

# Framework for the Design and Analysis of Congestion Revenue Rights

Minghai Liu and George Gross, *Fellow, IEEE*

**Abstract**—The capability to deal effectively with the uncertainty associated with locational marginal prices (*LMPs*) in congestion management schemes requires the development of appropriate financial tools. Congestion revenue rights (*CRR*) are hedging tools that provide the holder reimbursement of the congestion charges in the day-ahead market and thereby provide transmission service customers with price certainty. In this paper, we construct a framework for the design and analysis of the *CRR* by marrying finance theory notions with salient characteristics of electric power systems and electricity markets. The framework consists of three interconnected layers with one layer each to represent the models of the transmission network, the commodity markets and the *CRR* financial markets. The interaction between the layers is represented as information flows. The framework has sufficient scope to allow the analysis of a broad range of problems associated with ensuring price certainty for transmission services. The structural modularity of the framework provides the flexibility to analyze issues and design structures for the provision of transmission services in the competitive environment. We introduce a new notion of *CRR payoff parity* and a practical pricing scheme, which are used as the basis for the design of more liquid *CRR* markets. The application of the framework is further illustrated by the analysis of the conditions that guarantee the revenue adequacy for the *CRR* issuer.

**Index Terms** – Congestion management, congestion revenue rights, standard market design, locational marginal prices, fixed/firm/financial transmission rights, transmission congestion contracts.

## I. INTRODUCTION

THE advent of open access transmission and the spread of competitive markets in electricity have resulted in the growing prominence of transmission congestion. There is a growing realization that congestion is a major obstacle to vibrant competitive electricity markets. Various schemes from command and control to market based approaches have been proposed to manage congestion [1]-[4]. Congestion management is also at the heart of the standard market design (*SMD*) proposal of the U.S. Federal Energy Regulatory Commission (*FERC*) [4] of a uniform set of rules and designs for the U.S. electricity sector. The proposal uses locational marginal prices (*LMPs*) to identify congestion situations and to devise a

scheme for their management. Under the *SMD*, an independent entity is established to carry out the responsibilities for the operations and control of the transmission system as well as the various markets. We refer to this entity by the generic name of independent grid operator (*IGO*) to encompass various organizations such as independent system operator (*ISO*), transmission system operator (*TSO*), regional transmission organization (*RTO*) and independent transmission provider (*ITP*). At the very minimum, an integrated day-ahead market is operated by the *IGO* in which the *pool customers* buy (sell) energy from (to) the *IGO* and the *bilateral customers* – the entities that undertake bilateral transactions – obtain corresponding transmission services. The *LMPs* are determined in this market for each network node and the presence of congestion is signaled by the *LMP* differences. Congestion charges evaluated in terms of the *LMP* differences are collected by the *IGO* from the customers. Since the *LMPs* are unknown before the day-ahead market clears, such a scheme brings uncertainty to the amount of congestion charges faced by the transmission customers. In particular, *risk-averse* [5] customers may be unwilling to undertake transactions unless financial tools are available to hedge against such charges. Congestion revenue rights (*CRR*) [6]-[14] have been developed for this purpose.

*CRR* are financial tools issued by the *IGO* that provide the holder reimbursement of the congestion charges collected by the *IGO*<sup>1</sup>. Various *CRR*, including point-to-point rights such as the fixed/firm/financial transmission rights (*FTR*) [5], [6] and transmission congestion contracts (*TCC*) [14] and flow-based rights [8], have been proposed. Description of their deployment and the associated market rules has been given in [6]-[14], including a detailed market power analysis of a special class of *CRR* [9]. However, these descriptions focus mainly on the policy side of the *CRR* and many issues have still not been addressed. The growing awareness of the important role of the *CRR* in competitive electricity markets encourages more detailed discussion on this topic. Under such conditions, a framework that sets up a mathematical basis for the design and analysis of the *CRR* is of particular interest.

In this paper, we construct such a framework by marrying certain aspects of finance theory with salient characteristics of electric power systems and electricity markets. The framework consists of three interconnected layers. The *physical network layer* contains the model of the transmission network. The

---

The research was performed under the sponsorship of PSERC, CERTS and NSF.

The authors are with the Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, 1406 W. Green Street, Urbana, IL, 61801, USA (e-mail: mliu@uiuc.edu).

---

<sup>1</sup> A special class of *CRR* may be defined to have certain scheduling priorities associated with them. A detailed description and analysis of such *CRR* are given in [9]. In this paper, we focus on the basic financial features of *CRR*.

mathematical model for the day-ahead market constitutes the *commodity market layer*. The *financial market layer* represents the structure of *CRR* markets. The interaction between the layers is represented by information flows. The framework's structural modularity provides the flexibility to analyze issues and design structures for the provision of transmission services in the competitive environment. We focus our discussion on the point-to-point two-sided *CRR*. However, this framework is also applicable to the design and analysis of other types of *CRR* such as the flow-based rights [8] and point-to-point rights based on "use it or lose it" rules [9].

Due to the hedging ability of the *CRR*, the outcomes of the *CRR* markets impact, to a great extent, the customers' behavior in the commodity markets. Therefore, the liquidity of the *CRR* markets is important for competitive commodity markets. In the design of the framework, we take full consideration of this issue. We represent the *CRR* as a special financial derivative and derive the payoff parity principle and construct a practical pricing scheme. The application of the principle and the pricing scheme provides enhanced liquidity of *CRR* by establishing a workable basis for *CRR* secondary markets and a reconfiguration scheme. We also show the application of the framework to the *IGO* revenue adequacy, an important issue for the design of liquid *CRR* markets.

This paper contains seven additional sections. We devote section II to introduce the three-layer structure of the proposed framework. In sections III, IV and V, we present the mathematical description of each layer. The information flows in the framework are discussed in section VI. Applications of the framework to the analysis and design of the framework are illustrated in section VII. Section VIII provides a summary and suggestions for future work.

## II. STRUCTURE OF THE FRAMEWORK

A key requirement of the framework is to have the capability to deal with the complexity of issues comprehensively so as to address appropriately any issue concerning the transmission market design for competitive electricity markets. To meet this requirement, we design an interconnected three-layer framework structure consisting of the physical network, the commodity market and the financial market layers.

The physical network layer represents the transmission system. The relationship between the line power flows and the nodal injections are established and various network constraints are modeled. The characterization of congestion conditions is then given to complete the description of this layer.

The commodity market layer contains the model of the integrated day-ahead market. Bids/offers of the pool customers and transmission requests from the bilateral customers are represented and the *IGO* decision-making process is simulated by solving the so-called transmission scheduling problem (*TSP*). All the market outcomes including the energy sales, transmission schedules and *LMPs* are determined in the *TSP*. Congestion is managed by incorporating the network constraints in the *TSP*. The impacts of the congestion are also represented.

The models of the *CRR* and the *CRR* markets constitute the financial market layer. A novel feature of the model is the notion of the *CRR* payoff parity and a practical pricing scheme derived based on finance theory. This layer also contains a component for the issuance of *CRR*. This component need not be used once the *CRR* are issued.

