to the safe and stable subgraph may be too restrictive in the sense that the origin and recovery nodes may not be connected by a sequence of safe and stable edges. One can allow for transient nodes representing unstable nodes and unsafe transitions. Clearly, moderately unstable nodes are preferable to highly unstable ones essentially because of their longer time constants, which allow the control system with more time to react. The search for the optimal path should also impose the constraint to spend minimum time at the unstable transient nodes. Also when one or more unstable nodes are chained between two stable ones, one should optimize the sequence of events (switchings) in the chain separately, and then replace the whole chain with a more complex maneuver connecting them to stable end nodes of the chain. Although the optimization of events in a chain of unstable nodes is a mixed integer-programming problem [30,31], its complexity should be manageable especially if the number of unstable nodes is small, moreover, sub-optimal or approximate solutions could be more easily computed using neuro-dynamic programming or other aggregation methods [32].

## **2.2 Economic analysis**

As part of our economic analysis of transmission investment, we applied the insights of cooperative game theory to derive reasonable cost allocation rules to the problem of allocating the costs of a transmission enhancing investment, among the various participants in the power network. Given a transmission network where some of the lines are congested one may consider an investment aimed at alleviating congestion. As mentioned in the introduction, this investment may take the form of new transmission lines, additional control capabilities, etc. It is clear that from the social point of view, the investment will be worthwhile if the social benefits exceed its cost. On the other hand, it is not clear how that cost should be allocated among the agents should the investment be carried out. Although the social benefits of the investment may exceed the cost, each one of the individual agents' benefits may not be enough to cover the whole cost. Further, it is often the case that some of the agents, be them consumers or generators at some location,

are financially hurt by the investment. This may be the case when, due to the investment, the price at a given location increases, hurting the consumers there, or when the price at some location drops, thereby hurting the generators there. Moreover, the transmission expansion will certainly hurt the owners of the transmission network because the investment will bring about a decrease in congestion.

We chose to model an electricity cost *allocation problem* as a pair  $(C, \pi)$ , such that  $\pi = (\pi_1, \pi_2, \dots, \pi_n)$ ,

and  $0 \leq C \leq \sum_{i=1}^n \pi_i$ , where  $C$  is the cost of the investment

and  $\pi_i$  is agent  $i$ 's benefit from the investment. Note that the  $\pi_i$  can be negative. An *allocation* in  $(C, \pi)$  is a vector  $x = (x_1, x_2, \dots, x_n) \leq (\pi_1, \pi_2, \dots, \pi_n)$ , such that

$\sum_{i=1}^n x_i = C$  and  $x_i \geq 0, \forall i = 1, 2, \dots, n$ . The value  $x_i$  is

interpreted as agent  $i$ 's contribution to the financing of the cost  $C$ . The requirement  $x_i \leq \pi_i$  means that if agent  $i$  benefits from the investment, his contribution should be no greater than his benefit; if he suffers from the investment, he has to be compensated for his loss. Given an allocation  $x$ , the corresponding vector of *net benefits* is  $\pi - x$ . Our task is to find reasonable allocations. Specifically, we are interested in reasonable allocation rules that associate an allocation to each electricity cost allocation problem.

An *allocation rule* is a function  $f$  that maps each cost allocation problem  $(C, \pi)$  to one of its allocations. For example, the *Head tax rule* is the rule that maps each allocation problem  $(C, \pi)$  to the allocation  $H(C, \pi) = x$  where

$x_i = \min\{\lambda, \pi_i\}$  and  $\lambda \geq 0$  is chosen such that  $\sum_{i=1}^n x_i = C$ . This

allocation rule dictates that those who suffer from the investment should be fully compensated ( $x_i = \pi_i$ ), and those who benefit from the investment should contribute equal amounts to its financing, as long as nobody pays more than his benefit, in which case it pays his full benefit. For the purpose of finding allocation rules, we translate the electricity cost allocation problem into a cooperative game with transferable utility and apply well-known game-theoretic solution concepts to the game. Related work using a cooperative game theoretic approach are [33, 34, 35, 36].

A *cooperative game* consists of

1. a set  $N$  (the players), and
2. a function  $v: 2^N \rightarrow R$ , such that  $v(\Phi) = 0$ .  
( $2^N = \{S: S \subset N\}$ ).

A subset  $S$  of  $N$  is called a *coalition*;  $v(S)$  is called the worth of the coalition  $S$ . Intuitively,  $v(S)$  represents the total payoff that the coalition  $S$  can get by itself, without the help of other players. For the electricity scenario, we assume that each coalition is allowed to carry out the investment project as long as it pays the investment cost and compensates all the agents that suffer from the project. The idea is that each sufferer has the veto power, unless he is fully compensated for his loss.

