

# IS STRONG MODAL RESONANCE A PRECURSOR TO POWER SYSTEM OSCILLATIONS?

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**Abstract:** We suggest a new mechanism for interarea electric power system oscillations in which two oscillatory modes interact near a strong resonance to cause one of the modes to subsequently become unstable. The possibility of this mechanism for oscillations is shown by theory and computational examples. Theory suggests that passing near strong resonance can be expected in general power system models. The mechanism for oscillations is illustrated in 3 and 9 bus examples with detailed generator models.

## 1 Introduction

Power transactions are increasing in volume and variety in restructured electric power systems because of the large amounts of money to be made in exploiting geographic differences in power prices. Restructured power systems are expected to be operated at a greater variety of operating points and closer to their operating constraints. One operational constraint which already limits transactions under some conditions is the onset of low frequency interarea oscillations [4, 10, 11, 16].

We consider how changes in power system parameters could cause low frequency oscillations. For example, parameter changes such as bulk power transactions or generator redispatch change the power system equilibrium and hence change the system modes and possibly cause oscillations. The main contribution of the paper is to suggest, analyze and illustrate a mathematical mechanism for low frequency oscillations. Describing mechanisms which cause oscillations is an essential step in developing sound methods of operating the power system up to but not at the onset of oscillations.

The power system linearization and its modes vary as power system parameters change. Damped oscillatory modes can move close together and interact in such a way that one of the modes subsequently becomes unstable. An ideal version of this phenomenon occurs when two damped oscillatory modes coincide exactly. That is, the power system linearization has two conjugate complex pairs of eigenvalues which coincide in both

frequency and damping. This coincidence is called a resonance, or, especially in the context of Hamiltonian systems, a 1:1 resonance. If the linearization is not diagonalizable at the resonance, the resonance is called a *strong* resonance [18]. Otherwise, if the linearization is diagonalizable at the resonance, the resonance is called a *weak* resonance. Here we are most interested in strong resonance. At a strong resonance, the modes typically become extremely sensitive to parameter variations and the direction of movement of the eigenvalues turns through a right angle. For example, an eigenvalue that changes in frequency before the resonance can change in damping after the resonance and become oscillatory unstable as the damping changes through zero. The strong resonance is a precursor to the oscillatory instability in the sense that the resonance causes the eigenvalues to change the size and direction of their movement in such a way as to produce instability.

In practice the power system will not experience an exact strong resonance, but will pass close to such a resonance and the qualitative effects will be similar: the eigenvalues will move quickly and change direction as they interact and this can lead to oscillatory instability. Note that we are describing how a linearization of the power system model changes as a generator redispatch changes the equilibrium at which the linearization is evaluated.

Section 2 illustrates oscillatory instability caused by strong resonance with parameterized matrices. Computational results showing oscillatory instability caused by strong resonance in 3 and 9 bus power system models are presented in Section 3. Section 4 describes the general structure of strong resonance and relates this to what can be expected to be observed in general power system models. A method to predict eigenvalue movement near strong resonance is presented in section 5. Previous power systems work related to strong resonance includes analysis of the 1:1 resonance in Hamiltonian power system models by Kwatny [14], observation of mode patterns changing by Van Ness [23], the use of repeated poles to improve Prony analysis by Trudnowski, Johnson, and Hauer [20], and the studies of interarea oscillations at Ontario Hydro [13] and of the western North American power system [13, 21]. Seyranian has studied strong resonance in mechanics [18, 19]. These and other previous works are discussed more fully in section 6 and the paper concludes in section 7. This paper is an improved version of the conference paper [7].

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## 2 Illustration of strong resonance

This tutorial section illustrates strong resonance and near resonance in complex eigenvalues of parameterized matrices. Consider the matrix  $M_1$  parameterized by the

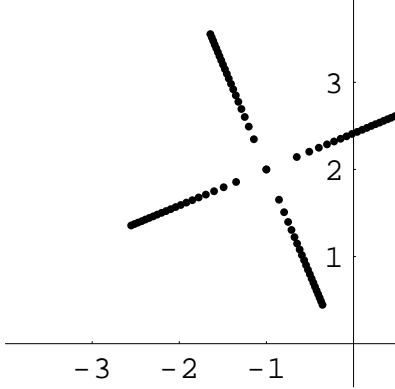


Figure 1: *Two modes of matrix  $M_1$  encountering strong resonance*  
real number  $\alpha$ :

$$M_1 = \begin{pmatrix} -1+2j & 1+j & 0 & 0 \\ \alpha & -1+2j & 0 & 0 \\ 0 & 0 & -1-2j & 1-j \\ 0 & 0 & \alpha & -1-2j \end{pmatrix}$$