The interactions between the three layers are through the information flows. These information flows are discussed in detail in section VI.

For the sake of simplicity, we define one hour as the smallest indecomposable unit of time and present the framework in terms of one specified hour. All the markets discussed in this paper are for the specified hour unless explicitly mentioned otherwise. We suppress the time notation in the paper. Appendix A provides a summary of the acronyms and the notation used in the paper. Appendix B reviews briefly the distribution factors used in the analysis. Appendix C provides the mathematical proofs for the analysis.

## III. THE PHYSICAL NETWORK LAYER

We consider a transmission network with  $N+1$  buses and  $L$  lines. We denote by  $\mathcal{N} \triangleq \{0, 1, 2, \dots, N\}$  the set of buses, with the bus 0 being the slack bus, and by  $\mathcal{L} \triangleq \{\ell_1, \ell_2, \dots, \ell_L\}$  the set of transmission lines and transformers that connect the buses in the set  $\mathcal{N}$ . We associate with each element  $\ell \in \mathcal{L}$  the ordered pair  $(i, j)$  and we write  $\ell = (i, j)$ . We adopt the convention that the direction of the flow on line  $\ell$  is from node  $i$  to node  $j$  so that  $f_\ell \geq 0$ , where  $f_\ell$  is the active power flow on line  $\ell$ . We define  $\underline{\mathbf{f}} \triangleq [f_1, f_2, \dots, f_L]^T$ . The series admittance of line  $\ell$  is  $g_\ell - jb_\ell$ . The net active power injection at node  $n \in \mathcal{N}$  is denoted by  $p_n$  and we define  $\underline{\mathbf{p}} \triangleq [p_1, p_2, \dots, p_N]^T$ .

We denote by  $\underline{\mathbf{B}}_d \triangleq \text{diag}\{b_1, b_2, \dots, b_L\}$  the  $L \times L$  diagonal branch susceptance matrix and by  $\underline{\mathbf{A}} \triangleq [\underline{\mathbf{a}}_1, \underline{\mathbf{a}}_2, \dots, \underline{\mathbf{a}}_L]^T$  the augmented branch-to-node incidence matrix with

$$\underline{\mathbf{a}}_\ell \triangleq [0 \dots 0 \overset{i}{1} 0 \dots 0 \overset{j}{-1} 0 \dots 0]^T \in \mathbb{R}^{N+1}.$$

Note that  $\underline{\mathbf{a}}_\ell$  includes an entry corresponding to the slack bus 0. Obviously, the algebraic sum of the columns of  $\underline{\mathbf{A}}$  vanishes:

$$\underline{\mathbf{A}}\mathbf{1}^{N+1} = \mathbf{0}, \quad (1)$$

where,  $\mathbf{1}^{N+1} = [1, 1, \dots, 1]^T \in \mathbb{R}^{N+1}$ . The augmented nodal susceptance matrix is

$$\underline{\mathbf{B}} \triangleq \underline{\mathbf{A}}^T \underline{\mathbf{B}}_d \underline{\mathbf{A}} \quad (2)$$

and  $\underline{\mathbf{B}}$  is singular since

$$\underline{\mathbf{B}}\mathbf{1}^{N+1} = \underline{\mathbf{A}}^T \underline{\mathbf{B}}_d \underline{\mathbf{A}}\mathbf{1}^{N+1} = \mathbf{0}. \quad (3)$$

Next, we obtain the reduced incidence matrix  $\underline{\mathbf{A}}$  from  $\underline{\mathbf{A}}$  by removing the row/column corresponding to the slack node

$$\underline{\mathbf{A}} \triangleq [\underline{\mathbf{a}}_1, \underline{\mathbf{a}}_2, \dots, \underline{\mathbf{a}}_L]^T \in \mathbb{R}^{L \times N}. \quad (4)$$

Each  $\underline{\mathbf{a}}_\ell \in \mathbb{R}^N$ ,  $\ell = 1, 2, \dots, L$  and  $\underline{\mathbf{A}}$  is full rank [15]. Analo-

gously, we partition

$$\underline{\mathbf{B}} = \begin{bmatrix} b_{00} & \underline{\mathbf{b}}_0^T \\ \underline{\mathbf{b}}_0 & \underline{\mathbf{B}} \end{bmatrix}. \quad (5)$$

The reduced nodal susceptance matrix

$$\underline{\mathbf{B}} \triangleq \underline{\mathbf{A}}^T \underline{\mathbf{B}}_d \underline{\mathbf{A}} \quad (6)$$

is nonsingular because  $\underline{\mathbf{B}}_d$  is nonsingular, as there is no line with 0 susceptance, and  $\underline{\mathbf{A}}$  is full rank.

Key characteristics of the transmission system may be described by the power flow equations and various constraints. Considering the irrelevancy of the reactive power flows to the *CRR* issues and the common use of the DC power flow model in the literature, we assume the power system to be lossless and the DC power flow conditions [15] to hold so that

$$\underline{\mathbf{p}} = \underline{\mathbf{B}}\underline{\boldsymbol{\theta}}, \quad (7)$$

where  $\underline{\boldsymbol{\theta}} \triangleq [\theta_1, \theta_2, \dots, \theta_N]^T$  is the vector of voltage angles at the network nodes. The scarcity of the transmission capability is represented by various limits under both the base case and contingency cases. For simplicity, in this paper, we only represent the active power line flow limits under the base case:

$$\underline{\mathbf{B}}_d \underline{\mathbf{A}} \underline{\boldsymbol{\theta}} \leq \underline{\mathbf{f}}^{\max}. \quad (8)$$

We call the line  $\ell$  congested whenever the corresponding inequality constraint becomes binding so that

$$b_\ell \underline{\mathbf{a}}_\ell^T \underline{\boldsymbol{\theta}} = f_\ell^{\max}. \quad (9)$$

We call the transmission system congested if there is (are) one or more congested line(s) in the network. The management of the physical congestion in a way so as to accommodate as many of the bilateral transactions and pool customers' needs is a key concern in the competitive environment.

#### IV. THE COMMODITY MARKET LAYER

In the day-ahead market, the pool customers submit their energy sale offers/purchase bids to the *IGO*. Without loss of generality, we assume one seller and one buyer at each node  $n \in \mathcal{N}$  and denote by  $\beta_n^s(p_n^s) / \beta_n^b(p_n^b)$ ,  $n = 0, 1, \dots, N$  the seller's offer/buyer's bid price as a function of the active power supply/consumption<sup>2</sup>. We assume  $\beta_n^s(p_n^s) / \beta_n^b(p_n^b)$  to be a continuous, differentiable and convex/concave function. We define  $\underline{\mathbf{p}}^s \triangleq [p_1^s, p_2^s, \dots, p_N^s]^T$  and  $\underline{\mathbf{p}}^b \triangleq [p_1^b, p_2^b, \dots, p_N^b]^T$ . For the bilateral customers, we assume all transactions to be basic and represent them by the set  $\mathcal{W} \triangleq \{\omega^1, \omega^2, \dots, \omega^W\}$ <sup>3</sup>, with each element denoted by the *ordered* triplet  $\omega^w \triangleq \{m^w, n^w, t^w\}$  representing a basic transaction with receipt point (*from* node)  $m^w$ , delivery point (*to* node)  $n^w$  in the amount  $t^w$  MW. For each transaction, the customer requests the corresponding transmission services from the *IGO*.

