

Contribution Allocation for Voltage Stability In Deregulated Power Systems

Gang M. Huang, *Senior Member, IEEE*, Kun Men

Abstract: With deregulation of power systems, it is of great importance to know who contributes to avoid a voltage collapse, which could be due to different parts of the power system—generator, control system and transmission part, etc. This paper focused on how to allocate the responsibility and contribution by bifurcation analysis. We investigate how parameters of the system influence the bifurcation points. Three bifurcations (the singularity induced bifurcation, saddle-node and Hopf bifurcation^[1]), and their relationship to several commonly used controllers are analyzed. Their parameters' impact on the bifurcation points is investigated in this paper, from which we found a way to allocate the contribution by analyzing the relative positions of the bifurcations. We also analyzed the influence of other parameters (parameters of transmission and generators) on the bifurcation points. A simple two-bus system is used in this paper to demonstrate our approach. Our result shows that bifurcation analysis is a credible way to allocate the contribution, and the analysis will help us design the controller and optimize the system to avoid the voltage collapse.

Keywords: Contribution, Bifurcation, Voltage Collapse, Exciting control system, Voltage Stability, stability margin

I. INTRODUCTION

The deregulated power system is based on transactions; each part of the unbundled systems (generator, control system and transmission part, etc.) has its own contribution to voltage stability. It is of great economic and security importance to allocate these contributions, so appropriate award can be allocated to encourage infrastructure investments. Analyzing the influence of the parameters of the system on voltage stability will also help us design and optimize the system.

This paper demonstrates that we can allocate the responsibility through bifurcation analysis. A new way is developed to allocate the responsibility in part III of this paper, and in part IV we also showed that how the parameters of the system will interact with the bifurcation points.

The authors gratefully acknowledge the support from Texas Advanced Technology Program and Energy Resource Research Program at Texas A&M University.

Gang M. Huang, Department of Electrical Engineering, Texas A & M University, College Station, TX 77843-3128 (huang@ee.tamu.edu)

Kun Men, Department of Electrical Engineering, Texas A & M University, College Station, TX 77843-3128 (menkun@ee.tamu.edu)

A simple one generator and one load bus system is used to demonstrate our approach. The system has three basic parts of the power system—generator, exciting control system and transmission, which will be used to demonstrate our unbundling of responsibility/contributions.

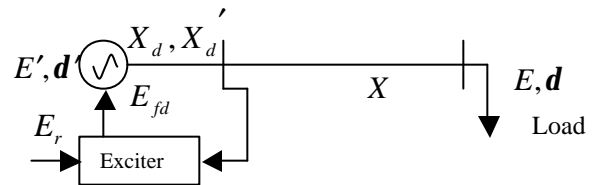


Fig. 1. Simple two bus system

In the above system, we assumed that the power factor of the load is constant as the load changes. We also assumed that the voltage dynamic is decoupled from the angle dynamic, which is well behaved, so the angle dynamic can be ignored at this scenario. The dynamics of this system can be modeled by parameter dependent differential-algebraic equations^[1] as:

$$\dot{x} = f(x, y, p), \quad f: \mathfrak{R}^{n+m+q} \rightarrow \mathfrak{R}^n \quad (1)$$

$$0 = g(x, y, p), \quad g: \mathfrak{R}^{n+m+q} \rightarrow \mathfrak{R}^m \quad (2)$$

$$x \in X \subset \mathfrak{R}^n, \quad y \in Y \subset \mathfrak{R}^m, \quad p \in P \subset \mathfrak{R}^q$$

The differential equation (1) represents the control system, the algebraic equation (2) represents the load flow equation. In this paper we focused on three commonly used controller—P-controller, PI-controller and PID controller of a voltage regulator, the load flow equation of this system and the mathematical model of these three types of controller are shown in part II.