$M_1$  is a complex matrix, but it is structured to be similar to a real matrix (note that the  $2 \times 2$  submatrices are complex conjugate). That is, a coordinate change  $T$  transforms  $M_1$  into a real matrix:

$$T M_1 T^{-1} = \begin{pmatrix} -1 & 1 & 2 & 1 \\ \alpha & -1 & 0 & 2 \\ -2 & -1 & -1 & 1 \\ 0 & -2 & \alpha & -1 \end{pmatrix}$$

where  $T = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ j & 0 & j & 0 \\ 0 & j & 0 & j \end{pmatrix}$

At  $\alpha = -2$ , the eigenvalues of  $M_1$  are  $-1.64 \pm 3.55j$  and  $-0.36 \pm 0.45j$ . As  $\alpha$  varies from  $-2$  to  $2$ , two of the eigenvalues of  $M_1$  vary as shown in Figure 1 (these eigenvalues are  $-1 + 2j \pm \sqrt{1+j}\sqrt{\alpha}$ ). Each eigenvalue shown in Figure 1 has a complex conjugate which moves correspondingly below the real axis. At  $\alpha = 0$ , the eigenvalues coincide at the strong resonance at  $-1 + 2j$ .  $M_1$  is not diagonalizable at the resonance. As  $\alpha$  increases through zero, the eigenvalues change direction by a right angle. The eigenvalue movement is fast near the resonance; indeed, exactly at the resonance the eigenvalues are infinitely sensitive to parameter variation. Note how one of the eigenvalues becomes unstable after the resonance.

Figure 1 is not typical because an exact strong resonance is encountered. It is more typical to come close to strong resonance as the parameter is varied. Consider a matrix  $M_2$  which is a perturbation of matrix  $M_1$ :

$$M_2 = M_1 + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad (1)$$

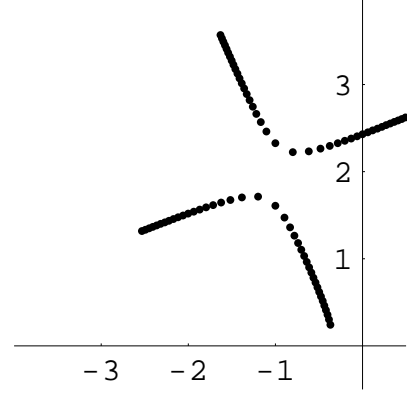


Figure 2: *Two modes of matrix  $M_2$  moving near strong resonance*

The eigenvalues of  $M_2$  vary as shown in Figure 2 as  $\alpha$  varies from  $-2$  to  $2$ . Note how the eigenvalues come close together and quickly turn approximately through a right angle. There is a marked effect of coming close to the resonance.

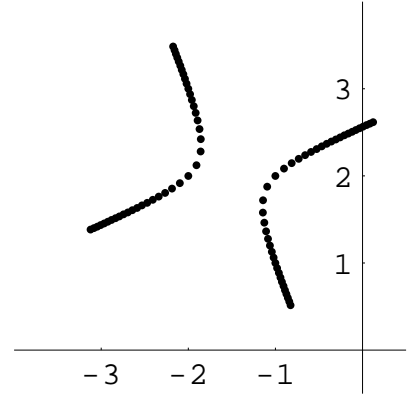


Figure 3: *Two modes of matrix  $M_3$  moving near strong resonance*

A different way to perturb  $M_1$  is the matrix

$$M_3 = M_1 + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (2)$$

The eigenvalues of  $M_3$  vary as shown in Figure 3 as  $\alpha$  varies from  $-2$  to  $2$ . The eigenvalue movements in Figure 2 and Figure 3 are both close to the eigenvalue movement in Figure 1, but a different eigenvalue becomes unstable in Figure 2 and Figure 3.

## 3 Power system computational results

This section shows examples of 3 bus and 9 bus power system models passing near strong resonance as generator power is redispatched. In both cases, the modal interaction near strong resonance leads to oscillatory instability. The 3 bus results first appeared in [8].

### 3.1 3 bus system

The 3 bus system shown in Figure 4 consists of generators at bus 1 and bus 3 and a constant power load at bus 2. The generator models are tenth order and the

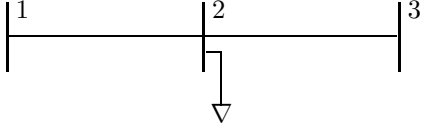


Figure 4: *Three bus power system*

system parameters are reported in Appendix B. As the generator dispatch is varied to increase the power supplied by bus 3, two damped complex eigenvalues vary as shown in Figure 5. The eigenvalues are initially at  $-0.4 \pm 8.3j$  and  $-0.9 \pm 4.3j$  and are stable. As the power supplied by bus 3 increases, the two eigenvalues approach one another, interact, and then one of the eigenvalues crosses the imaginary axis and becomes unstable.