The *IGO* collects all the pool bids, offers and the transmission requests of the bilateral customers and schedules the transmission services so as to maximize the total social welfare. The extent to which the transmission service requests of the bilateral customers are met depends on the customers' willingness to pay the charges for congestion. We assume all bilateral customers are willing to pay the charges – no matter how high – so that all their transactions are scheduled. The impact of these transactions is to introduce the active power injection  $p_n^t$  at each node  $n$  where

$$p_n^t = \sum_{w=1, m^w=n}^W t^w - \sum_{w=1, n^w=n}^W t^w, \quad n = 0, 1, 2, \dots, N. \quad (10)$$

We denote  $\underline{\mathbf{p}}^t \triangleq [p_1^t, p_2^t, \dots, p_N^t]^T$ . The *IGO*'s process to determine the successful bids/offers of the pool customers may be represented by the transmission scheduling problem (*TSP*) that maximizes the social welfare subject to the network constraints:

$$\mathit{TSP} \begin{cases} \max & s(p_0^s, p_0^b, \underline{\mathbf{p}}^s, \underline{\mathbf{p}}^b) = \sum_{n=0}^N \beta_n^b(p_n^b) - \beta_n^s(p_n^s) \\ \text{s.t.} & p_0^s - p_0^b + p_0^t = \underline{\mathbf{b}}_0^T \underline{\boldsymbol{\theta}} \quad \leftrightarrow \mu_0 \\ & \underline{\mathbf{p}}^s - \underline{\mathbf{p}}^b + \underline{\mathbf{p}}^t = \underline{\mathbf{B}}\underline{\boldsymbol{\theta}} \quad \leftrightarrow \underline{\boldsymbol{\mu}} \\ & \underline{\mathbf{B}}_d \underline{\mathbf{A}} \underline{\boldsymbol{\theta}} \leq \underline{\mathbf{f}}^{\max} \quad \leftrightarrow \underline{\boldsymbol{\lambda}} \end{cases} \quad (11)$$

Note that the inequality constraints may also be written as:

$$\sum_{n=1}^N (p_0^s - p_0^b) \psi_\ell^n \leq f_\ell^{\max} - \sum_{w=1}^W t^w \varphi_\ell^{o_w}, \quad \ell = 1, 2, \dots, L, \quad (12)$$

where  $\psi_\ell^n$  and  $\varphi_\ell^o$  are the injection shift factor (*ISF*) and the power transfer distribution factor (*PTDF*). Definitions of these factors are reviewed briefly in Appendix B. Clearly, since we assume no limits for the bilateral customers' willingness to pay for congestion, the transmission schedules for the bilateral customers, in effect, constrain the available transfer capability for the pool customers. In actual markets, bilateral customers may specify upper limits on their payments for congestion. In such cases, their transmission schedules become decision variables in the *TSP* model.

The day-ahead market is settled based on the optimal solutions of the *TSP*, which we assume to exist. The optimal values of the decision variables,  $(p_n^{b*}, p_n^{s*})$ , determine the quantities for the energy purchases/sales from/to the pool customers. Prices are determined based on the optimal values of the dual variables.  $\mu_n^*$  is the *LMP* at the node  $n$  of the network. A seller (buyer) at each node  $n$  is paid (pays) the *LMP*  $\mu_n^*$  by (to) the *IGO* for each MWh sold (bought) in the pool. The net income of the *IGO* from the pool,  $\sum_{n=0}^N \mu_n^* (p_n^{b*} - p_n^{s*})$ , is called

the *merchandising surplus* [9] and is nonnegative.  $\lambda_\ell^*$  measures the marginal change in social welfare with respect to change in the limiting capacity  $f_\ell^{\max}$  of line  $\ell$ . Note that  $\lambda_\ell^* \geq 0$  for  $\forall \ell \in \mathcal{L}$  and  $\lambda_\ell^* > 0$  implies that line  $\ell$  is con-

<sup>2</sup> Multiple sellers/buyers at a node may be represented by a single composite seller/composite buyer with its corresponding offer/bid function constructed by combining the offers/bids of the constituent sellers/buyers.

<sup>3</sup> In actual practice, a complex transaction is a linear combination of basic transactions.

gested. Consequently,  $\lambda_\ell^*$  is considered to be the congestion charges for each *MW* flow in line  $\ell$ . We denote by  $\tilde{\mathcal{L}} \subset \mathcal{L}$  the set of congested lines, then the total amount of congestion charges assessed from each transaction  $\omega^w = \{m^w, n^w, t^w\}$  is

$$\xi^w = \sum_{\ell \in \tilde{\mathcal{L}}} \lambda_\ell^* \varphi_\ell^{\omega^w} t^w. \quad (13)$$

The optimality conditions for (11) leads to the relationship

$$\mu_n^* - \mu_m^* = \sum_{\ell \in \tilde{\mathcal{L}}} \lambda_\ell^* \varphi_\ell^{\omega^w}. \quad (14)$$

The proof of this result is given in the Appendix C. It follows that the total congestion charges assessed from  $\omega^w$  is

$$\xi^w = (\mu_n^* - \mu_m^*) t^w. \quad (15)$$

In words,  $\xi^w$  is the product of its quantity and the *LMP* differences between its delivery node and its receipt node. Note that a congestion free system corresponds to empty  $\tilde{\mathcal{L}}$  so that

$$\mu_n^* - \mu_m^* = 0, \quad \forall m, n \in \mathcal{N}. \quad (16)$$

Absent congestion, all *LMPs* are equal and the congestion charges are 0. On the other hand, differences in the *LMPs* signal the presence of congestion in the system.

The rationale for this congestion charges comes from the evaluation of the congestion impacts on the social welfare. When congestion occurs, higher priced generation may be dispatched to meet the load and consequently, the social welfare decreases. This reduction may be considered as the costs imposed due to the limits in the transmission network. Therefore, we define the system congestion costs  $c$  to be the decrease in the social welfare due to the presence of the transmission constraints. Let  $s^*$  ( $\tilde{s}^*$ ) be the optimal value of the social welfare in the *TSP* with (without) the active power line flow limits, then

$$c \triangleq \tilde{s}^* - s^*. \quad (17)$$

Now,  $\tilde{s}^*$  is independent of the transaction amounts  $t^w$  but  $s^*$  is not so that

$$\frac{dc}{dt^w} = -\frac{ds^*}{dt^w} = \sum_{\ell \in \tilde{\mathcal{L}}} \varphi_\ell^{\omega^w} \lambda_\ell^* = \mu_n^* - \mu_m^*. \quad (18)$$

In other words, the marginal impact of the transaction  $\omega^w$  on the system congestion costs equals the *LMP* differences. This, then, is the economic rationale for the congestion charges.

