Analysis of Angle Stability Problems: A Transmission Protection Systems Perspective

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***Abstract*—Postfault rotor angle oscillations lead to power swings. Both unstable and stable swings can induce distance relay tripping. For unstable swings, a new computational procedure to locate all of the electrical centers is developed. It simplifies the work associated with visual screening of all the R-X plots. For stable swings, a generic three-tier hierarchy of stability-related norms defined by branch norm, fault norm, and system norm is proposed. Ranking by branch norm leads to ranking of power swings. Ranking by fault norm leads to ranking of faults or contingencies. Magnitude and rate of change of system norm can be used to detect an out-of-step condition. Results on a ten-machine system and a utility system with detailed models are also presented.**

***Index Terms*—Power system simulation, power system transient stability, power transmission protection, ranking.**

I. Introduction: Power Swings and Relaying Decision Uncertainty

POWER system protection at the transmission system level is based on distance relaying. The apparent impedance seen by a distance relay R (i.e., relay impedance) on a transmission line connecting nodes i and j, and having flow $P_{ij} + jQ_{ij}$ is given by

$$Z_R = \left[\frac{P_{ij}}{P_{ij}^2 + Q_{ij}^2} + j\frac{Q_{ij}}{P_{ij}^2 + Q_{ij}^2}\right]|V_i|^2 \qquad (1)$$

where all quantities refer to positive sequence values. Thus, the quadrant of Z_R depends only on the direction of P and Q flows. When $P_{ij} + jQ_{ij}$ is large and/or $|V_i|$ is small, then Z_R is small and the relay may trip.

Following a disturbance, the power swing observed by a distance relay refers to the relay impedance trajectory mapped on the R-X plane. Distance relays can trip on a power swing. Traditionally, if a swing *trajectory* enters zone-1, then it is considered to be severe enough to cause system instability [1]. The distance relay will also trip if the swing stays inside the backup zone for a time more than the corresponding time dial setting (TDS). In a stressed system, this can trigger cascade tripping. Tripping on a stable swing compromises *security* of the power system. Power swing blocking can either be applied selectively or to all of the distance relays. Blocking reduces *dependability* for three-phase faults during swings [2]. Traditionally, timers control swing detection and blocking logic. As stated in [3], "the setting of these timers is critical and depends on the speed of movement of the trajectory in the Z plane for various types of disturbances, whether the system has restoring forces or is being torn apart by the disturbances."

A straightforward way to find if a relay will trip after a disturbance is to incorporate the tripping logic in the transient stability program. However, it fails to give information on the criticality of a relay setting to the swing. Thus, conflict of dependability versus security [4] is not properly resolved. The bottleneck is the large simulation space defined by number of relays times the number of faults. This necessitates development of ranking schemes.

Some work based upon Lyapunov stability criterion has been reported in [5] to rank relays according to severity of the swings. Reference [6] uses *relay margin* as a measure of how close a relay is from issuing a trip command. For relays, which see swings within their relay characteristics, relay margin (RM) is used. It is the ratio of the time of longest consecutive stay of a swing in the zone to its TDS. For relays that do not see the swing, the relay space margin (RSM) is used. It is defined as the smallest distance between the relay characteristic and the swing trajectory in the R-X plane. To identify the most vulnerable relay, RM or RSM is used as a performance parameter. The most vulnerable relay corresponds to the one with the minimum ratio where the search space extends over all of the relays and time instants of simulation. Though not stated explicitly, work reported in [7] computes the relay margin by extensive simulations and modifies zone 2 or 3 setting to curtail relay operation on a stable swing.

A distance relay can also trip on load. In relaying parlance, this behavior is known as "load encroachment." Usually, loads have a large power factor. Hence, the relay characteristic is modified to avoid this region [8]. A line-tripping contingency can also lead to load encroachment. It can be detected by computing the relay impedance from a load flow analysis [7].

This paper investigates the following problems related to the performance of distance relaying under power swings.

A. Detection of Electrical Center

The natural separation of a system begins at the electrical centers. Therefore, locating all of the electrical center(s) in a system becomes an important issue. In this paper, a systematic approach to locate all electrical centers in a transmission network is proposed. These may not be the best locations to island the system and, hence, may require a *blocking* scheme.

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B. Relay Ranking Problem

If the postfault system is stable, then the question that arises is "are there any relays that can trip on a swing?" In this paper, a scheme is proposed to rank the distance relays in order of their vulnerability to tripping on swings. For this purpose, a new measure known as *branch norm* is introduced.

C. Fault or Contingency Ranking Problem

In a power system, there are infinite locations and ways by which a fault can occur. Therefore, the next problem is the identification of *credible* faults that result in either instability or low stability margin. For this purpose, a new measure known as *fault norm* is introduced.

D. Out-of-Step Detection Problem

From the protection system perspective, it is required to classify an ongoing swing as either stable or unstable. If the postfault system is unstable, then islanding is required. This classification has to be completed before the angular separation occurs. Because of the predictive nature of the problem, it becomes challenging. Distance relays only monitor *local* transmission line information and not the rotor angle oscillations. Therefore, they do not have adequate information to classify with certainty whether a swing is stable or unstable. With the developments in wide-area measurements (WAMs) [9], rotor angle oscillations can be monitored. This calls for development of new algorithms that can exploit the WAM potential. In this paper, to predict closeness to the out-of-step condition, a measure known as *system norm* is introduced. Magnitude of the system norm and its rate of change with respect to a system parameter provide information about the closeness to instability.

In this paper, angle stability-related norms are introduced through the trajectory sensitivity computation framework [10], [11]. They can handle any degree of complexity in terms of the modeling, such as differential-algebraic equations or hybrid systems. Alternative measures can be the circuit breaker critical clearing time ($t_{critical}$) or the minimum magnitude of the swing impedance (Z_{min}) seen by a relay. All stability norms measure angle stability information. Therefore, they can be correlated. These relationships are investigated.

The paper is organized as follows. Section II develops the procedure for locating all electrical centers in a system. Section III introduces branch impedance trajectory sensitivity (BITS) as a measure for angle stability margin assessment. A three-tier hierarchy of norms is proposed in Section IV. Results are presented in Section V. Section VI includes conclusions and discussion.

