Quantifying Transmission Reliability Margin

Jianfeng Zhang, Ian Dobson, Fernando L. Alvarado

Abstract— **In bulk electric power transfer capability computations, the transmission reliability margin accounts for uncertainties related to the transmission system conditions, contingencies, and parameter values. We propose a formula which quantifies transmission reliability margin based on transfer capability sensitivities and a probabilistic characterization of the various uncertainties. The formula is verified by comparison with results from two systems small enough to permit accurate Monte-Carlo simulations. The formula contributes to more accurate and defensible transfer capability calculations.**

Keywords— **transfer capability, sensitivity, uncertainty, power transmission reliability, power system security, power system availability**

I. Introduction

Bulk power transfer capability computations have many uses in electric power system operation and planning. In the operation of bilateral markets, available transfer capability is used to allocate reservations of transmission rights [15], [16], [17]. In the operation of pooled markets, transfer capability combined with bid information can be used to help allocate financial transmission rights or transmission congestion contracts. In both planning and operations, transmission capability can be used to assess power system security when local power sources are replaced by imported power. Finally, transfer capability can be used to provide capacity data for simplified power system models suitable for locational price forecasting. All of these applications are reviewed in [8].

In many of these applications, it is desirable to quantify the uncertainty in the transfer capability computation as a safety margin so that if the computed transfer capability minus the safety margin is used, it is likely that the power system will remain secure despite the uncertainty. The transmission reliability margin (TRM) accounts for the uncertainties associated with the transmission system. Deregulation of power systems has increased the need for defensible calculations of transfer capability and related quantities such as the transmission reliability margin.

This paper describes a straightforward method to estimate the transmission reliability margin. The method exploits formulas for first order sensitivity of transfer capability [12], [9], [7]. These formulas can be quickly and easily computed when transfer capability is determined. The formulas determine a linear model for changes in transfer capability in terms of changes in any of the power system parameters. This paper supposes that the uncertainty of the parameters can be estimated or measured and shows how to estimate the corresponding uncertainty in the transfer capability. A formula for transmission reliability margin is then developed based on the uncertainty in the transfer capability and the desired or agreed upon degree of safety.

J. Zhang, I. Dobson, and F. L. Alvarado are with the Electrical & Computer Engineering Department of the University of Wisconsin, Madison WI 53706 USA. email dobson@engr.wisc.edu. Support in part from NSF research centers-small firms collaborative R&D initiative grant EEC-9815325 and grant ECS-9988574 is gratefully acknowledged.

II. transfer capability and TRM

We summarize a generic transfer capability calculation [12], [8] and discuss transmission reliability margin.

The time horizon of the transfer capability calculation is established and a secure base case is chosen. A base case transfer including existing transmission commitments is chosen. Then a transfer limited case is determined. One method to determine the transfer limited case gradually increases the transfer starting at the base case until the first security violation is encountered. The real power transfer at the first security violation is the transfer capability. The calculation may be repeated for a short list of contingencies and the minimum of these transfer capabilities is used.

In our framework [12], the following limits are accounted for in the transfer capability computation:

- power flow or current limits (normal and emergency)
- voltage magnitude upper and lower limits (normal and emergency)
- voltage collapse limit

Our framework accounts directly only for limits which can be deduced from static model equations. Although oscillation and transient stability limits can be studied offline and approximated by surrogate power flow limits, the sensitivities of the surrogate power flow limits will not be the same as the sensitivities of the oscillation and transient stability limits. Thus our methods do not extend to the estimation of uncertainties associated with oscillation and transient stability limits.

According to the North American Electric Reliability Council [16], "The determination of ATC must accommodate reasonable uncertainties in system conditions and provide operating flexibility to ensure the secure operation of the interconnected network". There are two margins defined to allow for this uncertainty: The transmission reliability margin is defined in [16] as "that amount of transmission capability necessary to ensure that the interconnected transmission network is secure under a reasonable range of uncertainties in system conditions". The capacity benefit margin ensures access to generation from interconnected systems to meet generation requirements. The capacity benefit margin is calculated separately from the transmission reliability margin.