The comparability principle requires both the pool customers and the bilateral customers to pay congestion charges since the injections/withdrawals of the two classes of customers impact the total congestion costs. In fact, the congestion charges for the pool customers are *implicitly* included in the *LMPs* [6]. Note that, absent the line flow limit constraints, *LMPs* at all the  $N+1$  nodes are equal in a lossless system. We denote by  $\tilde{\mu}^*$  this uniform system-wide market-clearing price and consider it to be the *marginal price of energy* of the transmission-unconstrained system. In reality, however, presence of the transmission capability limits results in the nodal *LMPs*  $\mu_n^*$  that may be different from  $\tilde{\mu}^*$ . The difference  $\mu_n^* - \tilde{\mu}^*$  may be considered to be the *implicit congestion*

*charges* assessed for each *MW* withdrawal at node  $n$ . This difference represents the change in the total congestion costs  $c$  to serve an additional unit of load at node  $n$ . Since

$$\mu_n^* = \tilde{\mu}^* + (\mu_n^* - \tilde{\mu}^*),$$

the pool customers incur the congestion charges implicitly by paying/getting paid the *LMP* at the particular node. The total amount of such implicit congestion charges paid by the pool customers may be expressed as

$$\xi^{\text{pool}} = \sum_{n=0}^N (\mu_n^* - \tilde{\mu}^*) (p_n^{b^*} - p_n^{s^*}) = \sum_{n=0}^N \mu_n^* (p_n^{b^*} - p_n^{s^*}) \quad (19)$$

where

$$\sum_{n=0}^N \tilde{\mu}^* (p_n^{b^*} - p_n^{s^*}) = \tilde{\mu}^* \sum_{n=0}^N (p_n^{b^*} - p_n^{s^*}) = 0 \quad (20)$$

since the supply-demand balance in a lossless system holds. Note that the right hand side of (19) equals the *merchandising surplus* [9] of the pool. The *IGO* collects the congestion charges from pool customers implicitly by collecting the *merchandising surplus*.

The congestion charges are functions of the *LMPs* which are determined in the day-ahead commodity market. Bilateral contracts, on the other hand, are typically signed a month or even a year before. At that time, the *LMPs* are unknown and we represent the *LMP* at node  $n$  by the random variable  $\mu_n^*$ .

Consequently, the congestion charges the transactions may be assessed are uncertain. Naturally, risk-averse customers would like to have risk management tools that could provide price certainty on the congestion charges. *CRR* are designed for this purpose. In the next section, we describe the model of the *CRR* and the financial markets in which they are traded.

## V. THE FINANCIAL MARKET LAYER

*CRR* are financial instruments issued by the *IGO* that entitle the holder to be reimbursed for the congestion charges collected by the *IGO* in the day-ahead market. In this section, we establish the model for the *CRR* and the financial markets in which the *CRR* are issued and traded. We focus on the point-to-point *CRR*, but the analysis may also be applied to other types of *CRR*.

### A. *CRR* overview

The *CRR* are defined for a point of receipt (*from* node)  $m$ , a point of delivery (*to* node)  $n$ , a specified amount of transmission service  $\gamma$  in *MW* and a per *MW* premium  $\rho$ . We use the quadruplet

$$\Gamma \triangleq \{m, n, \gamma, \rho\}$$

to denote the *CRR*. The *CRR* are issued by the *IGO* – the issuer – to the transmission customers – the holders. A holder of  $\Gamma$  is entitled to receive from the *IGO* a payment of

$$\chi \triangleq (\mu_n^* - \mu_m^*) \gamma \quad (21)$$

where  $\mu_n^*$  and  $\mu_m^*$  are the *LMPs* determined in the day-ahead market. We refer to this payment as the *CRR payoff*. The payoff may be positive, negative or zero and is independent of the usage, i.e., this payment occurs whether or not the holder re-

quests any transmission services in the day-ahead market. As such, the *CRR* entails an obligation on both the holder and the issuer.

*CRR* may provide a full hedge against the congestion charges associated with the use of the transmission service. We consider a transaction  $\omega^w = \{m^w, n^w, t^w\}$ . A party involved in this transaction must pay the congestion charges of

$$\xi^w = (\mu_{n^w}^* - \mu_{m^w}^*) t^w \quad (22)$$

which are unknown prior to the clearing of the day-ahead market. If the party holds a *CRR*  $\Gamma = \{m, n, \gamma, \rho\}$  with  $m = m^w$ ,  $n = n^w$  and  $\gamma = t^w$ , then the *CRR* payoff,  $\chi = (\mu_n^* - \mu_m^*) \gamma$ , reimburses all the congestion charges independent of the day-ahead market outcomes. In this case, the *CRR* perfectly hedges the congestion charges.

Clearly, the *CRR* payoff is a function of random variables whose values are unknown until the day-ahead market clears. The *CRR* is therefore a financial derivative [5]. Indeed, the *CRR* embody the salient attributes of *forward contracts* – the financial derivatives that require the holder to buy and the issuer to sell the underlying asset at the specified time for the specified price [5]. For *CRR*, the particular attributes are

- the *maturity time* is specified as the time when the *CRR* are exercised;
- the payoff is the linear function of the value of the *underlying asset*, the variable  $(\mu_n^* - \mu_m^*) \gamma$ ;
- the *strike price* is 0.

We apply finance theory notions to derive the *payoff parity principle* for the *CRR*. Two sets of *CRR* are said to have payoff parity if they yield the same payoffs independent of the outcomes of the day-ahead market. We use the symbol “ $\Leftrightarrow$ ” to denote the payoff parity. By definition,  $\Omega' = \{\Gamma'_1, \Gamma'_2, \dots, \Gamma'_{K'}\} \Leftrightarrow \Omega'' = \{\Gamma''_1, \Gamma''_2, \dots, \Gamma''_{K''}\}$  if and only if  $\sum_{k=1}^{K'} \chi'_k \equiv \sum_{k=1}^{K''} \chi''_k$ . Two payoff parity conditions follow directly from the definition:

- quantity cumulation condition*: for two sets of *CRR*  $\Omega'$  and  $\Omega''$  with all the *CRR* having a common *from* node and a common *to* node, i.e.,  $m'_k = m''_k = m$  and  $n'_k = n''_k = n$ ,  $k = 1, 2, \dots, \max\{K', K''\}$ , if  $\sum_{k=1}^{K'} \gamma'_k = \sum_{k=1}^{K''} \gamma''_k$ , then  $\Omega' \Leftrightarrow \Omega''$ ;
- cycling condition*:  $\{\Gamma = \{m, n, \gamma, \rho\}\} \Leftrightarrow \{\Gamma'_1, \Gamma'_2, \dots, \Gamma'_{K'}\}$  if  $m'_1 = m$ ,  $n'_{K'} = n$ ,  $m'_{k+1} = n'_k$ ,  $k = 1, 2, \dots, K'-1$  and  $\gamma'_k = \gamma$  for all  $k$ .

The payoff parity principle is useful in designing liquid *CRR* markets as illustrated in the following sections.