II. Location of Electrical Center(s)

For power systems that behave as a two-area system under instability, distance relay operation under the out-of-step condition can be explained by considering the equivalent generators connected by a tie line. When the two generators fall out-of-step, they create a voltage zero point on the connecting line. This is known as the *electrical* center. A distance relay perceives it as a solid three-phase short circuit and trips the line [12].

Relays near an electrical center are highly sensitive to power swings. The natural splitting of the system due to operation of distance relays begins at the electrical center. Location of electrical center depends upon the fault location, its type, and network configuration, loads, generation, etc. Due to the high impedance of the synchronous machine and/ or unit transformer, an electrical center may also be located within a generator. Such a situation falls within the purview of generator out-of-step relaying. If an electrical center exists on a transmission line, then the corresponding power swing plotted in the R-X plane cuts the transmission line impedance [13]. At an electrical center

1) Ratio of the magnitude of relay impedance (Z_R) to line impedance (Z_L) is less than unity. This criterion will be referred as *magnitude* criterion.
2) The relay impedance angle (θ_R) is equal to the transmission line impedance angle (θ_l). This criterion will be referred as *angle* criterion.

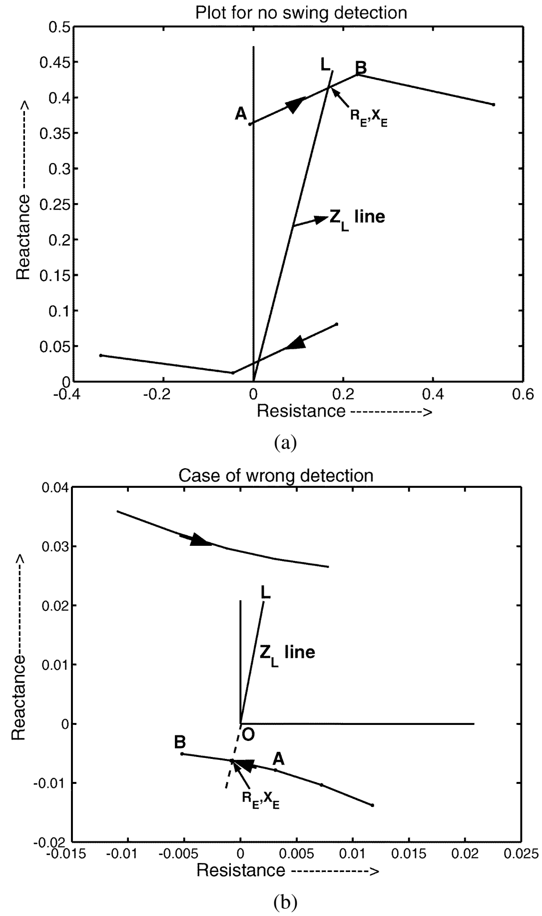

Fig. 1. (a) Case of no detection. (b) Case of wrong detection.

The transient stability program only simulates snapshots of the power system. Therefore, achieving the equality ($\theta_R = \theta_l$) is unlikely. In a program, a swing *cutting* the line impedance can be confirmed by checking that the sign of ($\theta_R - \theta_l$) reverses for two consecutive snapshots. Surprisingly, with this criterion, simulations bring out cases of nondetection and wrong detection of electrical center near the extremity of the line.

Illustrative examples observed on a test system are shown in Fig. 1(a) and (b). Swings as seen by the relays at the ends of the line have been overlapped in one figure. In Fig. 1(a), it is observed that the electrical center is located near the remote end of the transmission line. When the swing moves from A to B, the *cutting* criterion is satisfied but *magnitude* criterion fails to hold at B. Therefore, the electrical center is not identified. Hence, we conclude that in a discrete simulation, the magnitude constraint should be discarded. In Fig. 1(b), when the swing moves from A to B, both the *magnitude* and *cutting* criteria are satisfied. But this is a wrong detection because the swing movement is from the fourth to the third quadrant while the transmission line is located in the first quadrant. Consequently, *line segments* OL and AB do not intersect. Hence, the appropriate procedure would be to evaluate the exact point of intersection of the *lines* AB and OL and then check if the point is in the first quadrant. This will correctly identify the electrical center. The complete algorithm is detailed later. Results obtained from an Indian utility system are presented in Section V-B. This procedure can be integrated in a transient stability program. Electrical centers are associated with unstable swings. For stable swings, the angle stability margin assessment is required. It is discussed in the next section.

III. Bits and Angle Stability Measure

To evaluate the angle stability information with respect to a typical distance relaying parameter (e.g., branch reactance), we use *trajectory sensitivity of the rotor angles to the branch impedances*. This will be referred to as branch impedance trajectory sensitivity (BITS). This leads to the development of BITS-based norms. They provide a measure of rotor angle oscillation when a system parameter varies by a small amount.

Algorithm to Locate Electrical Center on a Transmission Line

At the snapshot "t," let $Z_t(R_t, X_t)$ and Z_{t-1} denote the relay impedance of the current point and the previous point, respectively, where "t" begins from the second instant of the postfault trajectory.

1. To check whether Z_t and Z_{t-1} are on opposite sides of the line (OL) defined by the transmission line impedance $Z_L(R_L + jX_L)$, Z_t and Z_{t-1} are projected on to the axis perpendicular to the transmission line impedance. Let α and β be the projections. Then

$$\alpha = -X_L * R_t + R_L * X_t \text{ and } \beta = -X_L * R_{t-1} + R_L * X_{t-1}.$$

2. If α and β have opposite signs, then and only then the points Z_t and Z_{t-1} are on the opposite sides. If either of them is zero, then the corresponding point (R_E, X_E) actually lies on the line OL. For the case when $\text{sign}(\alpha) \neq \text{sign}(\beta)$, coordinates of intersection $E(R_E, X_E)$ are found as follows:

$$\text{Let } m_R = \frac{X_t - X_{t-1}}{R_t - R_{t-1}} \quad m_L = \frac{X_L}{R_L} \text{ and } c = X_t - m_R * R_t.$$

Then (1) $R_E = c/(m_L - m_R)$ and (2) $X_E = m_L * R_E$

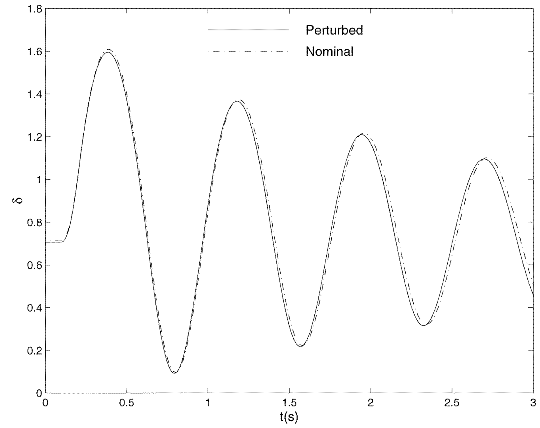

Fig. 2. Nominal and perturbed system δ response ($P_m = 1.0$ p.u.).