Since uncertainty increases as conditions are predicted further into the future, the transmission reliability margin will generally increase when it is calculated for times further into the future.

III. Quantifying TRM

A. Parameters and their uncertainty

The transfer capability is a function A of many parameters $p_1, p_2, \ldots, p_m$:

$$\text{transfer capability} = A(p_1, p_2, \ldots, p_m) \tag{1}$$

Uncertainty in the parameters p_i causes uncertainty in the transfer capability and it is assumed that this uncertainty in the transfer capability is the uncertainty to be quantified in the transmission reliability margin. The uncertain parameters p_i can include factors such as generation dispatch, customer demand, system parameters and system topology. The parameters are assumed to satisfy the following conditions:

1. Each parameter p_i is a random variable with known mean $\mu(p_i)$ and known variance $\sigma^2(p_i)$. These statistics are obtained from the historical record, statistical analysis and engineering judgment.
2. The parameters are statistically independent. This assumption is a constraint that can be met in practice by careful selection of the parameters [3].

B. Transfer capability sensitivity

We assume that the nominal transfer capacity has been calculated when all the parameters are at their mean values. The uncertainty U in the transfer capability due to the uncertainty in all the parameters is:

$$U = A(p_1, p_2, ..., p_m) - A(\mu(p_1), \mu(p_2), ..., \mu(p_m)) \tag{2}$$

The mean value of the uncertainty is zero:

$$\mu(U) = 0 \tag{3}$$

Approximating the changes in transfer capability linearly in (2) gives

$$U = \sum_{i=1}^{m} \frac{\partial A}{\partial p_i} (p_i - \mu(p_i)) \tag{4}$$

$\frac{\partial A}{\partial p_i}$ is the small signal sensitivity of the transfer capability to the parameter p_i evaluated at the nominal transfer capability.

When the transfer capability is limited by voltage magnitude or thermal limits, the sensitivity of the transfer capability to parameters can be computed using the formulas of [12], [9].

When the transfer capability is limited by voltage collapse, the sensitivity of the transfer capability to parameters can be computed using the formulas of [10]. (Topology changes can also be accommodated with limited accuracy using the fast contingency ranking techniques in [11].)

In each case a static, nonlinear power system model is used to evaluate the sensitivities. The computation of $\frac{\partial A}{\partial p_i}$ is very fast and the additional computational effort to compute $\frac{\partial A}{\partial p_i}$ for many parameters p_i is very small [10], [12], [9]. For example, the sensitivity of the transfer capability to all the line admittances can be calculated in less time than one load flow in large power system models [10], [12].

C. Approximate normality of U

Since the parameters are assumed to be independent,

$$\sigma^2(U) = \sum_{i=1}^{m} \sigma^2 \left(\frac{\partial A}{\partial p_i} (p_i - \mu(p_i)) \right) \tag{5}$$

$$= \sum_{i=1}^{m} \left(\frac{\partial A}{\partial p_i} \right)^2 \sigma^2(p_i) \tag{6}$$

and the standard deviation of U is

$$\sigma(U) = \sqrt{\sum_{i=1}^{m} \left(\frac{\partial A}{\partial p_i} \right)^2 \sigma^2(p_i)} \tag{7}$$

The central limit theorem asserts that, under suitable conditions which are discussed in the appendix, the sum of n independent random variables has an approximately normal distribution when n is large. Reference [13] states: "in practical cases, more often than not, $n = 10$ is a reasonably large number, while $n = 25$ is effectively infinite." Hence for practical power system problems with many parameters, we expect that the uncertainty U is approximately a normal random variable with mean zero and standard deviation given by (7). This approximation gives a basis on which to calculate the transmission reliability margin. The conditions described in the appendix are mild and require little knowledge of the distribution of the parameters.