Based on this idea, we associate each cost allocation problem  $(C, \pi)$  with the game  $(N, v)$ , where  $N = \{1, \dots, n\}$  is the set of players, and

$$v(S) = \max\{0, \sum_{i \in S} \pi_i - (C - \sum_{i \notin S} \min\{0, \pi_i\})\},$$

$\forall S \subseteq N$ .

That is, a coalition  $S$  can always do nothing, in which case it gets 0, or undertake the investment after compensating the agents outside the coalition for the losses they might suffer.

It will be recalled that the set of imputations of a cooperative game  $(N, v)$  is the set of vectors  $y$ , such that  $y \geq 0$  and  $\sum_{i \in N} y_i = v(N)$ . Since in our

game  $v(N) = \max\{0, \sum_{i \in N} \pi_i - C\}$ , the set of imputations

is the set of vectors  $y \geq 0$ , such

that  $\sum_{i \in N} y_i = \max\{0, \sum_{i \in N} \pi_i - C\}$ . Every imputation  $y$  of

the game  $(N, v)$  induces a cost allocation for the electricity cost allocation problem  $(C, \pi)$  as follows:  $x = \pi - y$ .

Endowed with the definitions above, we can now apply well-known game theoretic solutions concepts to the game induced by the cost allocation problem and obtain the corresponding cost allocation. Two of the most prominent solution concepts for cooperative games are the Shapley value and the Core. For the record, we provide here their definitions.

The *Shapley value* of a game  $(N, v)$  is given by a vector  $\varphi(v) = (x_1, \dots, x_n)$

where  $x_i = (1/n!) \sum_R [v(S_i \cup \{i\}) - v(S_i)]$ , where  $R$

runs over all the  $n!$  different orders on  $N$ , and  $S_i$  is the set of players preceding  $i$  in the order  $R$ .

The *Core* of the game  $(N, v)$  is the set of all imputations  $y$ , such that  $v(S) \leq \sum_{i \in S} y_i$  for all  $S \subset N$ .

The Shapley value of a game is a single payoff vector, while the core is a set of payoff vectors, which can be empty. The Shapley value may not be in the core, even when it is not empty. Intuitively the Shapley value represents a reasonable compromise, whereas the core represents a set of payoff vectors which are coalitionally stable. There is no general relationship between the two although it can be shown that for our cost allocation problems, the Shapley value always belong to the Core.

Given the simplicity of Shapley value's formula, it is not difficult to calculate its recommended cost allocations for any cost allocation problem. There is another prominent solution concept, the *nucleolus*, which unfortunately is not so easy to calculate. However, it turns out that for the class of cooperative games induced by cost allocation problems, the nucleolus is given by a relatively simple formula and

thus it can be easily calculated. This is due to the fact that cost allocation problems are intimately related to a class of problems that has been widely studied in economics. We refer to the class as Bankruptcy problems.

A *bankruptcy problem* is defined as a pair  $(E; d)$ , where  $d = (d_1, \dots, d_n)$ .

$$0 \leq d_1 \leq \dots \leq d_n \text{ and } 0 \leq E \leq d_1 + \dots + d_n.$$

$E \in R$  is the estate and  $d = (d_1, \dots, d_n) \in R^n$  denotes a vector claims or debts.

Given a bankruptcy problem  $(E; d)$ , one can define an *associated cooperative game*  $(N, w)$ , where  $N = \{1, 2, \dots, n\}$  and  $w(S) = \max\{0, E - \sum_{i \notin S} d_i\}$ . As

with electricity allocation problems, imputations to this associated game will induce allocations to the corresponding bankruptcy problem. The following observation will allow us to apply our knowledge of bankruptcy problems to the electricity cost allocation problems.

**Observation:** For each electricity cost allocation problem  $(C, \pi)$ , there is a bankruptcy problem  $(E; d)$ , such that both problems induce the same cooperative game.