3. If $0 \leq R_E \leq R_L$ and $0 \leq X_E \leq X_L$, then the electrical center exists on the primary transmission line of the relay.

A. Definition: Branch Impedance Trajectory Sensitivity

BITS of a branch l at a time instant t for a pair of machines (i, j) denoted by $S_l^{ij}(t)$ is defined as follows:

$$S_l^{ij}(t) = \frac{\partial \delta_{ij}(t)}{\partial x_l} \tag{2}$$

where the rotor angles are with respect to a common reference. In an offline transient stability simulation, the common reference is the rotor angle of the reference generator. In WAM application, synchronized phasor measuring units (PMUs) will enable computation of δ_{ij}. Clearly

$$S_l^{ij}(t) + S_l^{ji}(t) = 0.$$

Therefore, at least one of the sensitivities is non-negative. The application of BITS for a single-machine infinite bus system (SMIB) is shown below.

B. Motivational Example Using SMIB System

In a SMIB system, a self-clearing three-phase fault at the terminal of the machine is simulated. It is cleared after 0.1 s. The objective is to study the sensitivity of the rotor angle trajectory to small perturbations in the line reactance under different loading conditions.

Mechanical input power is considered to be a constant. With $P_M = 1.0$ pu, $M = 0.0159$ pu, $D = 0.01$ pu, line reactance $x^r = 0.5$ p.u., and fault clearing time $t_{clear} = 0.1$ s, the rotor angle response to the fault for the nominal value of x^r ($x_0^r = 0.5$ p.u.) and x^r perturbed by 1% ($x^r = 0.505$ p.u.) is shown in Fig. 2. BITS versus time is plotted in Fig. 3. The resultant swing in the P-δ plane is shown in Fig. 4. Note that one cannot distinguish between the two steady-state P-δ characteristics because a 1% increase in the reactance will approximately translate into a 1% reduction in the maximum steady-state power transfer capacity. As seen in Fig. 4, on the P-δ plane, both the systems (unperturbed and perturbed) have almost identical response viz., A-B-C-D-E-D-A-F-A. Hence, the trajectory sensitivity is small (Fig. 3). The peak value is 15, which implies that a 1% increase

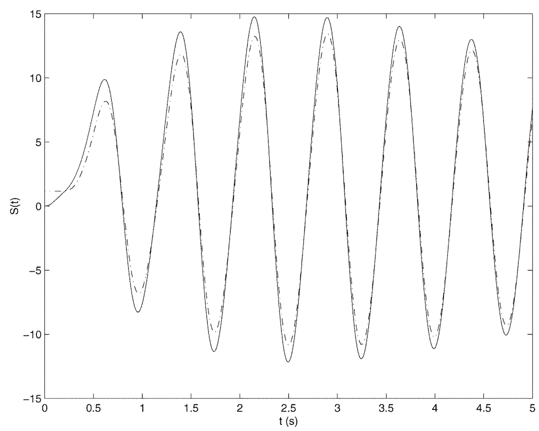

Fig. 3. Rotor angle trajectory sensitivity ($\mathrm{P_m} = 1.0$ p.u..).

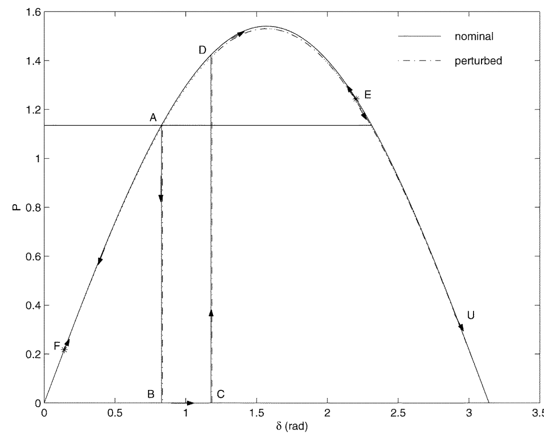

Fig. 5. Power swings in P-δ plane for $P_M = 1.135$ p.u.

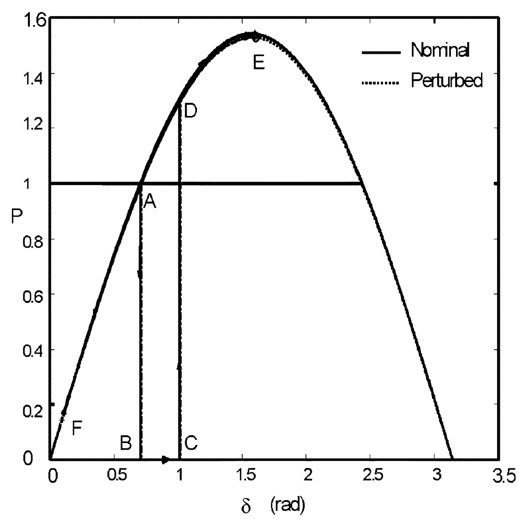

Fig. 4. Power swings in P-δ plane $\mathrm{P_m} = 1.0$ p.u.