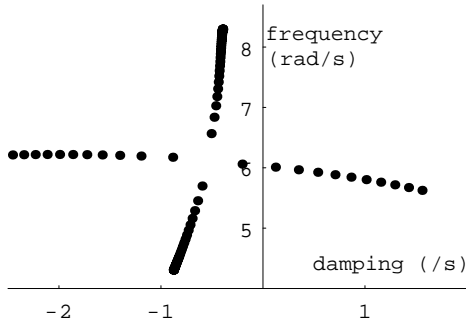


Figure 5: *Eigenvalues as dispatch varies;  $V_{ref} = 1.07$*

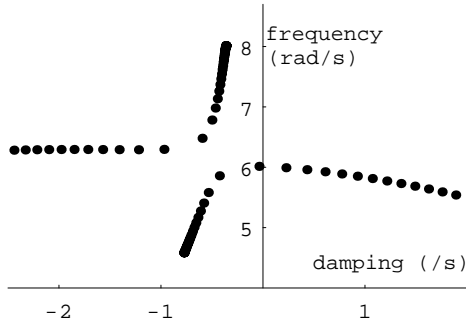


Figure 6: *Eigenvalues as dispatch varies;  $V_{ref} = 1.03$*

The case shown in Figure 5 is adjusted to show the eigenvalues coming close together and has  $V_{ref} = 1.07$ , where  $V_{ref}$  is the voltage reference set point of the generators at buses 1 and 3. Rerunning the case for decreased and increased  $V_{ref}$  is shown in Figures 6 and 7. Figures 6 and 7 show typical perturbations of the strong resonance. Observe that if one attempts to stabilize the unstable eigenvalue of Figure 6 by increasing  $V_{ref}$ , then this eigenvalue is indeed stabilized, but the other eigenvalue becomes unstable as shown in Figure 7. This shows the importance of examining both modes when trying to stabilize the system near strong resonance.

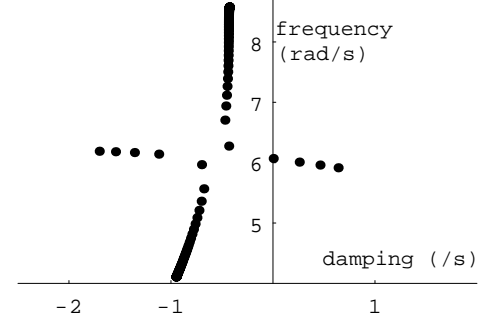


Figure 7: *Eigenvalues as dispatch varies;  $V_{ref} = 1.11$*

### 3.2 9 bus system

The form of the 9 bus system is based on the western North American power system from the text of Sauer and Pai [17]. There are 3 generators with 2 axis models and IEEE Type I exciters. More details may be found in Appendix B. Figure 8 shows the eigenvalue movement when real power generation at bus 2 is varied from 1.5 pu to 2.10 pu in steps of 0.05. Real power generation at bus 3 is fixed at 1.5 pu.

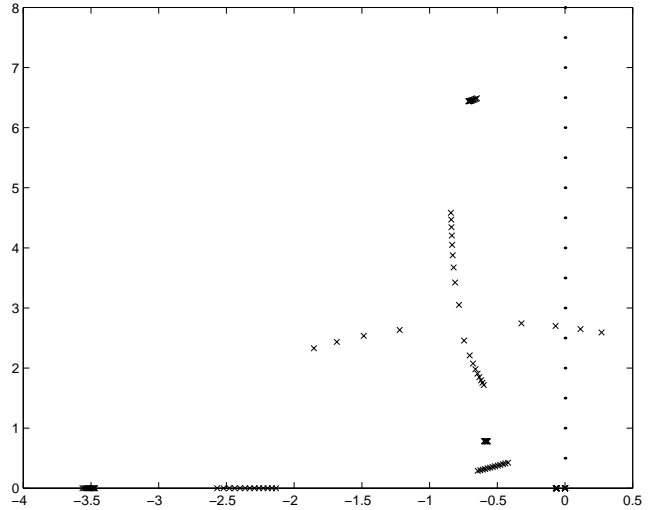


Figure 8: *9 bus eigenvalues as dispatch varies*

The eigenvalues pass near resonance and then one of the eigenvalues becomes oscillatory unstable. Note that the eigenvalues initially move together by a change mostly in frequency. It is the strong resonance which transforms this movement into a change in damping and hence instability. The eigenvalues move quickly near the resonance.

### 4 Structure near resonance and genericity

This section describes in general how two oscillatory modes of the Jacobian vary when they are near a strong or weak resonance and the genericity of these resonances. The detailed mathematics to support all these results is presented in Appendix A.

#### 4.1 Strong resonance

Near strong resonance the Jacobian is similar to a matrix which includes a  $4 \times 4$  submatrix  $M'_C$  describing the modes of interest:

$$M'_C = \begin{pmatrix} \lambda & 1 & 0 & 0 \\ \mu & \lambda & 0 & 0 \\ 0 & 0 & \lambda^* & 1 \\ 0 & 0 & \mu^* & \lambda^* \end{pmatrix} = \begin{pmatrix} M_C & 0 \\ 0 & M_C^* \end{pmatrix} \quad (3)$$

Here  $\lambda$  and  $\mu$  are complex numbers which are functions of the power system parameters. The eigenvalues of  $M'_C$  are the same as the eigenvalues of the Jacobian corresponding to the two oscillatory modes of interest.