There are cases in which the central limit theorem approximation does not work well: As stated in [13], "the separate random variables comprising the sum should not have too disparate variances: for example, in terms of variance none of them should be comparable with the sum of the rest." This can occur in (4) when there are a few parameters which heavily influence the transfer capability (large $\frac{\partial A}{\partial p_i}$) and the other parameters have little influence on the transfer capability (small $\frac{\partial A}{\partial p_i}$) and are insufficiently numerous. In these cases, accurate answers can be obtained by using the central limit theorem to estimate the combined effect of the numerous parameters of little influence as a normal random variable and then finding the distribution of U with the few influential parameters by Monte Carlo or other means (c.f. [4] in the context of probabilistic transfer capacity). This partial use of the central limit approximation dramatically reduces the dimension of the problem and the computational expense of solving it.

In all cases the central limit theorem approximation improves as the number of similar parameters increases and thus the approximation generally improves as the power system models become larger and more practical.

D. Formula for TRM

We want to define the transmission reliability margin large enough so that it accounts for the uncertainty in U with rare exceptions. More precisely, we want

$$\text{probability}\{-U \le \text{TRM}\} = P \tag{8}$$

where P is a given high probability. This can be achieved by choosing the transmission reliability margin to be a certain number K of standard deviations of U:

$$\text{TRM} = K\sigma(U) \tag{9}$$

K is chosen so that the probability that the normal random variable of mean zero and standard deviation 1 is less than K is P. (That is, $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{K} e^{-t^2/2}dt = P$.) It is straightforward to calculate K from P by consulting tables of the cumulative distribution function of a normal random variable [2]. For example, if it is decided that the transmission reliability margin should exceed the uncertainty $-U$ with probability P=95%, then $K = 1.65$. (That is, a normal random variable is less than 1.65 standard deviations greater than the mean 95% of the time.) If it is decided that the transmission reliability margin should exceed the uncertainty $-U$ with probability P=99%, then $K = 2.33$.

Combining (7) and (9) yields a formula for transmission reliability margin:

$$\text{TRM} = K\sqrt{\sum_{i=1}^{m}\left(\frac{\partial A}{\partial p_i}\right)^2 \sigma^2(p_i)} \tag{10}$$

In order to use formula (10) we need:
- A choice of uncertainty parameters $p_1, p_2, ..., p_m$ satisfying the three conditions above.
- The variance $\sigma^2(p_i)$ of each parameter.
- Calculation of the sensitivity $\frac{\partial A}{\partial p_i}$ of the transfer capability to each parameter p_i.

IV. Uncertainty parameters

The transfer capability is computed from a base case constructed from system information available at a given time. There is some uncertainty or inaccuracy in this computation. There is additional uncertainty for future transfer capabilities because the transfer capability computed at the base case does not reflect evolving system conditions or operating actions. These two classes of uncertainty are detailed in the following two subsections.

A. Uncertainties in the base case transfer capability

These uncertainties are:
- inaccurate or incorrect network parameters
- effects neglected in the data (e.g. the effect of ambient temperature on line loading limits)
- approximations in transfer capability computation

B. Uncertainties due to evolving conditions

These uncertainties are:
- ambient temperature and humidity (contributes to loading) and weather
- load changes not caused by temperature
- changes in network parameters
- change in dispatch
- topology changes. This is often referred to as "contingencies." The probabilities of these contingencies can be estimated.
- changes in scheduled transactions

These uncertainties generally increase when longer time frames are considered. While some of these uncertainties may be quite hard to characterize a priori, it is important to note that it would be practical to collect empirical data on the changes in base cases as time progresses. Then variances of the uncertain parameters corresponding to various time frames could be estimated.

It is important to satisfy the statistical independence assumption when modeling parameter uncertainty. For example, if the uncertainty of different loads has a common temperature component, then this temperature component should be a single parameter and the load variations should be modeled as a function of temperature.