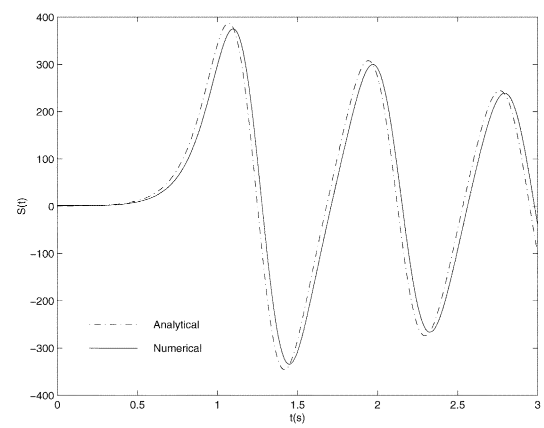

Fig. 6. Rotor angle trajectory sensitivity $P_M = 1.135$ p.u.

in the line reactance causes 4.29° increase in the rotor angle. Therefore, the following conclusion can be drawn for an unstressed system.

Unstressed System: W*ith a large stability margin (i.e., a large decelerating area), the response of the marginally perturbed system closely follows the unperturbed response. In other words, trajectory sensitivity $S(t)$ obtained by linearizing the response around $\delta(t, x_0^r)$ is small*

$$\delta(\mathrm{t}, \mathrm{x}_0^\mathrm{r} + \Delta \mathrm{x}^\mathrm{r}) = \delta(\mathrm{t}, \mathrm{x}_0^\mathrm{r}) + \mathrm{S}(\mathrm{t}) * \Delta \mathrm{x}^\mathrm{r}. \quad (3)$$

Stated more formally, if the system defined by (3) is stable, then (a) $\max_{\mathrm{t}} |S(t)| \leq M$ where M is a finite nonnegative number and (b) $\lim_{x^r \to x_0^r} \delta(t, x^r) = \delta(t, x_0^r)$. Part (a) follows from the bounded input bounded output criteria of stability. Consequently, $\|\delta(t, x^r) - \delta(t, x_0^r)\| \leq M\Delta x^r$. In the limit $\Delta x^r \to 0$ and, hence, part (b) follows:

Remark 1: For a stable response in the limit $\mathrm{x}^\mathrm{r} \to \mathrm{x}_0^\mathrm{r}$, the perturbed response $\delta(\mathrm{t}, \mathrm{x}^\mathrm{r})$ will be in phase with the unperturbed response $\delta(\mathrm{t}, \mathrm{x}_0^\mathrm{r})$. Hence, in the $\lim \mathrm{x}^\mathrm{r} \to \mathrm{x}_0^\mathrm{r}$, the trajectory sensitivity $\mathrm{S(t)}$ response will be collinear with the angle response $\delta(\mathrm{t}, \mathrm{x}_0)$. Consequently, the peaks and troughs of $\mathrm{S(t)}$ will coincide with the peaks and troughs of $\delta(\mathrm{t}, \mathrm{x}_0)$. We now investigate the behavior of a stressed system.

Stressed System: When the initial conditions are changed from $P_M = 1.0$ p.u. to $P_M = 1.135$ p.u., the response of the system (A-B-C-D-E-D-A-F) with reactance at the nominal value will be stable (Fig. 5). Now, the stability margin is low and the trajectory sensitivity is high. The peak value is 400 in Fig. 6, which is 26 times of unstressed system (Fig. 3). When a transient stability simulation is performed with 1% perturbation in the reactance value ($\mathrm{x}_\mathrm{new}^\mathrm{r} = 0.505$ p.u.), the system becomes unstable (Fig. 5) with trajectory A-B-C-D-E-U. When the trajectory sensitivity is computed with $\mathrm{x}^\mathrm{r} = 0.505$ p.u., it increases monotonically to infinity. Thus, when the system is operating close to the transient stability limit, a small positive perturbation in x induces transient instability. As machines go out of step, the trajectories of the unperturbed and perturbed system diverge. This leads to high BITS. Therefore, BITS can be used as a measure for system stability.

Trajectory sensitivities can be computed by modeling additional differential equations in a transient stability program [10]. These results agree with finite difference approximation of derivatives as shown in Figs. 3 and 6. Finite difference approximation does not require extra programming. As BITS can measure stability margin, it can be used for ranking. The next section develops this approach.

IV. Norms: A Measure for Angle Stability

In a multimachine system, given a fault, the search space for computing the BITS is $n_{\rm t} \times n_l \times C_{n_g}^2$ where $\rm n_t$, $\rm n_l$, and n_g are the number of snapshots, lines, and generators in the network, respectively. This is a very large number. To compress information, norms can be used. Norms facilitate ranking by assigning a single non-negative scalar to the behavior under investigation. For a fault "f," line "l" and time t, following norm, was introduced in [14] to compute the $\rm t_{critical}$

$$N_n(f,l,t) = \sqrt{\sum_{i=1}^{n_g-1}\left(\frac{\partial \alpha_i}{\partial x}\right)^2 + \sum_{i=1}^{n_g}\left(\frac{\partial \omega_i}{\partial x}\right)^2} \quad (4)$$

where ω_i and $\alpha_i = \delta_i - \delta_{ng}$ are evaluated at time "t." This norm compresses the information associated with multiple machine pairs. The following norms are now proposed to further compress the information in time dimension.

Branch Norm & Relay Ranking Problem: For a fault "f" the following functions associate a non-negative scalar with line "l."

$$N_\infty(f,l) = \max_{\rm t} N_n(f,l,t) \quad (5)$$

$$N_2(f,l) = \sqrt{\sum_{\rm t=1}^{n_{\rm t}} |N_n(f,l,t)|^2} \quad (6)$$

$$S_\infty(f,l) = \max_{ij}\max_{\rm t} S_{ij}^l(t). \quad (7)$$

These will be referred to as **branch norms.** Norm defined by (5) samples the peak $\mathrm{N}_n(f,l,t)$ over the time span $\rm t = 0 \ldots n_t$. The $\rm N_\infty$ norm emphasizes only peak behavior while $\rm N_2$ norm lays stress on the rms behavior of BITS given by (6). The norm defined by (7) samples the peak BITS over all machine pairs.

For a fault, branches (lines, transformers) can be ranked in the descending order of the branch norm. This will solve the relay-ranking problem. Therefore, in a distance relay coordination program, performance evaluation on power swings can be restricted to only those relays that protect the first few ranked lines. The following norms further reduce the dimension associated with the number of transmission lines.