The behavior of  $M'_C$  is governed by the submatrix

$$M_C = \begin{pmatrix} \lambda & 1 \\ \mu & \lambda \end{pmatrix} \quad (4)$$

The eigenvalues of  $M_C$  are

$$\lambda_1 = \lambda + \sqrt{\mu} \quad \text{and} \quad \lambda_2 = \lambda - \sqrt{\mu} \quad (5)$$

Therefore the eigenvalues of  $M'_C$  are

$$\lambda \pm \sqrt{\mu} \quad \text{and} \quad (\lambda \pm \sqrt{\mu})^*$$

and these are the eigenvalues of the Jacobian corresponding to the modes of interest. The idea is to study these modes by examining the eigenvalues and eigenvectors of  $M_C$ .

The eigenvalues of  $M_C$  coincide at  $\lambda$  when  $\mu = 0$  and this is the condition for strong resonance.  $M_C$  is nondiagonalizable at resonance (alternative terms for 'nondiagonalizable' are 'nonsemisimple' and 'nondefective'). The sensitivity of these eigenvalues to the real or imaginary part of  $\mu$  is  $\frac{\pm 1}{2\sqrt{\mu}}$ , which tends to infinity as  $\mu$  tends to zero. As  $\mu$  moves in the complex plane on a smooth curve through 0 with nonzero speed, the argument of  $\sqrt{\mu}$  jumps by  $90^\circ$  so that the direction of the eigenvalue movement changes by  $90^\circ$ .

The right and left eigenvectors of  $M_C$  are

$$\begin{pmatrix} 1 \\ \pm\sqrt{\mu} \end{pmatrix} \quad \text{and} \quad (\pm\sqrt{\mu}, 1)$$

At the strong resonance at  $\mu = 0$ , the eigenvectors are infinitely sensitive to changes in  $\mu$ , the right and left eigenvectors are orthogonal, and there is a single right eigenvector together with a generalized right eigenvector. As  $\mu$  tends to zero and the resonance is approached, the two right eigenvectors become aligned and tend to the right eigenvector at  $\mu = 0$ . Thus the system modes approach each other as  $\mu$  tends to zero. The dependence of this approach on  $\sqrt{\mu}$  shows that this approach is initially slow and then very quick near  $\mu = 0$ .

If the system is near strong resonance, then the following are typical:

- The eigenvalues and eigenvectors are very sensitive to parameter variations

- A general parameter variation causes the direction of eigenvalue movement in the complex plane to turn quickly through approximately a right angle.
- The right and left eigenvectors are nearly orthogonal.
- The right eigenvectors of the two modes are nearly aligned. This implies that the pattern of oscillation of the two modes is similar.

#### 4.2 Mode coupling at strong resonance

We examine the time and frequency domain solutions at strong resonance when  $\mu = 0$ . Assume that  $\lambda = \sigma \pm j\omega$  with  $\sigma < 0$ . The time domain solutions to the linear differential equations with matrix (3) are linear combinations of  $te^{\sigma t} \cos \omega t$ ,  $te^{\sigma t} \sin \omega t$ ,  $e^{\sigma t} \cos \omega t$  and  $e^{\sigma t} \sin \omega t$ . Some perturbations mainly excite the  $te^{\sigma t} \cos \omega t$  and  $te^{\sigma t} \sin \omega t$  solutions and these perturbations will cause oscillations that grow before exponentially decaying to zero. Figure 9 shows the frequency domain description in block diagram form.

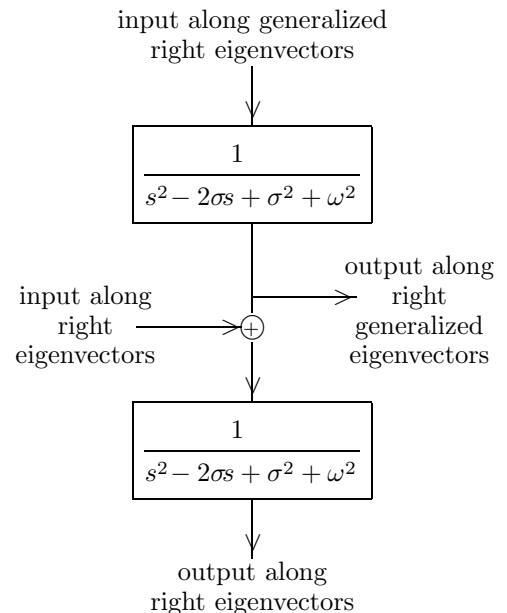


Figure 9: Modes at strong resonance

Observe the input/output combination passing vertically down the page in Figure 9 in which the output of the first damped oscillator feeds the second damped oscillator. This one-way mode coupling has interesting consequences for the power system behavior.