The reduced Jacobian matrix^[1] of the system can be written as:

$$F_x = \begin{bmatrix} f_x - f_y g_y^{-1} g_x \end{bmatrix} \quad (3)$$

Through the analysis of the eigenvalue of F_x , we demonstrate the influence of the control system in part III. We focus on three types of controllers for regulators and the exciters, and observe that three types of bifurcation usually occurred: Hopf bifurcation, saddle-node and singularity induced bifurcation^[11,12], correspondingly we denote these three types of bifurcation as A, B and C in this paper. Then we show how different controllers and their parameters impact on the locations of A, B and C on the PV curve. We found that there are three basic patterns as follows. 1) $A < B < C$; 2) $A < C < B$; 3) A disappear and $B < C$ when different controllers

and different control parameters are used. Accordingly, we find a way to allocate the contribution and responsibility between the controller and transmission by analyzing the eigenvalues of F_x . We demonstrate that our method is easy and credible.

Based on the analysis in section III, we further demonstrate how the parameters of the system impact on the bifurcation points and the maximum loadability point, Pmax, in section IV, where we demonstrate the influence of controller, transmission and generator on voltage stability.

II MATHEMATIC MODEL for DIFFERENT CONTROL SYSTEM

2.1 algebraic equations of load flow^[2]

$$\begin{cases} P = \frac{E' E}{x} \sin \mathbf{d}' \\ Q = \frac{-E^2 + EE' \cos \mathbf{d}'}{x} \end{cases} \quad (4)$$

$$\begin{cases} P = \frac{E_g E}{x} \sin \mathbf{d}_g \\ Q = \frac{-E^2 + EE_g \cos \mathbf{d}_g}{x} \end{cases} \quad (5)$$

Equation (4) can be simplified as :

$$0 = E'^2 E^2 - (x'P)^2 - (x'Q + E^2)^2 \quad (6)$$

Here equation (6) is the $g(x,y,p)$ in equation (2).

2.2 differential equations of controller

In this paper, we just focus on three types of commonly used controller—P-controller, PI-controller and PID-controller, their mathematic model are shown as below:

1) P-controller^[3]:

$$\dot{E}' = \frac{1}{T_{d0}} \cdot \left[-\frac{x+x_d}{x'} E' + \frac{x_d-x_d'}{x'} \cdot \frac{(E^2+x'Q)}{E'} + E_{fd} \right] \quad (7)$$

$$\dot{E}_{fd} = \frac{1}{T} \left\{ -(E_{fd} - E_{fd}^0) - K_p \left[\frac{1}{E} \sqrt{(xP)^2 + (xQ + E^2)^2} - E_r \right] \right\} \quad (8)$$

Here equation (7)~(8) are the $f(x,y,p)$ in equation (1).

2) PI-controller^[8,10]

$$\dot{E}' = \frac{1}{T_{d0}} \cdot \left[-\frac{x+x_d}{x'} E' + \frac{x_d-x_d'}{x'} \cdot \frac{(E^2+x'Q)}{E'} + E_{fd} \right] \quad (9)$$

$$\dot{E}_{fd} = \frac{1}{T} \left\{ -(E_{fd} - E_{fd}^0) - E_{pi} / T_i - K_p \left(\frac{1}{E} \sqrt{(xP)^2 + (xQ + E^2)^2} - E_r \right) \right\} \quad (10)$$

$$\dot{E}_{pi} = \frac{1}{E} \sqrt{(xP)^2 + (xQ + E^2)^2} - E_r \quad (11)$$

Here equation (9)~(11) are the $f(x,y,p)$ in equation (1).