Fault Norm & Fault Ranking Problem: For a fault "f," the following functions associate a non-negative scalar viz. maximum of all branch norms with a fault

$$N_\infty(f) = \max_{l} N_\infty(f,l) \quad (8)$$

$$S_\infty(f) = \max_{l} S_\infty(f,l). \quad (9)$$

$N_2(f)$ can be defined in a similar way. $S_\infty(f)$ samples the peak rotor angle BITS for a given fault in the $n_{\rm t} \times n_l \times C_{n_g}^2$ space. Henceforth, these will be referred to as **fault norms**.

For a system, faults can be ranked in the descending order of fault norm. This will solve the fault or contingency ranking problem. The rank-1 fault corresponds to the worst contingency. Therefore, noncritical faults need not be investigated for distance relay coordination.

System Norm and Out-of-Step Protection: For a set of faults, the following functions associate a non-negative scalar viz. maximum of all fault norms with the system

$$S_{\max} = \max_{f} S_\infty(f) \quad (10)$$

$$N_{\max} = \max_{f} N_\infty(f). \quad (11)$$

Henceforth, they will be referred to as **system norms**. For example, $\rm S_{max}$ samples peak BITS among all of the faults in the system. The magnitude and the rate of change of the system norm with respect to a system parameter can be used to classify a swing as stable or unstable. As the system approaches instability, the system norm should increase monotonically to infinity.

A. Other Norms for Relaying

Norm being a measure can be defined in multiple ways. The BITS-based norms are appealing because of the direct correlation with the rotor angle swings. Alternative measures (norms) that can be defined are as follows:

$$\mathrm{Z_{min}}(l,f) = \min_{\rm t} |\mathrm{Z_{min}}(l,f,t)| \quad (12)$$

$$\mathrm{Z_{min}}(f) = \min_{l} \mathrm{Z_{min}}(l,f) \quad (13)$$

$$\mathrm{Z_{min}} = \min_{f} \mathrm{Z_{min}}(f). \quad (14)$$

For a fault and a transmission line, (12) associates a single non-negative scalar with the power swing observed by the relay. A transmission line has two relays, one at each end of the line. The relay that looks in the direction of fault should be used for $\rm Z_{min}$ calculation. $\mathrm{Z_{min}}(l,f)$ is an alternative for the *branch norm*. For a fault, (13) associates a single scalar, which is the minimum distance to the origin reached by the swing in R-X plane in the entire transmission system. Therefore, it can be classified as a *fault norm*. Equation (14) associates a similar scalar for all of the faults considered in the system. Hence, it can be classified as a *system norm*. Another alternative could be based on RSM. Yet another alternative for fault norm can be the critical clearing time of a fault. Then, the system norm will be the minimum critical clearing time in the entire system.

Remark 2: Branch norm, fault norm, and system norm provide a three-tier hierarchy for organization of information. For a fault, evaluation of all branch norms leads to the fault norm. Evaluation of all fault norms leads to the system norm.

B. Relationship Between Various Measures

All three categories of the norms viz. BITS, $\rm Z_{min}$, and $\rm t_{critical}$ measure the same behavioral pattern. Hence, a correlation should exist between them. For example, the evaluation of a BITS norm ((5) and (7)) requires computation of a maximum whereas the corresponding $\rm Z_{min}$ norm ((12)) seeks the minimum. Hence, as the criticality of a fault increases, the BITS norm should increase while the $\rm Z_{min}$ norm should decrease.

This indicates an approximate inverse relationship between BITS and Z_{min}. From [12, (4)], for a SMIB system

$$|Z_{\mathrm{R}}(t, x_o^{\mathrm{r}})| = \frac{|Z_{\mathrm{L}}|}{2\sin\left(\frac{\delta(t, x_0^{\mathrm{r}})}{2}\right)}. \tag{15}$$

From (15), at $\delta = \delta_{\max}$, $Z_{\mathrm{R}}(t, x_o^{\mathrm{r}})$ is minimum. By remark 1, at $\delta(t) = \delta_{\max}$, $S(t)$ is also at its maximum. Hence, there is an inverse relationship between BITS norm and Z_{min} of a line. This permits development of a *generic* ranking scheme.

A Generic Ranking Scheme for Relay and Fault Ranking

1. **Initialization Step:** List the set of faults **F** of interest, in consultation with utility engineers. In the extremity, all lines can be considered. It implies at least as many transient stability simulations. Later on, *a priori* knowledge can be used to prune the list. In this work, only three-phase faults at the end of a line are considered. By introducing *a virtual* node, faults in between the end nodes can also be simulated.

2. **Relay Ranking Step:** For each fault "f" from the set **F**,
 1) Compute the branch norm for all lines of interest. The norm can be based on BITS Z_{min} or RSM.
 2) Rank the branch norms and list the branch norm ranking. The branch norm ranked 1 is the fault norm associated with the fault "f."

3. **Fault Ranking Step:** Rank the fault norms and list the fault ranking. Fault norm ranked 1 is the system norm.

V. Results

The proposed approach has been tested on three machines, 9-bus system [15]; ten machines, 39-bus system [16]; and a utility system in India. The relay coordination was done using an object-oriented relay coordination program [17], [18]. For illustration purpose, all relays are modeled by mho characteristics. The shape of the relay characteristic only affects the RSM/RM calculations.

A. Ten-Machine Example System

The single line diagram of the system is shown in Fig. 7. All of the machines are represented as classical models. The trajectory sensitivities are computed by considering the additional sensitivity equations along with the system equations [14]. A few illustrative cases are now discussed.

Ranking by Branch Norm: A three-phase fault is created on line 28–29 close to bus 28. Henceforth, it will be referred as fault F1. The fault is cleared in 0.06 s by tripping the line 28–29 simultaneously from both the ends. This is illustrative of the high-speed protection system operation on the EHV network using distance relays and carrier communication [19]. Table I captures the normalized branch norm indices for a subset of lines, which have high, medium, and low sensitivities. It also summarizes BITS norms for various loading conditions (90 – 110%). Normalization with respect to the peak N_∞ norm (column 2 of Table I) brings out the relative criticality of various lines.