Consider two modes which initially are local to separate areas of the power system. The modes are initially decoupled so that disturbances in one area will only affect the mode in that area. Now suppose that parameters change so that the two modes interact by encountering a strong resonance. As the strong resonance is approached, the mode eigenvectors will converge so that the modes are no longer confined to their respective areas. Moreover, at the strong resonance, a disturbance along the generalized eigenvector can excite the eigenvector mode (but not vice versa). We expect that qualitatively similar mode coupling effects can occur for systems that pass near strong resonance.

### 4.3 Genericity of strong resonance

The rarity of strong resonance can be examined using its codimension [24], which is, roughly speaking, the number of independent parameters that need to be varied to typically encounter the resonance. The coincidence of two pairs of complex eigenvalues of a matrix at  $\lambda = \sigma \pm j\omega$  typically happens with Jordan canonical form

$$\begin{pmatrix} \lambda & 1 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda^* & 1 \\ 0 & 0 & 0 & \lambda^* \end{pmatrix} \quad (6)$$

A strong resonance of the form (6) without regard to the value of  $\lambda$  occurs in the matrix (3) when the complex parameter  $\mu = 0$ . Since this requires both the real and imaginary part of  $\mu$  to be zero, this is a codimension 2 event. (On the other hand, the occurrence of a strong resonance of the form (6) for a *particular* value of  $\lambda = \lambda_0$  requires both  $\lambda = \lambda_0$  and  $\mu = 0$  and is a codimension 4 event.)

Thus strong resonance is a codimension 2 event and it can be typically encountered when varying two parameters. Strong resonance will not be typically encountered when varying one parameter, but it is still possible to pass near to strong resonance when varying one parameter.

### 4.4 Weak resonance

At weak resonance of two complex modes, the Jacobian is similar to a matrix which includes a  $4 \times 4$  submatrix

$$\begin{pmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda^* & 0 \\ 0 & 0 & 0 & \lambda^* \end{pmatrix} \quad (7)$$

Weak resonance is a codimension 6 event. Thus we do not expect weak resonance to occur in a generic set of equations such as those that might be expected when modeling a power system with no special structure, even if two parameters were varied. However, weak resonance can occur with some special structure: For example, consider two power systems which are decoupled from each other. The eigenvalues of the entire system belong to either one power system or the other. If parameters vary so that an eigenvalue of one power system coincides with an eigenvalue of the other power system, then these two eigenvalues will not interact as parameters vary and this is weak resonance. Another example of special structure which can yield weak resonance is when a power system study is done with a bilaterally symmetric power system model.

At weak resonance, there is ambiguity in associating eigenvectors with one of the modes that is resonating because any nontrivial combination of the eigenvectors is also an eigenvector. Moreover, the eigenvectors and eigenvalues are ill conditioned in that some parameter changes cause sudden changes in the eigenvectors and eigenvalues. In particular, there are strong resonances arbitrarily close to a weak resonance [18].

Suppose that two modes which are local to separate areas of the power system and thus decoupled encounter

a weak resonance. Then the modes remain decoupled at the weak resonance. For example, a disturbance confined to one area will only excite the local mode of that area.

### 4.5 Typical resonance in power system models

The analysis of genericity in sections 4.3 and 4.4 raises the question of the extent to which practical power system models are non-generic or have ‘special structure’. It seems clear that special structure such as bilateral symmetry or perfect decoupling due to the power system areas being completely disconnected is not expected in practical power systems models. Moreover, a sensible initial working assumption is that practical power system models are generic. However, it is a possibility that in some cases there could be sufficient decoupling between power system areas to make the areas approximately decoupled. In these cases the power system could pass near to a weak resonance. This would also imply passing near a strong resonance, since there are strong resonances arbitrarily close to a weak resonance. However, not all perturbations of the weak resonance involve the strong resonance and, moreover, it is possible that the strong resonance could be observed only in a detailed analysis whereas the weak resonance would determine the approximate overall behavior. More work is needed to clarify whether a weak resonance is likely to occur in a practical power system model and what would be expected to be observed near a weak resonance.

Another consideration is the genericity of the parameter changes being considered. Parameter changes such as power redispatch strongly affect the equilibrium and are expected to generically change the power system linearization. On the other hand, it is not clear whether changing a control system gain corresponds to a generic parameter change. Control systems are designed to affect particular modes and changes in control gains often have little or no effect on the equilibrium.

## 5 Predicting eigenvalue movement near strong resonance

Let  $\lambda_1$  be a lightly damped system eigenvalue. Formulas to compute the first order eigenvalue sensitivity  $\frac{\partial \lambda_1}{\partial p}$  with respect to changes in any parameter  $p$  are very useful in determining the robustness of  $\lambda_1$  and in detecting whether  $\lambda_1$  is a critical mode that can readily become unstable as parameters change [6, 22]. (For parameters such as generator redispatch, the effect of the equilibrium movement must be accounted for by Hessian terms in the formulas computing these sensitivities.)