### B. *CRR* issuance

*CRR* are issued in a centralized auction. In the auction, customers submit bids that indicate the *from* node, *to* node and

desired quantity of the requested *FTR* and the maximum premium they are willing to pay for it. The *IGO* determines the successful bids by maximizing its total income subject to the *simultaneous feasibility test (SFT)*. The *SFT* considers *CRR*  $\Gamma_k = \{m_k, n_k, \gamma_k, \rho_k\}$  to correspond to a fictitious basic transaction  $\tilde{\omega}_k \triangleq \{m_k, n_k, \gamma_k\}$  and check whether the transmission system can support all such fictitious transactions under the base case and all the contingency conditions. For simplicity, we consider the active power line flow limits under the base case only. We denote by  $\Omega = \{\Gamma_1, \Gamma_2, \dots, \Gamma_K\}$  the set of *CRR* that are issued by the *IGO*, then, the *SFT* is formulated as

$$\sum_{k=1}^K \gamma_k \varphi_\ell^{\tilde{\omega}_k} \leq f_\ell^{\max} \quad \forall \ell \in \mathcal{L}. \quad (23)$$

We can extend the formulation in (23) to include the impacts of contingencies. For each contingency case, we add a number of constraints corresponding to the binding line limits. For each contingency, the *PTDFs* of the changed network are used. The *SFT* specification ensures the *IGO* issues no more *CRR* than what the transmission system can accommodate under the base and the considered contingency conditions.

### C. *CRR* valuation / pricing

Once the *CRR* are issued by the *IGO*, they can be subsequently traded in secondary markets. Liquid trading requires a practical scheme to value and price the *CRR*. We design such a scheme by incorporating finance theory.

*CRR* are an example of financial derivatives since they are forward contracts. In the *CRR* analysis, we assume all the customers to be risk neutral and neglect the time value of the money due to the short time horizon. Then, the value of the *CRR* financial derivative is simply the expected value of its payoff [5]. The payoff for the *CRR*  $\Gamma = \{m, n, \gamma, \rho\}$  is  $(\mu_n^* - \mu_m^*) \gamma$ . Therefore, the value of the *CRR* is

$$E\left\{(\mu_n^* - \mu_m^*) \gamma\right\} = \left(E\left\{\mu_n^*\right\} - E\left\{\mu_m^*\right\}\right) \gamma. \quad (24)$$

We can estimate the expected values of the *LMPs*,  $E\left\{\mu_n^*\right\}$ , using historical data. If the *CRR* markets are perfectly competitive, the *CRR* prices equal the value of the *CRR* [5]. In reality, however, the *CRR* markets are not perfectly competitive and the *CRR* prices may be different from their values. In this case, we adopt the following assumptions:

- the market is free of *arbitrage* [5], i.e., the opportunity that a market player can construct a *portfolio* so that the *portfolio profit*  $\pi$  satisfies the probability conditions:  $P\{\pi > 0\} > 0$  and  $P\{\pi < 0\} = 0$
- where,  $P\{\bullet\}$  indicates the probability of the event  $\{\bullet\}$ ;
- the absence of arbitrage ensures that there is no opportunity for market players to make money without taking any risks;
- the futures markets for energy exist so that the futures energy prices are available at every node  $n \in \mathcal{N}$ ;

Then, the price  $\rho$  for each *MW* of the *CRR*  $\Gamma = \{m, n, \gamma, \rho\}$  equals the difference in the futures energy prices  $v_n$  and  $v_m$  at node  $n$  and node  $m$ , respectively:

$$\rho = v_n - v_m. \quad (25)$$

The proof of (25) is provided in the Appendix C.

Several conclusions follow directly from (25). The price  $\rho$  may be positive, zero or negative. Also, for two *CRR*,  $\Gamma = \{m, n, \gamma, \rho\}$  and  $\Gamma' = \{m', n', \gamma', \rho'\}$ , if  $m = m'$  and  $n = n'$ , then,  $\rho = \rho'$ . Therefore, all the *CRR* with the same receipt node and same delivery node must have the same price. For the set of *CRR*  $\{\Gamma'_1, \Gamma'_2, \dots, \Gamma'_{K'}\}$  with a continuous path of nodes forming a loop, i.e.,  $m'_{k+1} = n'_k$ ,  $k = 1, 2, \dots, K' - 1$  and  $n'_K = m'_1$ , then  $\sum_{k=1}^{K'} \rho'_k = 0$ .

This pricing scheme and the *CRR* payoff parity principle can be used to design liquid *CRR* secondary markets as illustrated in section VII.

## VI. THE INFORMATION FLOWS

In sections III, IV and V, we have introduced the three layers that constitute the framework. In this section, we describe the interaction/interconnection between these layers through the various information flows depicted in Fig. 1.

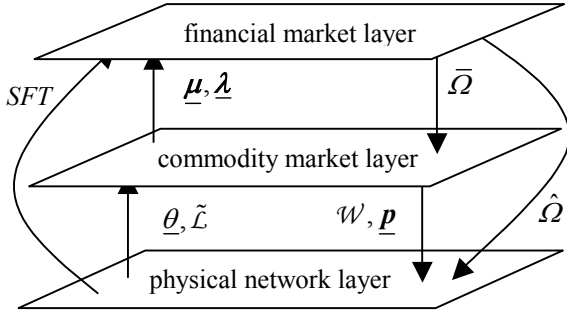


Figure 1: The three-layer framework structure

We start from the financial market layer. The inputs of the *CRR* markets are the *CRR* requests from the customers. The decision making process of the *IGO* requires the involvement of the *SFT*, which is a result of the repetitive information exchange between the financial market layer and the network layer. Tentative *CRR* set  $\hat{\Omega}$  is sent to the network layer to check its feasibility. The result is fed back to the financial layer. If infeasible, the issuance quantities are modified and the new *CRR* set is sent to be checked. Such iteration continues till the feasible solution  $\bar{\Omega}$  is found.

As we have pointed out, the outcomes of the financial markets may impact the customers' behavior in the commodity market. We denote by  $\bar{\Omega}$  the set of *CRR* held by the customers after *CRR* issuance and secondary markets.  $\bar{\Omega}$  is sent to the commodity market layer and impacts the bids/offers and the bilateral transactions – the inputs of the *TSP*.

Due to the involvement of the physical constraints, the solu-

tion scheme of the *TSP* may also be considered as an interactive process between the commodity market layer and the network layer. Transmission schedules including bilateral transactions  $\mathcal{W}$  and nodal injection and withdrawals represented by  $\underline{p}^s, \underline{p}^b$  are sent to the network layer to check the feasibility. The resulting system status  $\underline{\theta}$  and the congested lines  $\tilde{L}$  is fed back to determine the *LMPs*. The *LMPs* are then sent to the financial market layer to compute the *CRR* payoffs.

The information flows serve to interconnect the three layers into the integrated framework proposed in this paper.

## VII. APPLICATIONS OF THE FRAMEWORK

Since the basic model of the *CRR* and the financial markets are included in the proposed framework, many critical issues related to the deployment of *CRR* may directly be investigated using the framework. For example, the framework may be applied to analyze the market power associated with *CRR*. Reference [9] has performed such investigations on simple two and three node systems. Our framework makes possible the analysis on more realistic sized systems.