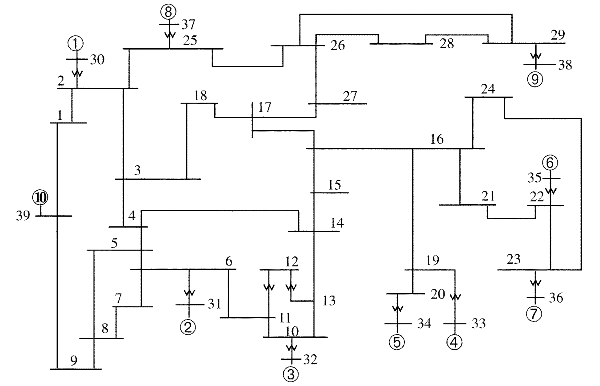

Fig. 7. Ten-machine system.

TABLE I
RANKING BY BRANCH NORM FOR FAULT F1

Normalized N_∞ with 110 % loading				Absolute N_∞ BITS for different loading conditions			
Line	N_∞	RSM	Z_{min}	Line	% Loading 90	100	110
29-26	1	.023	.066	29-26	90.27	145.1	30822
26-27	.163	.176	.184	26-27	25.69	31.74	5022
26-25	.151	.187	.258	26-25	9.41	20.02	4656
26-28	.011	.484	.508	26-28	5.88	6.81	343.0

As line 29–26 has rank 1, it can be characterized as the weakest link. The normalized indexes for all other lines are much lower. Therefore, other lines are far less vulnerable to tripping on swing. To confirm these inferences, RSM is computed for relays on these lines. The RSMs are tabulated in the column 3 of Table I. Lines are designated in Table I in such a way that relay at the *from end* looks in the direction of the fault. The RSM computation corresponds to the relay at the *from end*. The minimum magnitude of relay impedance (Z_{min}) is shown in column 4. It is seen that both norms rank consistently with the BITS norm.

Another case that was investigated corresponds to a three-phase fault on line 4–5 close to bus 4 with t_{clear} of 0.1 s (fault F2). The corresponding results are summarized in Table II. Tables I and II indicate that the branch ranking does not change with system loading.

Location of Electrical Center: For the fault F1, as line 29–26 is the weakest link in the postfault system, it has a large possibility of developing the electrical center. To confirm the location of the electrical center, t_{clear} was increased from 0.06 s to 0.07 s. This induced instability. The closeness of t_{clear} to the critical clearing time also indicates that the existing system is working close to the stability limit. Under instability, the system splits into two groups. By a transient stability simulation, it is confirmed that the electrical center is on the line 29–26.

TABLE II
RANKING BY BRANCH NORM FOR FAULT F2

Normalized N_∞ with 100% loading				Absolute N_∞ BITS for different loading conditions			
Line	N_∞	RSM	Z_{min}	Line	% Loading		
					90	100	110
1-2	1	.160	.190	1-2	53.73	57.46	60.63
1-39	.968	.169	.204	1-39	51.15	55.62	58.06
2-25	.619	.234	.204	2-25	29.18	35.60	53.06
14-15	.541	.320	.331	14-15	26.55	31.07	36.3

TABLE III
RANKING BY FAULT NORM WITH 110% LOADING

Faulted Line	Fault near bus	$N_\infty(f)$ Normalized	Fault norm Z_{min}	Fault Norm $t_{critical}$
28 – 29	28 (F1)	1	0.0658	0.07
21 –22	22	0.002	0.1494	0.23
17 –18	17	0.002	0.1617	0.25
4 –5	4 (F2)	0.002	0.1900	0.29

Ranking by Fault Norm: From Tables I and II, the fault norm associated with the fault F1 at 110% system loading is 30 822 and the corresponding fault norm for F2 is 60.63. Table III ranks four faults in the descending order of their BITS fault norm. It is seen that the BITS norm ranks consistently with the Z_{min} norm which, in turn, ranks consistently with the $t_{critical}$ norm.

BITS as a Transient Stability Margin Indicator: Tables I and II indicate that with an increase in the system loading, the trajectory sensitivities also increase. This indicates a reduction in the stability margin. Fault F1 is the critical fault for the system because it exhibits a very steep rise in the fault norm, with an increase in the system loading. In contrast, it can be seen that for the fault F2, the fault norm is low and its gradient is very small. This indicates that for this fault, the loading conditions are still far away from the system stability boundary. In fact, the critical clearing time for this fault with the nominal loading condition is 0.29 s. Hence, the fault F2 can be classified as noncritical. *Critical fault loading* can be defined as the system load that induces postfault separation even with normal fault clearing time. *Critical system loading* will be the minimum of critical fault loading over the set of faults considered. It can be computed from the critical fault loading of the fault ranked one.

B. Indian Utility System Example

The data have been adapted from the western regional grid of India. The system consists of 283 busses, 391 transmission lines, 123 transformers, 55 generators, 189 loads, and 28-shunt reactors. The system load is about 15 000 MW. The entire system is simulated in the transient stability program developed at IIT Bombay. The generators are represented in detail–up to 2.2 model (one field winding and one damper winding on the d-axis, and two damper windings on the q-axis) [20]–[22]. Parameters in per unit of a 247-MVA machine, on its own base, are given in the appendix. Only a part of the 400-kV network is considered for relay coordination and performance evaluation on power swings. It is shown in Fig. 8. It corresponds to the 400-kV network of a state electricity board in the grid. The relay settings (zone 1 and 2) obtained from the relay coordination program matched closely with the utility settings. All of the settings are detailed in [7]. A few interesting cases are now discussed.

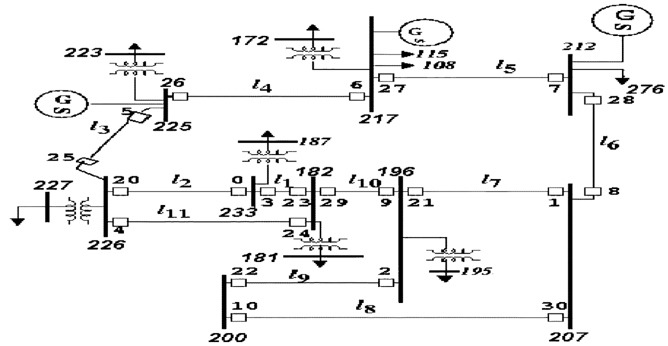

Fig. 8. Indian utility system (drawn to scale).