3) PID-controller

For a PID controller^[12],

$$K_p + \frac{1}{T_i s} + K_D s \quad (12)$$

We know that $K_D s$ is not practical^[12], so if T_D is small enough, we can use equation (13)^[6,7] to replace equation (12):

$$K_p + \frac{1}{T_i s} + \frac{K_D s}{1 + T_D s} \quad (13)$$

then the control system can be expressed as below:

$$\dot{E}' = \frac{1}{T_{d0}} \cdot \left[-\frac{x+x_d}{x'} E' + \frac{x_d-x_d'}{x'} \cdot \frac{(E^2+x'Q)}{E'} + E_{fd} \right] \quad (14)$$

$$\dot{E}_{fd} = \frac{1}{T} \left\{ -(E_{fd} - E_{fd}^0) - E_{pi} / T_i - E_D - \left(K_p + \frac{K_D}{T_D} \right) \left(\frac{1}{E} \sqrt{(xP)^2 + (xQ + E^2)^2} - E_r \right) \right\} \quad (15)$$

$$\dot{E}_{pi} = \frac{1}{E} \sqrt{(xP)^2 + (xQ + E^2)^2} - E_r \quad (16)$$

$$\dot{E}_D = -\frac{K_D}{T_D^2} \left(\frac{1}{E} \sqrt{(xP)^2 + (xQ + E^2)^2} - E_r \right) - \frac{E_D}{T_D} \quad (17)$$

Here equation (14)~(17) are the $f(x,y,p)$ in equation (1).

II. CONTRIBUTION ANALYSIS for DIFFERENT EXCITATION CONTROLLERS

Demonstrating Example:

$$T_{d0} = 5, T = 1.5, x_d = 1.2, x = 0.1, x_d' = 0.2, Q = 0.5P, E_r = 1.0$$

(Note: all cases shown in this paper will use the same system parameters unless specified otherwise.)

Firstly we will show that if there is no voltage regulator control, the PV curve of the system will be changed drastically:

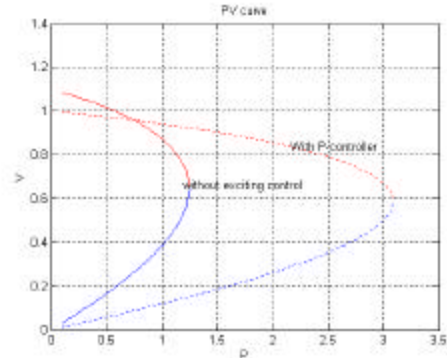


Fig.2 The PV curve with and without controller

When there is no voltage controller, E' is constant (here we it equals to 1.1). The PV curve is obtained by solving equation (6). When we use a controller to keep the $E_g = E_r$, the PV curve is obtained by solving equation (5). Fig 2 shows that the Pmax will increase, which implies that the regulator will increase the stability margin. Note that the regulating range of a voltage regulator is decided by its exciter size^[13,14]. We have shown that when the exciter hits the limit, the Pmax deteriorates.

In the following, we will analyze the impacts of different controllers of the regulator on the voltage stability of the system:

3.1 P-controllers in the regulator

In this paper we keep E_G as constant as P changes (P is the load) for the voltage regulation function; thus, the E_{fd0} in equation (8) is not constant, and will vary with the load P .

Bifurcation point C is induced by singularity, where $\det(g_y)$ equals to zero at point C. When $E_G \equiv E_r$, g_y is decided by the load flow equation, so C point is only induced by transmission system.

However, point B will vary with the change of K_p ; $B \rightarrow P_{max}$ with $K_p \rightarrow \infty$; and $B > C$ when $K_p > 5.25$; $B \approx A$ when $K_p = 1.895$; when $K_p < 1.895$, A will disappear; and $B \rightarrow 0.735$ with $K_p \rightarrow 0$. Fig. 3 shows how the locations of bifurcation A, B and C vary with the change of K_p :

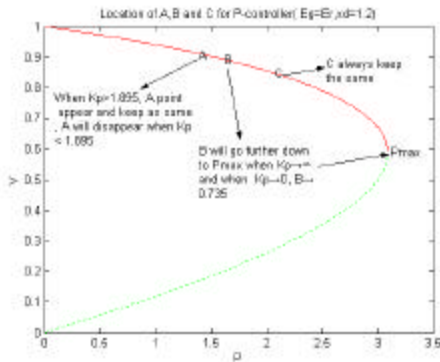


Fig.3 The location of A, B and C

When $K_p = 2.5, 5, 10$, the locations of the bifurcation A, B and C and the eigenvalues of reduced matrix are found by using equations (3), (6)~(8) as shown in Fig 4~6:

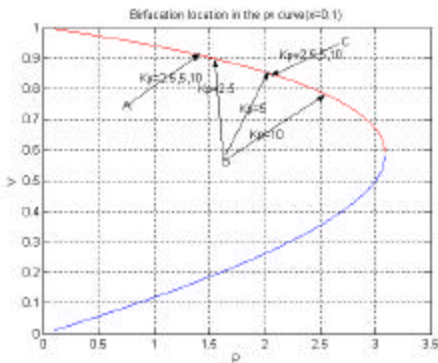


Fig 4, the locations of the bifurcations when $K_p = 2.5, 5, 10$

In Fig 5, note that K_p has little influence on one of the eigenvalues (denoted by E_{iT}), while at Fig 6. K_p has a substantial impact on the other eigenvalue (denoted by E_{iC}).

When $K_p = 1.8$, Fig 9 shows the location of B and C in PV curve. Note that A disappeared. From Figures 4 to 9 we can see that the eigenvalue E_{iT} is strongly related to the load flow, while the eigenvalue E_{iC} is strongly influenced by the controller.

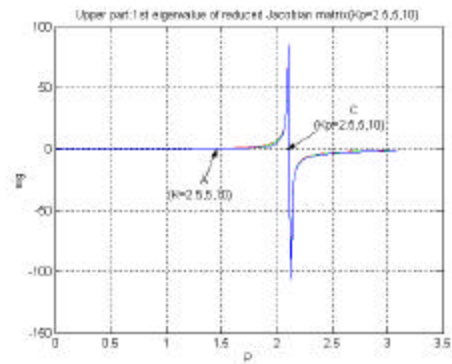


Fig 5, the eigenvalue which is slightly influenced by K_p

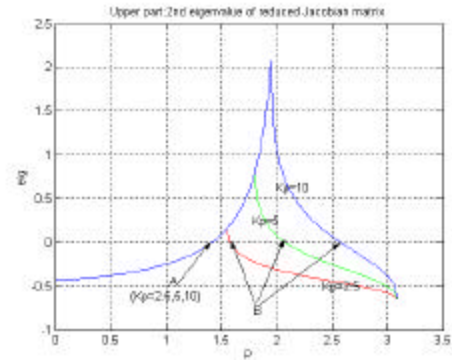


Fig 6, the eigenvalue that is strongly influenced by K_p

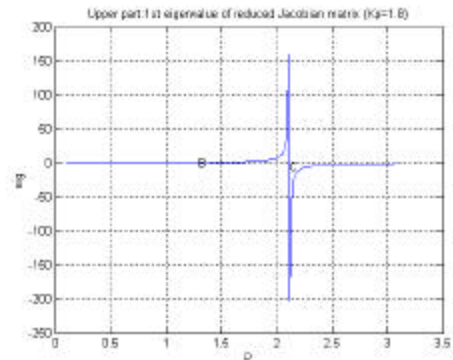


Fig 7, the eigenvalue E_{iT} ($K_p = 1.8$)

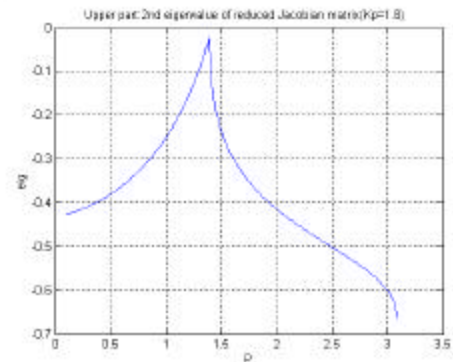


Fig 8, the eigenvalue E_{iC} ($K_p = 1.8$)