Now suppose that eigenvalues  $\lambda_1$  and  $\lambda_2$  are close to a strong resonance. Then the nonlinear and rapidly changing movement of the eigenvalues near the resonance will make  $\frac{\partial \lambda_1}{\partial p}$  and  $\frac{\partial \lambda_2}{\partial p}$  very poor estimates of the eigenvalue movements for any sizable changes in  $p$ . This section shows how better estimates can be obtained by exploiting the structure near strong resonance.

An important general observation is that  $\lambda$  and  $\mu$  of

(4) can be calculated from numerical eigenvalue results. Inversion of (5) yields

$$\lambda = (\lambda_1 + \lambda_2)/2 \quad (8)$$

$$\mu = (\lambda_1 - \lambda_2)^2/4 \quad (9)$$

$\lambda$  is the average eigenvalue and  $\mu$  describes the detuning from exact strong resonance.

Figure 10 shows how  $\lambda$  and  $\mu$  computed from (9) vary for the case shown in Figure 8. The approximately linear variation of  $\lambda$  and  $\mu$  in Figure 10 motivates the following method of estimating eigenvalue movement. The sensitivity of  $\lambda$  and  $\mu$  with respect to  $p$  can be calculated from the sensitivities of  $\lambda_1$  and  $\lambda_2$ :

$$\frac{\partial \lambda}{\partial p} = \frac{1}{2} \left( \frac{\partial \lambda_1}{\partial p} + \frac{\partial \lambda_2}{\partial p} \right) \quad (10)$$

$$\frac{\partial \mu}{\partial p} = \frac{\lambda_1 - \lambda_2}{2} \left( \frac{\partial \lambda_1}{\partial p} - \frac{\partial \lambda_2}{\partial p} \right) \quad (11)$$

First order estimates of the changes in  $\lambda$  and  $\mu$  are made using (10) and (11) and then estimates of the eigenvalue movements are obtained using (5). Figure 11 shows a good match between the estimated and actual eigenvalue movements.

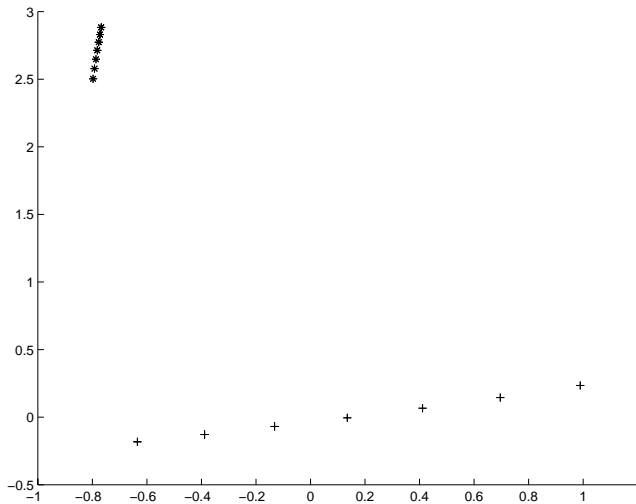


Figure 10: Variation in  $\lambda$  and  $\mu$  near strong resonance;  $*$  =  $\lambda$ ,  $+$  =  $\mu$ .

## 6 Related Previous Work

This section describes previous work and its relation to the strong resonance phenomenon described in this paper.

Kwatny [14, 15] studies the flutter instability in power system models with Hamiltonian structure. A stable equilibrium of a Hamiltonian power system model necessarily has all eigenvalues on the imaginary axis. One generic way for stability to be lost as a parameter varies is the flutter instability, or Hamiltonian Hopf bifurcation. In the flutter instability, two modes move along the imaginary axis, coalesce in an exact strong resonance, and split at right angles to move into the

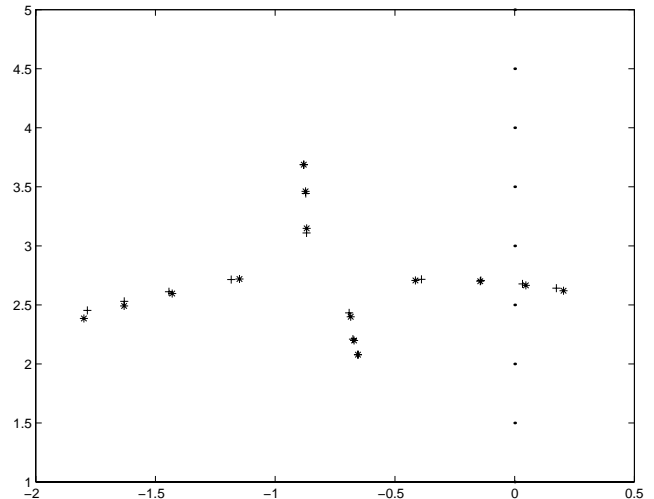


Figure 11: Predicting eigenvalue movement near resonance;  $+$  = predicted eigenvalues,  $*$  = actual eigenvalues.