That is, although electricity cost allocation problems and bankruptcy problems are different objects, they induce the same cooperative games. This allows us to take advantage of the literature on bankruptcy problems to define potentially interesting cost allocation rules. For instance, based on [37] we can calculate the allocation induced by the nucleolus on any cost allocation problem. For two-agent allocation problems  $(C, (\pi_1, \pi_2))$ , the allocation is given by

$$\begin{aligned} & \left(\frac{C}{2}, \frac{C}{2}\right), \text{ if } \pi_1, \pi_2 \geq C; \\ & \left(C - \frac{\max\{0, \pi_2\}}{2}, \frac{\max\{0, \pi_2\}}{2}\right), \text{ if } \pi_1 \geq C > \pi_2; \\ & \left(\frac{\max\{0, \pi_1\}}{2}, C - \frac{\max\{0, \pi_1\}}{2}\right), \text{ if } \pi_1 < C \leq \pi_2; \\ & \left(\frac{\pi_1 - \pi_2 + C}{2}, \frac{\pi_2 - \pi_1 + C}{2}\right), \text{ if } \pi_1, \pi_2 < C. \end{aligned}$$

For general cost allocation problems, the allocation is given by a consistent extension of the above two-agent rule.

### 3. OTHER PROGRESS

In addition to the work reported in the previous section, we have progressed in several other areas.

#### 3.1 Benchmark test systems

We proposed three benchmark test systems, each of which is intended to reflect characteristics of the Western Electric Coordinating Council (WECC) system; we referred to these three systems as Testbeds 1, 2, and 3, in increasing size and complexity [3]. Testbed 1 was to be a simplified 4-bus model, Testbed 2 a 179-bus model, and Testbed 3 a full 6000 bus planning model provided by Bonneville Power Administration. We have developed Testbed 1, but in order to test concepts and illustrate ideas and algorithms, we decided to increase the size to 9 buses. This is the model of Fig. 2. We believe the model to be a matured model for the purpose of the control design work, but we do intend to utilize this model in the economic design effort of the project as well.

#### 3.2 Interdisciplinary work

Interdisciplinary work occurs in the project between the economics side and the engineering side via student group presentations and project meetings. We expect this interaction to increase as we progress. We are just beginning to identify analytic connections between the two project thrusts. One step we will take to facilitate this is to have both groups using the model of Fig. 2. The ultimate vision for this project is the development and layout of an integrated approach for performing the engineering analysis and design together with the cost allocation rules and framework.

#### 3.3 Models

New models developed in our work include the automaton and the cooperative games, as described in Section 2.

#### 3.4 Education

Our project is now fully staffed with three funded PhD students. Haifeng Liu is an EE PhD student specializing in power systems and controls; he began study at ISU in August, 2003. Licheng Jin is an EE PhD student specializing in controls and power systems; she began study at ISU in June, 2004. Wenzhou Shang is an Economics PhD student specializing in electricity markets and game theory; she began study at ISU in August, 2002.

We have proposed and had approved a senior-level elective course, EE/Econ 458, which will be offered in Fall 2004. The title of the course is "Economic systems for electric power planning." The course will emphasize electricity market operation, power system controls technologies, planning approaches, and optimization methods. The course is open to seniors in both programs and will be taught jointly by Professors Kumar, McCalley, and Volij. The course presently has 30 on-campus enrollments. It will also be offered to off-campus students via video-streaming. One faculty from Botswana and one faculty from Nigeria will also participate in the course. We will offer this course every year for the next three years hope to see it

become a standard senior year elective for both programs thereafter.

### 3.5 Publications

We have published two conference papers [4, 38] in the past year.

### 3.6 Industry

John Paserba is a well-known industry expert in power system dynamics employed by Mitsubishi Electric, a leader in the IEEE Power Engineering Society Committee on Power System Dynamics, and the industry advisor to this project. We have communicated with him about project progress, and a progress report will be sent to him to solicit his input. We believe that not only will his participation enrich our work but it will also provide a promising means of introducing the fruits of our work to the industry.

In addition, as mentioned in Section 3.1, Bonneville Power Administration (BPA) has agreed to supply us with a 6000 bus model of the WECC system. Our contact at BPA is Bill Mittelstadt, one of BPA's principle engineers. Illustration on the WECC model, together with interaction with BPA, will provide another promising transition avenue.

## 4. FUTURE WORK

Work on control system design for the immediate future will include algorithm development and testing for capacitor amount determination (see Section 2.1.2) and switching sequence determination (see Section 2.1.3). Longer-term efforts include continuous control development and integration of discrete and continuous control plans.

We plan to axiomatically analyze the different cost allocation rules. Specifically, we want to distinguish between different rules according to the properties that they satisfy. We also plan to analyze the strategic interaction that is induced by various market mechanisms. Special attention will be given to dynamic settings with some uncertainty.

Our immediate educational efforts include preparing and delivering EE/Econ 458 for fall 2004 (see Section 3.4).

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## BIOGRAPHIES

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