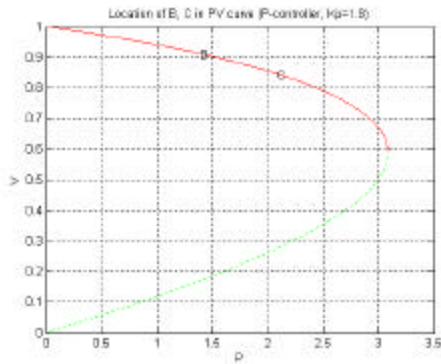


Fig 9, the location of B and C when $K_p=1.8$

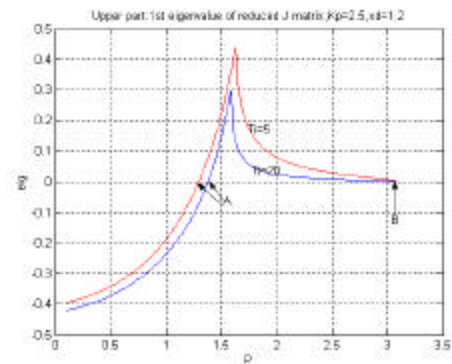


Fig 11, the eigenvalue EigC (PI-controller)

- We also can conclude that there are three basic patterns:
- 1) $A < B < C$. When $P \in (A, B)$, both the eigenvalue EigT and EigC are positive; when $P \in (B, C)$, only the eigenvalue EigT is positive.
 - 2) $A < C < B$. When $P \in (A, C)$, both the eigenvalue EigT and EigC are positive; when $P \in (C, B)$, only the eigenvalue EigC is positive.
 - 3) A disappear and $B < C$. Only the eigenvalue EigT is positive when $P \in (B, C)$

3.2 PI-controller in a regulator ($K_p = 2.5, T_I = 5.0 / T_I = 20$)

Using equation (3), (6) and (9)~(11), we can obtain three eigenvalues of the system: EigC (EigC is influenced by K_p and T_I , and is mainly influenced by T_I) and EigT, the other eigenvalue is always negative, and we also found bifurcation point A, B and C.

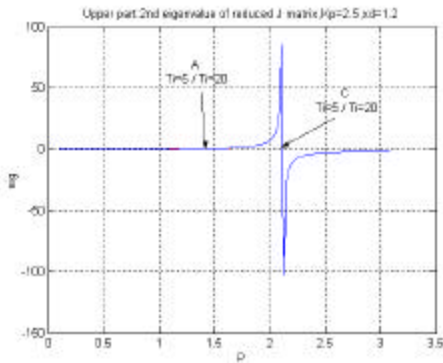


Fig 10, the eigenvalue Eig T (PI-controller)

From Fig 10~12, we can conclude that PI controller behaves very similar to the P-controller case as $K_p \rightarrow \infty$. When $P \in (0, A)$, all eigenvalues are negative; when $P \in (A, C)$, both eigenvalue EigC and EigT are positive; when $P \in (C, B)$, only the eigenvalue EigC is positive. Accordingly, it follows the basic pattern 2 as described in 3.1. The difference is that A point appear earlier in the PV curve, here $A=1.401$, (in P-controller, $A=1.4166$). Thus, the dynamic stability margin decrease when the I-component is introduced. However, PI-controller will enhance the dynamic response after a small disturbance of P from our simulation.