right and left halves of the complex plane. The Hamiltonian power system model in [14] represents electromechanical mode phenomena with simple swing models for the generators. Kwatny [14] gives a 3 bus example of the flutter instability and emphasizes that the flutter instability is generic in one parameter Hamiltonian systems. It is also possible to add uniform damping to the conservative model in order to shift the Hamiltonian eigenvalue locus a fixed amount leftwards in the complex plane [15]. Then two eigenvalues (necessarily of the same damping) approach other in frequency, coalesce in an exact strong resonance and then split apart in damping. One of these eigenvalues can then cross the imaginary axis in a Hopf bifurcation to cause an oscillation. This is clearly a special case of strong resonance causing an oscillation. The Hamiltonian plus uniform damping model structure constrains the eigenvalues to move either vertically along the line of constant damping or horizontally and causes the resonance to be exact.

Van Ness [23] analyzes a 1976 incident of 1 Hz oscillations at Powerton station with a 60 machine model of the midwestern American power system with 9 machines represented in detail. The paper seems successful in reproducing the essential features of the incident by eigenanalysis of the model. Figure 7 of [23] examines the effect of a variation of power and excitation at Powerton unit 6. The eigenvector associated with a dominant eigenvalue shows significant changes near the instability that are attributed to a resonant interaction with another nearby mode. Movement in the real part of close eigenvalues when the excitation is lowered ‘seems to be due to a coupling effect which has been observed in the model’. Unfortunately the data is sparse; only one change in each of the power or excitation is presented and firm conclusions about the nature of the resonant interaction cannot be made. However, the features shared between the account of the eigenanalysis of [23] and strong resonance are suggestive.

Klein and Rogers et al. at Ontario Hydro [13] analyze local modes and an interarea mode in a symmet-

ric power system model with 2 areas and 4 machines. The symmetry is bilateral: each of the 2 areas has the same machines and transmission lines. However, the base case is a stressed case with area 1 exporting power to area 2 over a single weak tie line. The two local modes have eigenvalues that are practically equal, and each of the computed local modes has substantial components across the entire system. A small decrease in the machine inertias in area 2 causes the local modes to change substantially to have significant components only in their respective areas. Klein and Rogers attribute these results to the nonuniqueness of eigenvectors associated with a weak resonance. Although similar eigenvector changes could be found near a strong resonance, one might argue that a weak resonance could be expected here because of the high degree of system symmetry. (A perfect bilateral symmetry would cause a weak resonance and exclude strong resonance between symmetric modes.) We do not expect perfect bilateral symmetry in a practical power system. A perfect bilateral symmetry in a power system may require symmetry of both the network and the operating point: The 4 bus computational example in the thesis of Jones [12] shows a strong resonance in a power system with bilateral symmetry in the network but an asymmetric operating condition.

Hamdan [9] studies the conditioning of the eigenvalue and eigenvectors of a system very similar to that of [13]. The eigenvectors become ill conditioned near resonance and singular value measurements of the proximity to a weak resonance (‘sep’ function) suggest that the system does pass near a weak resonance.

Klein and Rogers et al. [13] also discuss the modes near 0.7 Hz of the western North American power system. The Kemano generating unit in British Columbia can have high participation not only in a local mode of 0.77 Hz but also in modes involving the Southwest United States of 0.74 and 0.76 Hz. Klein and Rogers regard this modal interaction as unusual, distinguish it from the phenomenon observed in their symmetric power system model and conclude that ‘Oscillations in one part of the system can excite units in another part of the system due to resonance’. Mansour [21] shows large oscillations at Kemano due to disturbances in the the Southwest United States. It would be interesting to determine if this modal interaction can be explained by a nearby strong resonance.

Trudnowski, Johnson, and Hauer [20] use a strong resonance assumption to improve Prony analysis identification of transfer functions from noisy ringdown data. Closely spaced poles with large residues of nearly opposite sign are replaced by two poles in an exact strong resonance at the average of the previous pole positions. Trudnowski, Johnson, and Hauer show that this improves the estimates of the pole positions in a 27 bus, 17 generator example which captures some features of the western North American power system. This result is supportive of the occurrence of strong resonance in power systems.

DeMarco [5] describes how increased loading of tie lines can cause a low frequency mode to decrease in

frequency until the complex conjugate eigenvalues coalesce at the real axis and then split along the real axis so that one eigenvalue passes through the origin and steady state stability is lost in a collapse. This strong resonance of two real eigenvalues is sometimes called a node-focus point or a critical damping of the two modes. DeMarco demonstrates the phenomenon in a 14 bus system. Ajjarapu [1] also describes this phenomena and demonstrates it in a 3 bus system. The phenomenon is strong resonance of real eigenvalues as a precursor to steady state instability and is clearly analogous to strong resonance in the complex plane causing oscillatory loss of stability.