V. Simulation test results

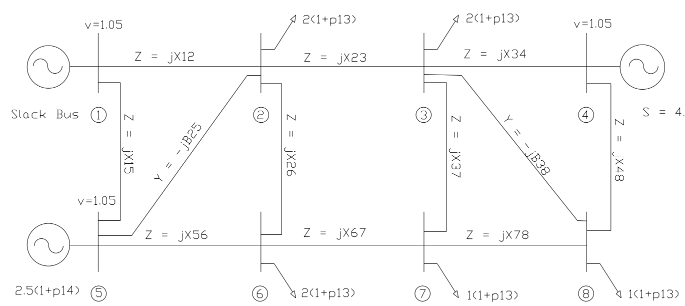

Fig. 1. 8 bus test system

This section tests the transmission reliability margin formula by comparing it with Monte Carlo simulations in two examples. The examples are chosen to be small enough that comprehensive validation against the formula is practical. However, the formula is applicable to extremely large systems and situations with numerous parameters. In these larger examples, validation against extensive Monte-Carlo analysis is impractical.

The first example uses the 8 bus system shown in Figure 1. The transfer capability from area 1 (buses 1,2,5,6) to area 2 (buses 3,4,7,8) is limited by the power flow limit on the line between bus 2 and 3. The parameters are listed in Table I.

The base case of the system assumes all parameters at their mean values. At the base system, the transfer capability is 2.8253 (with no contingency). Sensitivity of the transfer capability to these parameters can be calculated with no difficulty. Given a desired high probability P, the transmission reliability margin defined in (8) is calculated using formula (10). Table II lists transmission reliability margins with respect to different probabilities P. 10,000 samples are used in the Monte Carlo simulation.

The second example uses the IEEE 118 bus system. There are 186 lines and the real power flow limit was assumed to be 1.0 p.u. at all lines except that the real power flow limit for line 54 was assumed to be 3.0 p.u.. We consider a point to point power transfer from bus 6 to bus 45.

TABLE I
PARAMETER DISTRIBUTIONS

parameter	distribution
line susceptance B25	binary; prob{B25=5.0}=0.95, prob{B25=0}=0.05
line susceptance B38	binary; prob{B38=2.5}=0.95, prob{B38=0}=0.05
line impedance X12	uniform; μ=0.1, σ=0.0029
line impedance X23	uniform; μ=0.2, σ=0.0058
line impedance X34	uniform; μ=0.1, σ=0.0029
line impedance X15	uniform; μ=0.1, σ=0.0029
line impedance X26	uniform; μ=0.1, σ=0.0029
line impedance X37	uniform; μ=0.1, σ=0.0029
line impedance X48	uniform; μ=0.1, σ=0.0029
line impedance X56	uniform; μ=0.1, σ=0.0029
line impedance X67	uniform; μ=0.2, σ=0.0058
line impedance X78	uniform; μ=0.1, σ=0.0029
system loading p13	normal; μ=0.0, σ=0.1
bus 5 generation p14	normal; μ=0.0, σ=0.1
line 2-4 flow limit	normal; μ=1.5, σ=0.1
line 6-7 flow limit	normal; μ=1.5, σ=0.1

TABLE II
TRM FOR 8 BUS SYSTEM

P	90%	95%	99%	99.5%
TRM formula	0.6012	0.7750	1.0944	1.2118
Monte Carlo	0.6027	0.7846	1.1083	1.2171

The uncertain parameters are the power injections to all buses. The power injections are assumed to have a uniform distribution around 5% of their nominal values.

An AC power flow model was used. At the base case, the transfer capability is 1.8821 p.u.. Given a desired probability P, transmission reliability margin defined in (8) is calculated using formula (10). Table III lists transmission reliability margins with respect to different probabilities P. 10,000 samples were used in the Monte Carlo simulation.

In both the 8 and 118 bus examples, the Monte Carlo results confirm the TRM estimates from formula (10).

VI. Probabilistic transfer capacity

Our approach is not limited to the determination of transmission reliability margin. Since our approach yields an approximately normal distribution of transfer capability uncertainty U and an estimate (7) of the standard deviation of U, this is an alternative way to find the probabilistic transfer capacity as presented in [3], [4], [14], [18], [19]. The probabilistic transfer capacity can be used for system planning, system analysis, contract design and market analysis. Reference [14] suggests promising applications of probabilistic transfer capacity in the new market environment.