TABLE IV
(A) CASE 1: RANKING BY BRANCH NORM (WITH $R_f = 0.01$ p.u.). (B) CASE 1: RANKING BY BRANCH NORM (WITH $R_f = 0$ p.u.).

(A)

Line No. (Relay No.)	Normalized BITS norm		RSM / RM	Z_{min}
	N_∞	N_2		
6 (28)	1	1	0.3867 *(RM)*	0.0755
4 (26)	0.7665	0.7780	0.0562	0.0788
3 (25)	0.5184	0.5240	0.0717	0.0884
7 (1)	0.4573	0.4857	0.0962	0.1014

(B)

Line No. (Relay No.)	Normalized BITS norm		RSM	Z_{min}
	N_∞	N_2		
6 (28)	1	1	0.00012	0.09563
4 (26)	0.8997	0.9710	0.08466	0.10703
3 (25)	0.7399	0.7231	0.09375	0.11559
7 (1)	0.5493	0.6339	0.09460	0.12230

Ranking by Branch Norm:

a) Case 1: Three-Phase Fault on Line 1, Near Bus 182, Fault-Clearing Time is 0.1 s. Fault Resistance is 0.01 p.u.: Table IV(a) tabulates the result for a fault on line 1, near bus 182 with a fault clearing time of 0.1 s and fault resistance of 0.01 p.u. It is observed that normalized ranking based on BITS branch norm is consistent with the RSM. Further, the swing is severe enough to enter zone 3 of relay 28. The duration of stay of the swing in zone 3 is less than the TDS setting of relay 28. Therefore, the relay does not trip. This is indicated by the relay margin being less than unity. This relay is also ranked first by the Z_{min} branch norm. The swing characteristic on the R-X plane for relays 25 and 28 are shown in Figs. 9 and 10, respectively. They confirm the results of Table IV(a). Table IV(b) summarizes the results for the same fault with $R_f = 0$ p.u.. Again, it is observed that the branch norm by BITS, RSM, and Z_{min} indicators give consistent results. However, the RSM in row 1 is now close to zero, which indicates that the swing is just approaching zone 3. Also, both N_∞ and N_2 norms give similar ranking.

b) Case 2: Additional Case Studies: Table V summarizes the results for two more cases. Case 2A corresponds to a three-phase fault on line 10 near bus 196, and case 2B corresponds

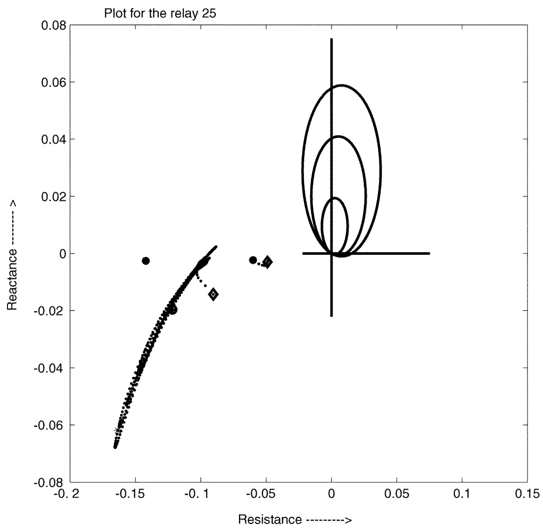

Fig. 9. Swing characteristics in the R-X plane of relay 25.

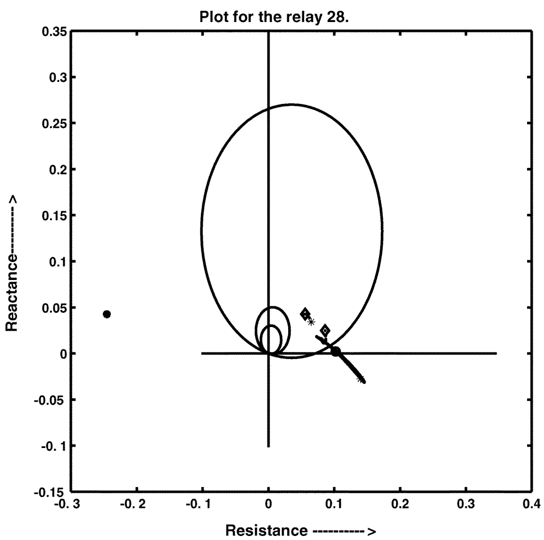

Fig. 10. Swing characteristics of relay 28.

TABLE V
RANKING BY BRANCH NORM: ADDITIONAL CASES

Case 2A			Case 2B		
Line No. (Relay No.)	Normalized N_∞ norm	Z_{min}	Line No. (Relay No.)	Normalized N_∞ norm	Z_{min}
4 (26)	1	0.0851	6 (28)	1	0.0893
3 (25)	0.7335	0.0933	4 (26)	0.6696	0.1017
6 (28)	0.7208	0.0962	3 (25)	0.6162	0.1097
1 (23)	0.6389	0.5474	7(1)	0.5728	0.1192

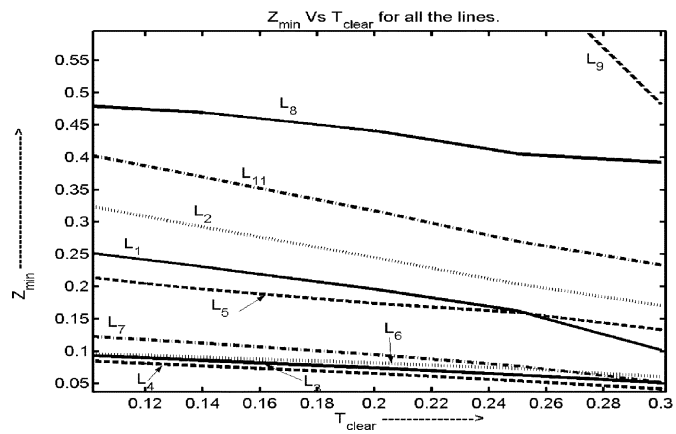

Fig. 11. $\mathrm{Z}_{\mathrm{min}}$ versus $\mathrm{t}_{\mathrm{clear}}$ for case 2A.