The structural modularity of the framework provides flexibility for its applications to the design of the *CRR*. The models in each layer are general so that different market designs may be accommodated. Also, since the interaction between the layers is exogenous to each layer and represented by the information flows, we may modify any layer without impacting the models in the other two layers. This flexibility provides great convenience for the design of various types of the *CRR*. For example, if the flow-based *CRR* are concerned, we may modify the financial market layer by applying the analogous analysis while no modification is required to the network and commodity market layers. In fact, many results in the financial market layer, such as the payoff parity principle, are still valid.

In this section, we present two examples to illustrate the applications of the framework to the analysis and design of the *CRR*, respectively. We end the section with a discussion of the application of the framework in actual practice for a prescribed period of time.

### A. Revenue adequacy for the *IGO*

Revenue adequacy is a critical issue in the deployment of *CRR*. Using the framework, we can prove that the proposed market model allows the *IGO* to ensure its revenue adequacy.

The *IGO* collects congestion charges from both the pool and bilateral customers. The total revenue is:

$$\xi^{total} = \sum_{n=0}^N \mu_n^* (p_n^{b*} - p_n^{s*}) + \sum_{w=1}^W (\mu_{n_w}^* - \mu_{m_w}^*) t^w. \quad (26)$$

This revenue is then the source used to make the *CRR* payoffs to the holders. Let  $\bar{\Omega} = \{\Gamma_1, \Gamma_2, \dots, \Gamma_{\bar{K}}\}$  be the set of *CRR* held by the customers after *CRR* issuance and secondary markets, then, the total payoff is

$$\chi^{total} = \sum_{k=1}^{\bar{K}} (\mu_{n_k}^* - \mu_{m_k}^*) \gamma_k. \quad (27)$$

We claim the *IGO* revenue adequacy is guaranteed, i.e.,

$$\chi^{total} \leq \xi^{total}. \quad (28)$$

The equality holds if

$$\sum_{k=1}^{\bar{K}} \gamma_k \varphi_\ell^{\tilde{w}_k} = f_\ell^{max}, \quad \forall \ell \in \tilde{\mathcal{L}}. \quad (29)$$

In other words, if all the constraints in the *SFT* corresponding to the lines that are congested in the day-ahead market are binding, the *IGO*'s incomes equal its payments. Otherwise, the *IGO*'s net income is always positive.

We prove this proposition using the models in the framework. Although this issue belongs to the financial market, the proof requires equation (14), which is derived based on the *TSP* model in the commodity market layer and the physical constraints from the network layer. The total *CRR* payoffs may then be rewritten as

$$\chi^{total} = \sum_{k=1}^{\bar{K}} \sum_{\ell \in \tilde{\mathcal{L}}} \gamma_k \varphi_\ell^{\tilde{w}_k} \lambda_\ell^* = \sum_{\ell \in \tilde{\mathcal{L}}} \lambda_\ell^* \sum_{k=1}^{\bar{K}} \gamma_k \varphi_\ell^{\tilde{w}_k}. \quad (30)$$

Also, the relationship in (C3) leads to

$$\begin{aligned} \sum_{n=0}^N \mu_n^* (p_n^{b*} - p_n^{s*}) &= \sum_{n=0}^N \left( \mu_0^* - \sum_{\ell \in \tilde{\mathcal{L}}} \psi_\ell^n \lambda_\ell^* \right) (p_n^{b*} - p_n^{s*}) \\ &= - \sum_{n=0}^N \sum_{\ell \in \tilde{\mathcal{L}}} \psi_\ell^n \lambda_\ell^* (p_n^{b*} - p_n^{s*}) \\ &= \sum_{\ell \in \tilde{\mathcal{L}}} \lambda_\ell^* \sum_{n=0}^N \psi_\ell^n (p_n^{s*} - p_n^{b*}) \end{aligned} \quad (31)$$

and

$$\sum_{w=1}^W (\mu_{n^w}^* - \mu_{m^w}^*) t^w = \sum_{w=1}^W t^w \sum_{\ell \in \tilde{\mathcal{L}}} \lambda_\ell^* \varphi_\ell^{\omega^w} = \sum_{\ell \in \tilde{\mathcal{L}}} \lambda_\ell^* \sum_{w=1}^W \varphi_\ell^{\omega^w} t^w \quad (32)$$

therefore,

$$\xi^{total} = \sum_{\ell \in \tilde{\mathcal{L}}} \lambda_\ell^* \left( \sum_{n=0}^N \psi_\ell^n (p_n^{s*} - p_n^{b*}) + \sum_{w=1}^W \varphi_\ell^{\omega^w} t^w \right) = \sum_{\ell \in \tilde{\mathcal{L}}} \lambda_\ell^* f_\ell^{max} \quad (33)$$

since the term in the bracket equals the active power flow in line  $\ell$  [16]. Note that the financial market layer is designed so that the *CRR* satisfy the *SFT* conditions

$$\sum_{k=1}^{\bar{K}} \gamma_k \varphi_\ell^{\tilde{w}_k} \leq f_\ell^{max} \quad \forall \ell \in \mathcal{L}. \quad (34)$$

We conclude, by comparing (33) with (30), that

$$\chi^{total} \leq \xi^{total}.$$

In practice, however, the *PTDFs* used in the *SFT* may be different from the *PTDFs* used in the day-ahead market due to changes in the network [16]. In such cases, equation (30) does not hold and the *IGO*'s revenue adequacy is not guaranteed.

### B. Design of *CRR* secondary markets and reconfiguration scheme

The centralized *CRR* issuance requires the involvement of the *IGO* to run the *SFT* and consequently limits the liquidity of the market. Successful use of the *CRR* requires secondary markets in which the *CRR* issued by the *IGO* can be subsequently traded. The secondary market models proposed in the literature only allows the trade of existing *CRR* to avoid the involvement of the *IGO* [4]. Such design, however, diminishes the merit of the secondary markets. We propose a sec-

ondary market model based on the payoff parity principle given in the framework. The holder of an existing *CRR*  $\Gamma = \{m, n, \gamma, \rho\}$  may sell a set of *CRR*  $\Omega' = \{\Gamma'_1, \Gamma'_2, \dots, \Gamma'_k\}$  as long as  $\{\Gamma\} \Leftrightarrow \Omega'$ . We can prove that the parity *CRR* set must still satisfy the *SFT* conditions and, therefore, no *IGO* involvement is necessary. Such design improves the flexibility for the *CRR* trades, while avoiding the involvement of the *IGO* in the secondary markets.