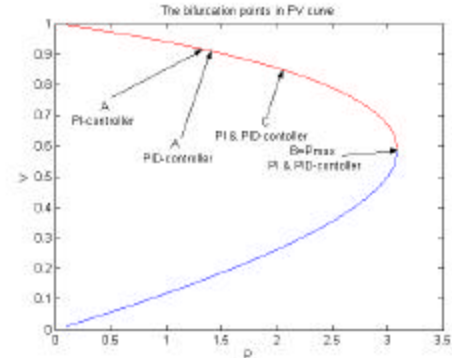


Fig 12, the location of A, B and C in PV curve for a PI/PID controller

3.3 PID-controllers in a regulator. ($K_p=2.5, T_I=5.0, K_D=1, T_D=0.01 / T_D = 0.005$)

Using equation (3), (6) and (14)~(17), we obtain four eigenvalues of the system: EigC and EigT, and other two negative eigenvalues. We also find three bifurcation points, where $A < C < B$. When $P \in (A, C)$, both EigC and EigT are positive; when $P \in (C, B)$, only EigC is positive. This follows a similar pattern 2 as discussed in 3.1.

From Fig 12, we can see that PID behaves similar to a PI-controller, except that A point appears later ($A=1.4154$). This implies that although I-component can decrease the dynamic stability margin, the D-component can compensate the decrease. On the other hand, the dynamic response remains as good as a PI controller.

Through sections 3.1, 3.2, 3.3 we can conclude that the three basic ordering patterns of bifurcation points A B C as discussed in 3.1 are generally true for all controllers. Our experience indicates that no other ordering of A B C is possible. Accordingly, we can draw the conclusion:

- 1) $A < B < C$. When $P \in (A, B)$, both EigC and EigT are positive; when $P \in (B, C)$, only the EigT is positive. From the parameter analysis, we can conclude that the voltage collapse is due to both controller and transmission when $P \in (A, B)$. The voltage collapse is only caused by transmission part when $P \in (B, C)$. In this case, $[A, C]$ is the unstable area, and A defines the dynamic stability margin boundary.
- 2) $A < C < B$. When $P \in (A, C)$, both EigC and EigT are positive; when $P \in (C, B)$, only the EigC is positive. From the parameter analysis we can conclude that the voltage collapse is due to both controller and transmission when $P \in$

(A,C). The voltage collapse is caused by controller when $P \in (C,B)$. In this case, $[A, B]$ is the unstable area, and A defines the dynamic stability margin boundary.

3) A disappear and $B < C$, only the EigT is positive when $P \in (B,C)$. Thus, the voltage collapse is only due to transmission when $P \in (B,C)$. In this case, $[B, C]$ is the unstable area, and B is the dynamic stability margin boundary.

4) In conclusion, the tuning of the control parameters will influence A and thus the dynamic stability margin of the system. The contribution with and without the controller/regulator can be easily assessed. More impacts analysis will be substantiated in the following section.

5) The analysis also explains why a dynamic stability margin is smaller than the static analysis as observed by our research team in [13].

IV. CONTRIBUTIONS OF OTHER SYSTEM PARAMETERS ON STABILITY

Following the same argument, we can also unbundle the voltage collapse responsibility of the system by studying how the parameters of the three parts of the system (generator, controller and transmission part) affect the bifurcation patterns:

4.1 The parameters of the control system:

a) Three types of regulators:

As discussed in 3.1, impacts of K_p in the P-controller are summarized as follows. K_p does not influence C, but have impacts on B and A. When $K_p \rightarrow \infty$, $B \rightarrow P_{max}$; and when $K_p \rightarrow 0$, $B \rightarrow 0.735$; when $K_p=1.895$, $A \approx B$; when $K_p < 1.895$, A disappear; when $K_p=5.25$, $B \approx C$.

For PI and PID controllers, we did simulation with different parameters of the controller and the result summarized in table 1: (**Note**: for PI and PID controller, A defines the dynamic stability margin, and the unstable area is $[A, B]$.)