TABLE III
TRM FOR 118 BUS SYSTEM

P	90%	95%	99%	99.5%
TRM formula	0.0803	0.1036	0.1462	0.1619
Monte Carlo	0.0795	0.1027	0.1427	0.1585

VII. Conclusions

This paper has presented a way to estimate transmission reliability margin with a formula. The formula requires estimates of the uncertainty in independent parameters, the evaluation of transfer capability sensitivities, and specification of the degree of safety. The transfer capability sensitivities with respect to many parameters are easy and quick to evaluate once the transfer capability is determined [10], [12]. This ability to quickly obtain sensitivities with respect to many parameters makes it practical to account for the effects of many uncertain parameters in large power system models. The validity of the formula has been confirmed by comparison with Monte Carlo runs on 8 and 118 bus systems. However, the central limit theorem approximation used to derive the formula improves as the number of parameters increases so that the formula is most applicable to larger power system models for which Monte Carlo comparisons are impractical. No cost information is used in the formula.

The approach includes estimating the statistics of the uncertainty in the transfer capability and thus gives an alternative way to obtain a probabilistic transfer capacity with a more formal way of accounting for uncertainty than is ordinarily used in such calculations. The formula provides a defensible and transparent way to estimate transmission reliability margin; in particular, the degree of safety assumed and the sources of uncertainty are apparent in the calculation. The improved estimate of transmission reliability margin will improve the accuracy of transfer capabilities and could be helpful in resolving the tradeoff between security and maximizing transfer capability. The sensitivities used in the calculation highlight which uncertain parameters are important. Indeed, the calculation provides one way to put a value on reducing parameter uncertainty because a given reduction in uncertainty yields a calculable reduction in transmission reliability margin and this can be related to the profit made in an increased transfer.

Appendix

Let $X_1, X_2, ..., X_m$ be independent, zero mean random variables and write $s_m^2 = \sum_{k=1}^m \sigma^2(X_k)$ for the variance of $\sum_{k=1}^m X_k$. The approximate normality of $\sum_{k=1}^m X_k$ requires a central limit theorem. (Note that the most straightforward version of the central limit theorem does not apply because we do not assume that $X_1, X_2, ..., X_m$ are identically distributed.) A special case of the Lindeberg theorem [1] states that if

$$\lim_{m\to\infty} \sum_{k=1}^{m} \frac{1}{s_m^2} \int_{|X_k|>\epsilon s_m} X_k^2 \, dF = 0 \tag{11}$$

holds for all positive ϵ then $\frac{1}{s_m}\sum_{k=1}^{m} X_k$ converges in distribution to a normal random variable of mean zero and variance unity.

One useful class of random variables satisfying the Lindeberg condition (11) is random variables which are both uniformly bounded and whose variance uniformly exceeds some positive constant. It is also possible to augment the random variables in this class with some normal random variables.

References

[1] P. Billingsley, *Probability and measure*, second edition, Wiley 1986, theorem 27.2, page 369.

[2] W.H. Beyer, ed., *CRC standard mathematical tables*, 27th edition, CRC Press, Boca Raton, Florida USA, 1984.

[3] M. Chau, M. Heyeck, "Probabilistic transfer capability - An enhancement to transmission system performance appraisal methods", *IEEE Trans. on Power Apparatus and Systems*, Vol. PAS-104, No. 9, August 1985, pp. 2159-2167.

[4] R. M. Maliszewski, M. Chau, "Application of probabilistic transfer capability analysis in transmission system performance studies", *CIGRE*, 38-01, Aug.28-Sept.3, 1988.

[5] Y.S. Chow, H. Teicher, *Probability theory: independence, interchangeability, martingales*, second edition, Springer Verlag, NY, 1988.

[6] A.S. Debs, *Modern Power Systems Control and Operation*, Kluwer Academic, Boston, 1988.