Another scheme to increase the *CRR* liquidity is the *CRR* reconfiguration. The holder of the existing *CRR*  $\Gamma = \{m, n, \gamma, \rho\}$  may return them to the *IGO* in exchange for the *CRR*  $\Gamma' = \{m', n', \gamma', \rho'\}$  whose *from* and *to* nodes are different from  $\Gamma$ . The pricing scheme presented in the financial market layer provides the means for implementing such exchanges. We design a reconfiguration scheme, making use of the free of arbitrage assumption in the financial market layer. The new quantity  $\gamma'$  of the reconfigured *CRR* is determined so that the prices of the new and replaced *CRR* are identical. Since the reconfigured *CRR* must also satisfy the *SFT*, information flows from the network layer is required. The *SFT* yields two possible situations:

- as long as the *SFT* conditions are not violated, we set  $\gamma'$  such that

$$\gamma' \rho' = \gamma \rho, \quad (35)$$

so that

$$\gamma' = \gamma \left( \frac{V_n - V_m}{V_{n'} - V_{m'}} \right). \quad (36)$$

- if, on the other hand, the *SFT* conditions are violated, we set  $\gamma'$  to be the maximum *MW* amount  $\bar{p}'$  without violating the *SFT* conditions and exchange only that fraction of the existing *CRR* to result in  $\bar{p}'$ .

Since this scheme ensures that the existing and reconfigured *CRR* have identical prices, it ensures that no arbitrage opportunity may arise.

### C. General application of the framework

The discussion so far has focused on the framework for a specified hour, but its application for markets of a specified longer duration is straightforward. For example, we may use this framework to price the *CRR* that are issued for a longer period of time, say  $[T_1, T_2]$ . Such *CRR* may be considered as a family of *CRR*,  $\{\Gamma_t : t \in [T_1, T_2]\}$ , with one *CRR*  $\Gamma_t = \{m_t, n_t, \gamma_t, \rho_t\}$  for each hour  $t$ . We determine the price  $\rho_t$  for each  $\Gamma_t$  using (25), then, the price of the *CRR* for period  $[T_1, T_2]$  is  $\sum_{t=T_1}^{T_2} \rho_t$ . In similar ways, other issues of interest over longer periods can be studied.

## VIII. CONCLUSION

In this paper, we have presented a mathematical framework

for the design and analysis of the *CRR*. The framework consists of three interconnected layers with one layer each to represent the models of the transmission network, the commodity markets and the financial markets. The framework effectively exploits the structural characteristics of the interrelationships between the physical network, the commodity markets and the financial markets for *CRR*. Novel notions, such as the *CRR* payoff parity principle and pricing schemes are introduced and applied to enhance the liquidity of the *CRR* markets.

The framework represents a good start for the study of *CRR* and their application. There are, however, a number of extensions possible. The development of more detailed dynamic stochastic models that explicitly incorporate the impacts of various random factors to the outcomes of the day-ahead market, such as *LMPs*, is a natural extension. Since the contingencies in the transmission system may impact the behaviors of the transmission customers, incorporation of the contingency analysis results is another extension of the work reported here. A major long-term challenge is the inclusion of losses and nonlinear effects in transmission considerations. The iterative interaction between the layers via the information flows is another topic for future work with a focus on the investigation of the convergence properties of the iterations. Progress on the work on these topics will be reported in future papers.

#### IX. APPENDIX A: ACRONYMS AND NOTATION

<i>LMP</i>	: locational marginal price
<i>CRR</i>	: congestion revenue rights
<i>IGO</i>	: independent grid operator
<i>TSP</i>	: transmission scheduling problem
<i>ISF</i>	: injection shift factor
<i>PTDF</i>	: power transfer distribution factor
<i>SFT</i>	: simultaneous feasibility test
$\underline{f} \triangleq [f_1, f_2, \dots, f_L]^T$	: vector of active power line flow
$\underline{p} \triangleq [p_1, p_2, \dots, p_N]^T$	: vector of active power nodal injections
$\underline{B}_d$	: the branch susceptance matrix
$\underline{A} \triangleq [\underline{a}_1, \underline{a}_2, \dots, \underline{a}_L]^T$	: the reduced incidence matrix
$\underline{B}$	: the reduced nodal susceptance matrix
$\beta_n^s(p_n^s) / \beta_n^b(p_n^b)$	: the seller's offer/buyer's bid price
$\omega^w \triangleq \{m^w, n^w, t^w\}$	: the basic transaction $w$
$\mu_n^*$	: <i>LMP</i> at node $n$ with value $\mu_n^*$
$\lambda_\ell^*$	: per <i>MW</i> congestion charges for line $\ell$
$\Gamma \triangleq \{m, n, \gamma, \rho\}$	: <i>CRR</i> in the amount $\gamma$ <i>MW</i> with injection (withdrawal) at node $m$ ( $n$ ) and the per <i>MW</i> premium $\rho$
$\zeta$	: <i>CRR</i> payoff with value $\zeta$
$v_n$	: the futures energy price at node $n$
$\underline{\Psi} = [\psi_\ell^n]$	: <i>ISF</i> matrix
$\varphi_\ell^\omega$	: <i>PTDF</i> of line $\ell$ with respect to basic transaction $\omega$

#### X. APPENDIX B: THE *ISF* AND THE *PTDF*

We review briefly the definition of the injection shift factor (*ISF*) and power transfer distribution factor (*PTDF*). The derivation, nature and applications of these factors are treated in detail in [16].

The *ISF*  $\psi_\ell^n$  of a line  $\ell \in \mathcal{L}$  with respect to a change in injection at node  $n \in \mathcal{N}, n \neq 0$  is the approximate sensitivity of the active power flow  $f_\ell$  on line  $\ell$  with respect to the *MW* injection  $p_n$  at node  $n$ . The *ISF* matrix  $\underline{\Psi}$  is the  $L \times N$  matrix with each  $\psi_\ell^n$  in row  $\ell$ , column  $n$  of  $\underline{\Psi}$ . Under the DC power flow model

$$\underline{\Psi} \triangleq \partial \underline{f} / \partial \underline{p} = \underline{B}_d \underline{A} \underline{B}^{-1}. \quad (\text{B1})$$

The proof is given in [16].

The *PTDF*  $\varphi_\ell^\omega$  is the sensitivity of the active power flow  $f_\ell$  on line  $\ell$  with respect to the *MW* amount  $t^w$  of the basic transaction  $\omega^w = \{m^w, n^w, t^w\}$ . The relationship

$$\varphi_\ell^\omega \triangleq \partial f_\ell / \partial t^w = \psi_\ell^{m^w} - \psi_\ell^{n^w} \quad (\text{B2})$$

is established in [16].