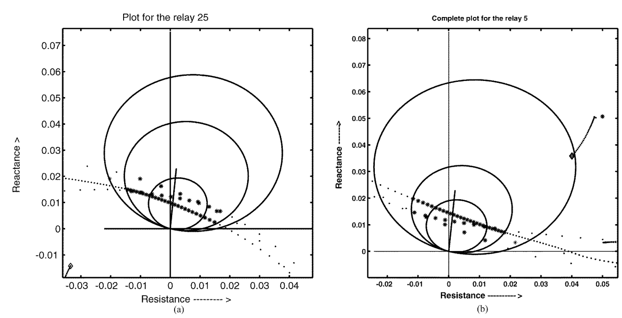

Fig. 12. Electrical center as seen by relays 25 and 5.

to a three-phase fault on line 2 near bus 233. Again, it can be seen that the ranking by N_∞ branch norm matches with the $\mathrm{Z}_{\mathrm{min}}$ norm. Though not tabulated in Table V, consistent rankings were observed with the N_2 norm. For the case 2A, $\mathrm{Z}_{\mathrm{min}}$ versus clearing time of the relays looking toward the fault is plotted in Fig. 11. It is observed that $\mathrm{Z}_{\mathrm{min}}$ branch norms reduce linearly as the clearing time increases. Hence, the relay ranking is independent of the $\mathrm{t}_{\mathrm{clear}}$.

Location of Electrical Centers: When the duration of the fault on line 10 near bus 196 is increased to create system instability, it is observed that the system splits into three groups. The application of proposed algorithm (Section II) identified 16 electrical centers. Three electrical centers on line l_3, l_7, and l_8, were located on the subnetwork shown in Fig. 8. With a clearing time of 0.35 s, the electrical center seen on line 3 by the relays 25 and 5 is shown in Fig. 12(a) and (b), respectively.

Ranking by Fault Norm: Results of normalized BITS fault norm and $\mathrm{Z}_{\mathrm{min}}$ fault norm for various faults are summarized in Table VI. It is seen that they provide consistent ranking. The system norm corresponds to the branch norm of line 6 for fault on line 1.

System Norm and Out-of-Step Relaying: It is observed that in the case 2A, as the clearing time increases, the peak swing also increases from 128° at $\mathrm{t}_{\mathrm{clear}} = 0.1$ s to 180° at 0.2 s. In a multimachine system, a 180° swing may recover. This can be deduced by the negative gradient of system norm at $\mathrm{t}_{\mathrm{clear}} = 0.2$ s (Fig. 13). Beyond 0.25 s, a swing of about 180° as well as a steep rise in system norm is encountered. Consequently, stability margin decreases rapidly and the system loses stability at 0.3 s. To conclude, out-of-step condition can be detected by the system norm and its rate of change.

C. Comparative Evaluation of Indicators

The results show that RSM $\mathrm{Z}_{\mathrm{min}}$, BITS, and $\mathrm{t}_{\mathrm{critical}}$ norms are consistent. Tables I–VI indicate an approximate inverse relationship between BITS norm with $\mathrm{Z}_{\mathrm{min}}$ and $\mathrm{t}_{\mathrm{critical}}$ norms. Advantage of the RSM indicator is that it accounts for the relay

TABLE VI
RANKING BY FAULT NORM

Faulted Line	Fault near bus	Normalized N_∞	N_∞ on line	Z_{min}
1 (R_f=0.01)	182	1	6	0.0755
10	196	0.4621	4	0.0851
2	233	0.3871	6	0.0893
1 (R_f=0.0)	182	0.2234	6	0.0956

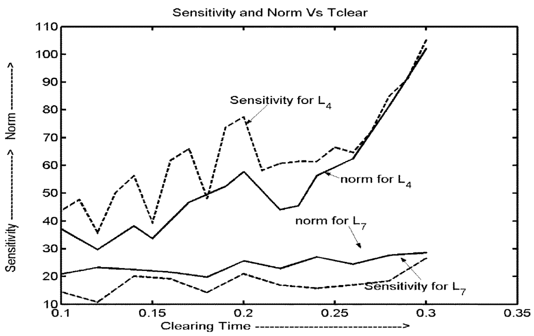

Fig. 13. Norm versus clearing time for case 2A.

characteristics. On the other hand, Z_{min}, BITS, and $t_{critical}$ indicators are more generic. RSM and Z_{min} computation requires only one transient stability simulation per fault. Postfault processing is less with the Z_{min} indicator than the RSM indicator. In the case of the BITS indicator, an additional transient stability run per branch norm is required. Consequently, in a relay coordination program, Z_{min} indicator can be used as the primary ranking tool.

Note: In Fig. 13, norm N_∞ for l_4 has been drawn to a scale of 1/10. Hence, 110 indicates N_{max} of 1100. Also, N_∞ for l_7 has been drawn to a scale of 1/8.

VI. CONCLUSIONS

Relay coordination programs primarily ascertain the *dependability* of the relaying scheme. An important *security* requirement is that distance relays should not trip on power swings. In the case of swings due to system instability, it is important to locate all electrical centers in the system. Therefore, a systematic procedure that complements the transient stability simulation has been developed.

Protection engineers should also evaluate the performance of distance relaying under stable power swings. As the simulation space is large (*curse of dimensionality*), ranking tools are desired. Hence, a three-tier hierarchy of norms viz., branch norm, fault norm, and system norm is proposed to solve the relay ranking, fault ranking, and out-of-step detection problem. Ranking of faults identifies critical faults, while ranking of relays identifies critical relays. Together, they pinpoint the faults and the relays that have to be investigated on power swings.

APPENDIX

The machine parameters for a 247-MVA generator in per unit on the machine base are as follows.

$X_d = 2.225$, $X'_d = 0.305$, $X''_d = 0.214$, $T'_{do} = 7.00$, $T''_{do} = 0.040$, $X_q = 2.11$, $X'_q = 0.75$, $X''_q = 0.25$, $T'_{qo} = 2.00$, $T''_{qo} = 0.170$, Inertia constant $= 2.80$ MJ/MVA, AVR/static exciter: Gain $= 200$, Time constant $= 0.05$ s.

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