[7] I. Dobson, S. Greene, R. Rajaraman, F.L. Alvarado, C.L. DeMarco, M. Glavic, A. DeSouza, R. Zimmerman, R.J. Thomas et al., *Transfer capability calculator and tutorial*, web site at http://www.pserc.cornell.edu/tcc/, 2001.

[8] I. Dobson, S. Greene, R. Rajaraman, C.L. DeMarco, F.L. Alvarado, M. Glavic, J. Zhang, R. Zimmerman, Electric power transfer capability: concepts, applications, sensitivity, uncertainty, PSerc publication 00-34, November 2001, available from [7] and http://www.pserc.wisc.edu/.

[9] S. Greene, "Margin and sensitivity methods for security analysis of electric power systems", PhD thesis, ECE dept., University of Wisconsin, Madison WI USA, 1998.

[10] S. Greene, I. Dobson, F.L. Alvarado, "Sensitivity of the loading margin to voltage collapse with respect to arbitrary parameters", *IEEE Trans. Power Systems*, vol. 12, no. 1, February 1997, pp. 262-272.

[11] S. Greene, I. Dobson, F.L. Alvarado, "Contingency analysis for voltage collapse via sensitivities from a single nose curve", *IEEE Transactions on Power Systems*, vol. 14, no. 1, February 1999, pp. 232-240.

[12] S. Greene, I. Dobson, F.L. Alvarado, "Sensitivity of transfer capability margins with a fast formula", *IEEE Transactions on Power Systems*, vol. 17, no. 1, pp. 34-40, February 2002.

[13] J. M. Hammersley, D. C. Handscomb, *Monte Carlo Methods*, John Wiley & Sons Inc., NY, 1965.

[14] Hyde M. Merrill, "Exploiting the random nature of transmission capacity", *Public Utilities Fortnightly*, Sep 15, 1998.

[15] Transmission Transfer Capability Task Force, "Transmission transfer capability", North American Electric Reliability Council, Princeton, New Jersey, May 1995.

[16] Transmission Transfer Capability Task Force, "Available transmission capability definitions and determination", North American Electric Reliability Council, Princeton, New Jersey, June 1996.

[17] P.W. Sauer, "Technical challenges of computing available transfer capability (ATC) in electric power systems", 30th Hawaii International Conference on System Science, Maui, Hawaii, January 1997.

[18] Feng Xia, A. P. Sakis Meliopoulos, "A methodology for probabilistic simultaneous transfer capability analysis", *IEEE Trans. Power Systems*, vol. 11, no. 3, August 1996, pp. 1269-1276.

[19] A. P. Sakis Meliopoulos, Sun Wook Kang, George Kokkinides, "Probabilistic transfer capability assessment in a deregulated environment", Proceedings of the 33rd Hawaii International Conference on System Sciences, Maui, Hawaii, January 2000.

Jianfeng Zhang (S) was educated at Zhejiang University (BS Process Control 1989) and the University of Wisconsin-Madison (MS Chemical Engineering 1997, MS Electrical Engineering 1998) and worked from 1989 to 1995 as a control engineer for China Textile Industrial Engineering Institute. He is currently a PhD candidate in Electrical Engineering at the University of Wisconsin. His current research area is power systems.

Ian Dobson (SM) was educated at Cambridge (BA Math 1978) and Cornell (PhD Electrical Engineering 1989) and worked from 1978 to 1983 as a systems analyst for the British firm EASAMS Ltd. In 1989 he joined the University of Wisconsin-Madison, and is now a professor in electrical and computer engineering. His current interests are applications of nonlinear dynamics, electric power system instabilities, power electronics and self-organized criticality.

Fernando Alvarado (F) obtained the PhD from the University of Michigan in 1972 and is now professor at the University of Wisconsin-Madison in the department of electrical and computer engineering. His main areas of interest are computer applications to power systems and large scale problems and the interplay between system operational reliability and markets and market design. He is the Vice Chair of the IEEE-USA energy policy committee.