#### XI. APPENDIX C: PROOFS

(i) *Proof of (14)*

We write the Lagrangian of the *TSP* formulation as

$$\begin{aligned} \mathcal{L}(p_0^s, p_0^b, \underline{p}^s, \underline{p}^b, \underline{\theta}, \underline{\mu}, \underline{\lambda}) = & s(p_0^s, p_0^b, \underline{p}^s, \underline{p}^b) - \underline{\mu}^T (\underline{B}\underline{\theta} - \underline{p}) \\ & - \mu_0 (\underline{b}_0^T \underline{\theta} - p_0) - \underline{\lambda}^T (\underline{B}_d \underline{A} \underline{\theta} - \underline{f}^{max}) \end{aligned}$$

where  $\underline{p} \triangleq \underline{p}^s - \underline{p}^b + \underline{p}^t$ ,  $p_0 \triangleq p_0^s - p_0^b + p_0^t$ . At the optimum,

$$\left. \frac{\partial \mathcal{L}}{\partial \underline{\theta}} \right|_{\theta^*} = -\underline{B}^T \underline{\mu}^* - \mu_0^* \underline{b}_0 - \underline{A}^T \underline{B}_d \underline{\lambda}^* = \underline{0}. \quad (\text{C1})$$

(C1) and (3) yield

$$\underline{\mu}^* = -\mu_0^* (\underline{B}^T)^{-1} \underline{b}_0 - (\underline{B}^T)^{-1} \underline{A}^T \underline{B}_d \underline{\lambda}^* = \mu_0^* \underline{I}^N - \underline{\Psi}^T \underline{\lambda}^* \quad (\text{C2})$$

Therefore,

$$\mu_n^* = \mu_0^* - \sum_{\ell \in \mathcal{L}} \psi_\ell^n \lambda_\ell^* \quad n = 1, 2, \dots, N \quad (\text{C3})$$

so that

$$\mu_n^* - \mu_{m^w}^* = \sum_{\ell \in \mathcal{L}} (\psi_\ell^{m^w} - \psi_\ell^{n^w}) \lambda_\ell^* = \sum_{\ell \in \mathcal{L}} \varphi_\ell^\omega \lambda_\ell^*.$$

(ii) *Proof of (25)*

We prove (25) by contradiction. If  $\rho < v_n - v_m$ , a customer may create a portfolio by

- buying the *CRR*  $\Gamma = \{m, n, \gamma, \rho\}$  for the costs  $\rho \gamma$
- selling  $\gamma$  futures contract for energy at node  $n$  at price  $v_n$
- buying  $\gamma$  futures contract for energy at node  $m$  at price  $v_m$

The costs to construct this portfolio are:

$$(\rho + v_m - v_n) \gamma < 0.$$

In the day-ahead market, the customer exercises the futures contracts for the energy and obtains the transmission service from the *IGO*. The payoff of the *CRR* cancels out the congestion charges. Therefore, no net payment occurs in the day-ahead market. Consequently, the customer ends up with a positive net profit  $(v_n - v_m - \rho)\gamma$  with 100% probability. This is an arbitrage opportunity which violates the arbitrage free assumption. Therefore,  $\rho < v_n - v_m$  cannot be true. Similarly, we may prove the existence of the arbitrage opportunity when  $\rho > v_n - v_m$ , which also leads to a contradiction. Hence, the only choice for ensuring the arbitrage free condition is to have

$$\rho = v_n - v_m.$$

## XII. REFERENCES

- [1] E. Bompard, P. Correia, G. Gross, and M. Amelin, "Congestion-Management Schemes: a Comparative Analysis Under a Unified Framework", *IEEE Trans. Power Systems*, vol. 18, No. 1, pp. 346-352, Feb. 2003.
- [2] H. Singh, H. Shangyou and A. Papalexopoulos, "Transmission Congestion management in Competitive Electricity Markets", *IEEE Trans. Power Systems*, vol. 13, No. 2, pp. 672-680, May 1998.
- [3] North American Electric Reliability Council, "Appendix 9C – Transmission Loading Relief Procedure", [Online]. Available: <http://www.nerc.com>.
- [4] United States of America Federal Energy Regulatory Commission, "Remedying Undue Discrimination Through Open Access Transmission Service and Standard Electricity Market Design", [Online]. Available: <http://www.ferc.gov/Electric/RTO/Mrkt-Strct-comments/nopr/WebNOPR.pdf>.
- [5] R. J. Rendleman, Jr., "Applied Derivatives: Options, Futures, and Swaps", Blackwell Publishers, 2002. pp. 155-207.
- [6] W. Hogan, "Contract Networks for Electric Power Transmission", *Journal of Regulatory Economics*, vol. 4, pp. 211-242, 1992.
- [7] PJM Interconnection L.L.C., "PJM Manual for Fixed Transmission Rights", [Online]. Available: <http://pubs.pjm.com/dynaweb/PJMpulp/m06>.
- [8] H. Chao, S. Peck, S. Oren and R. Wilson, "Flow-based Transmission Rights and Congestion Management", *Electricity Journal*, Oct. 2000, pp. 38-58.
- [9] P. L. Joskow and Jean Tirole, "Transmission rights and market power on electric power networks", *RAND Journal of Economics*, vol. 31, No. 3, pp. 450-487, 2000.
- [10] S. Oren, P. Spiller, P. Varaiya and F. F. Wu, "Nodal Prices and Transmission Rights: A Critical Appraisal." *Electricity Journal*, Vol. 8 (1995), pp. 24-35.
- [11] L. E. Ruff, "Flowgates, Contingency-Constrained Dispatch, and Transmission Rights", *Electricity Journal*, Vol. 14, No. 1, Jan./Feb. 2001, pp. 34-55.
- [12] R. P. O'Neill, U. Helman, B. F. Hobbs, W. R. Stewart, and M. H. Rothkopf, "A Joint Energy and Transmission Rights Auction: Proposal and Properties", Federal Energy Regulatory Commission, Working Paper, February 2002.
- [13] J. Bushnell, "Transmission Rights and Market Power." *Electricity Journal*, Vol. 12 (1999), pp. 77-85.
- [14] New York ISO, "Centralized Transmission Congestion Contract (TCC) Auction Manual", [Online]. Available: <http://www.nyiso.com/services/documents/tie/pdf/tccrev7.pdf>.
- [15] A. Wood and B. Wollenberg, "Power Generation Operation and Control", 2<sup>nd</sup> edition, New York, John Wiley & Sons, pp. 105-108.
- [16] M. Liu and G. Gross, "Effectiveness of the Distribution Factor Approximations Used in Congestion Modeling", In *Proceedings of the 14th power systems computation conference*, Seville, 24-28 June, 2002.
- [17] D. G. Luenberger, "Linear and Nonlinear Programming", (2nd ed.) Reading, Mass. Addison-Wesley, 1984, pp. 314-315.
- [18] P. Nitu, G. Gross, R. Entriken, G. Anders and E. Janossy, "Risk Assessment and Financial Management Tutorial", IEEE Power Engineering Society Summer Meeting 2002.

## XIII. BIOGRAPHIES



**George Gross** (F' 88) is Professor of Electrical and Computer Engineering and Professor, Institute of Government and Public Affairs, at the University of Illinois at Urbana-Champaign. His current research and teaching activities are in the areas of power system analysis, planning, economics and operations and utility regulatory policy and industry restructuring. His undergraduate work was completed at McGill University, and he earned his graduate degrees from the University of California, Berkeley. He was previously employed by Pacific Gas and Electric Company in various technical, policy and management positions.



**Minghai Liu** received his B.S. and M.S. degrees in Electrical Engineering from Tsinghua University, Beijing, China, in 1997 and 2000, respectively. Since August, 2000, he has been a graduate student and a research assistant at the University of Illinois at Urbana-Champaign. His research interests focus on topics in power system operations, control, optimization and economics.