TABLE 1-a

Xd	K _p	T _I	K _D	T _D	A	B	C				
1.2	2.5	1	0	0	P=0.98	P=3.09	P=2.115				
					P=1.167						
					P=1.273						
					P=1.236						
					P=1.404						
					1			0.001	P=1.4105		
								0.1	P=1.415		
								0.01	P=1.416		
					3			0	0	0	P=1.1625
											P=1.2839
											P=0.5
											P=0.648
											P=0.685
											P=0.726
											8
15											
100											

From table 1-a, we can see that K_p only has influence on A, which is different from P-controller. Here B and C remain the

same. (With a constant power factor, we prove that B always appear at Pmax when an I-controller is used in the regulator.) A will increase with bigger T_I and bigger K_D . T_D has little influence on A, but too large or too small T_D will decrease A. All parameters of PI and PID controllers have no impacts on B and C.

The steady-state stability margin C is independent of the controller as long as the voltage regulator has enough excitation capacity to regulate the terminal voltage.

b) The influence of the exciter

The equation (7) represents the exciter. Now we investigate the influence of the T_{do}' on the stability margin: (In this case the regulator is P-controller, $K_p=2.5$)

TABLE 1-b

T_{do}'	Pmax	A	B	C
3	3.09	1.248	1.584	2.116
5	3.09	1.416	1.584	2.116
7	3.09	1.534	1.584	2.116

From table 1-b we can see that T_{do}' will influence the A point, but it have no influence on B and C. We verified the fact for all three controllers. A will increase with bigger T_{do}' , that means the dynamic stability margin will increase with bigger T_{do}' . However, the size of the exciter will limit the range of E_{fd} and thus the voltage regulation range [13].

4.2 The parameters of a generator:

Now we investigate the influence of the x_d on the stability margin:

a) P-controller

TABLE 2-a

x_d	Pmax	A	B	C
1.2	3.09	1.416	1.584	2.116
0.3	3.09	2.016	2.719	2.116

b) PI-controller

TABLE 2-b

x_d	Pmax	A	B	C
1.2	3.09	1.297	3.09	2.116
0.3	3.09	2.005	3.09	2.116

c) PID-controller

TABLE 2-c

x_d	Pmax	A	B	C
1.2	3.09	1.415	3.09	2.116
0.3	3.09	2.0158	3.09	2.116

Through the table 2-a, b and c, we can see that x_d has influence on A point; and if we use Pcontroller, it also influence the B point. It implies that the dynamic stability margin will increase with smaller x_d . C, which defines the steady-state stability margin, it is independent with x_d .

4.3 The parameters of the transmission system:

Now we investigate how x influences stability margin :

a) *P-controller*

TABLE 3-a

x	Pmax	A	B	C
0.1	3.09	1.416	1.584	2.116
0.12	2.58	1.360	1.509	1.966

b) *PI-controller*

TABLE 3-b

x	Pmax	A	B	C
0.1	3.09	1.297	3.09	2.116
0.12	2.58	1.2506	2.58	1.966

c) *PID-controller*

TABLE 3-c

x	Pmax	A	B	C
0.1	3.09	1.415	3.09	2.116
0.12	2.58	1.3589	2.58	1.966

Through the analysis of table 3-a, b and c, we see that x has influence on all of the three bifurcation points. Both the dynamic and steady stability margin will increase with smaller x .

V. CONCLUSION

This paper develops a way to unbundle the contribution of voltage stability to generator owners, transmission owners and excitation control owners. We showed that how the parameters of the three parts of the system (generator, controller and transmission) affect the bifurcation patterns, which enable us to design and optimize the system better and at the same time to allocate the contributions. In the same token, an investment award system can be built to award investments on the corresponding power infrastructure.

A simple two-bus system is used in this paper to demonstrate our approach, which shows our approach is easy and credible.

VI. ACKNOWLEDGEMENTS

The authors gratefully acknowledge the support from Texas Advanced Technology Program, PSERC and Energy Resource Research Program at Texas A&M University.

VII. REFERENCES

- [1] Garng Huang, Liang Zhao, Xuefeng Song, "A New Bifurcation Analysis for Power System Dynamic Voltage Stability Studies". Accept by IEEE 2002 PES, Winter meeting NY.
- [2] V.Venkatasubramanian, H.Schaettler and J.Zaborazky, "Voltage Dynamics: Study of a Generator with Voltage Control, Transmission, and Matched MW Load", IEEE Transaction on Automatic Control, Vol 37, No.11, November 1992. p1717-1733
- [3] IEEE Committee Report, "Excitation System Model for Power System Stability Studies", IEEE Trans. Power Apparatus Syst., Vol.PAS-100, No.2, Feb.1981. P494-509.
- [4] P. Kokotovic, H.K.Khalil, and J. O'Reilly. "Singular Perturbation Methods in Control: Analysis and Design". Academic Press, 1986
- [5] Costas Vournas, "Voltage Stability of Electric Power Systems", Kluwer Academic Publishers. 1998.

- [6] K. Kim, M.J. Basler and A.Godhwani. "Supplemental Control in a Modern Digital Excitation System". Pannel Session for the 2000 IEEE Power Meeting, Singapore, 2000
- [7] Richard C. Schaefer and Kiyong Kim, "Excitation Control of the Synchronous Generator", IEEE industry Applications Magazine, March/April 2001, pp.37-43.
- [8] Riko Safari c, Karel Jezemik and Dusan Borojevic, "PI-Controller for Avoiding Integrator Wind-up in System with Input Saturation", IEEE 1991. pp:803-806
- [9] Carson W. Taylor, "Power System Voltage Stability", McGraw-Hill, Inc. 1994.
- [10] Yixin Ni, "Dynamic Power System Theorem and Analysis", Tsinghua University publisher.
- [11] V.Venkatasubramanian, H.Schaettler and J.Zaborazky, "Local Bifurcations Feasibility Regions in Differential-Algebraic systems", IEEE Transactions on Automatic Control, VOL 40, NO. 12. Dec. 1995. pp:1992-2013
- [12] Kailath, T.. 1980. "Linear System". Englewood Cliffs: Prentice-Hall.
- [13] G Huang, H Zhang, "Dynamic Voltage Stability reserve studies for deregulated environment ", Proc. IEEE/PES Summer Meeting, July 2001, Canada.
- [14] Garng Huang, Tong Zhu, "TCSC as a transient voltage stabilizing controller," in *Proc. of Power Engineering Society Winter Meeting*, 2001, IEEE, vol. 2, pp628-633, Columbus, OH, Jan. 2001.

VIII. BIOGRAPHIES

Dr. Garng M. Huang received his B.S. and M.S. in E.E. from national Chiao Tung University, Hsinchu, Taiwan, R.O.C. in 1975, 1977 respectively. He received his doctorate degree in System Science and Mathematics from Washington University, St. Louis in 1980. He had been teaching there since then until 1984. He joined Texas A&M University, Department of Electrical Engineering in 1984. He is currently a professor and the director of graduate studies there. He have been working on many funded research projects, such as Emergency Control of Large Interconnected Power System, HVDC Systems, Restoration of Large Scale Power Systems, On-line Detection of System Instabilities and On-line stabilization of Large Power System, Fast Parallel/Distributed Textured Algorithm for Data Network Routing Problem, etc. His current interest is the large-scale systems theory, large scale parallel/distributed computing and control and their applications.

Dr. Huang is a senior Member of IEEE, and a Registered Professional Engineer of Texas. He has served as the Technical Committee Chairman of Energy System Control Committee and an associated editor in the IEEE Automatic Control Society; he has also been serving in a number of committees and subcommittees of IEEE PAS Society. Dr. Huang has published more than one hundred papers and reports in the areas of nonlinear, distributed control systems, parallel/distributed computing and their applications to power systems, data networks and flexible structures.

Kun Men received his BS degree in Electrical Engineering from Xi'an Jiaotong University and MS degrees in Electrical Engineering from Tsinghua University, Beijing, China, respectively in 1996 and 2001. He is presently a Ph.D. student at the Department of Electrical Engineering, Texas A&M University, College Station, Texas.