There is a large amount of very useful previous work addressing the tuning of control system gains to avoid oscillations which we do not attempt to review here.

The strong resonance and its implications for stability is known in mechanics. Seyranian [18] gives a perturbation analysis of eigenvalue movement caused by parameter changes near both strong and weak resonance. Of particular interest is the analysis showing how passage through a weak resonance can be perturbed to obtain strong resonances. Seyranian [19] considers strong resonance of a parameterized linear oscillatory system. The eigenvalue movements near resonance are shown to be hyperbolas to first order and a procedure for calculating the hyperbolas from the eigenstructure is given. The role of the resonance as a precursor to instability and in altering which mode goes unstable is described and two applications in mechanics are presented.

Recent work in the thesis of Jones [12], building on the conference version of this paper [7], advances the computational examples of strong resonance. Jones shows strong resonance near 0.7 Hz of two well damped electromechanical modes of a 19 machine dynamic model of the western North American power system. Since both modes are well damped, no oscillatory loss of stability is caused. The results confirm the approximate coincidence of mode shapes near the strong resonance and the predicted effects of perturbing the resonance.

## 7 Discussion and Conclusion

This paper demonstrates strong resonance as a precursor to oscillatory instability in 3 and 9 bus power systems as power dispatch is varied. Mathematical analysis confirms the observed qualitative features of the eigenvalue and eigenvector movement near strong resonance. Near strong resonance, eigenvalues move quickly and turn through approximately 90°. Thus if the eigenvalues are initially approaching each other in frequency, then they will quickly separate in damping after the resonance. One of these eigenvalues can cross the imaginary axis and cause an oscillation. This new mechanism for power system oscillations can be seen as a generalization of Kwatny’s flutter instability of Hamiltonian power system models [14, 15] to a general power system model.

The new mechanism for power system oscillations requires some change of perspective: instead of only examining the damping of a single mode, one must also

consider the possibility that two modes interact near a strong resonance to cause the oscillations. If two modes do interact in this way, then attempting to explain and predict the eigenvalue movement or attempting to damp the oscillation by only examining the mode that crosses the imaginary axis can easily fail. The new mechanism does not preclude the possibility of a single isolated mode changing in damping as a cause of oscillations; rather, the new mechanism points out an alternative way in which the interaction of two modes causes one of the modes to reduce its damping and become unstable.

With the notable exceptions of the Hamiltonian work of Kwatny [14, 15] the transfer function identification work by Trudnowski et al. [20], and the recent work by Jones [12], the possibility of strong resonance of oscillatory modes seems to have been neglected in electric power systems analysis. However, theory suggests that a typical power system model can pass close to strong resonance as a parameter is varied and that encountering strong resonance is more likely than encountering a weak resonance. More work is needed to determine whether practical power systems have any special structure that could make approximate weak resonance more likely. Nevertheless, we do suggest that effects due to nearby strong resonance do occur in practical power systems. Artificially symmetric power system models may fail to give resonance results representative of practical power systems.

As two eigenvalues approach strong resonance, the corresponding eigenvectors also converge. This is one way to explain how power system modes which are initially associated with different power system areas become coupled. It will be interesting to try to verify these explanations in power system examples such as the 0.7 Hz western North American power system modes in which some sort of resonance has long been suspected of causing ‘anomalous’ results.

Is strong modal resonance a precursor to power systems oscillations? The theoretical and simulation evidence in this paper strongly suggests that a strong resonance can be a precursor to oscillations and that nearby strong resonance is a possible explanation whenever electric power systems have closely spaced modes interacting.

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### Appendix A: Generic structure near resonance

This appendix describes the generic structure of two modes of a general power system model near resonance using the matrix deformation theory explained in Wiggins [24] and Arnold [2].

We begin with a general dynamic power system model and obtain a parameterized Jacobian  $J(\alpha)$ . Assume that the power system is modeled by parameterized differential-algebraic equations which are analytic

in the state and the parameters  $\alpha \in \mathbb{R}^p$ . Further suppose that the derivative of the algebraic equations with respect to the algebraic variables is nonsingular at the equilibrium. Then we can locally solve the algebraic equations for the algebraic variables via the implicit function theorem and obtain analytic differential equations in a neighborhood of the equilibrium. Suppose that the Jacobian of the differential equations at the equilibrium is nonsingular. Then the equilibrium is an analytic function of the parameters and evaluating the Jacobian at the equilibrium yields a real parameterized matrix  $J(\alpha)$ . The Jacobian  $J(\alpha)$  is an analytic function of the parameters  $\alpha$  in some open set  $U$ .

Suppose that at  $\alpha = \alpha_0 \in U$ , exactly two complex eigenvalues coincide at  $\lambda_0 = \sigma_0 + j\omega_0$ , where  $\omega_0 \neq 0$ . It follows that the complex conjugates of these eigenvalues also coincide at  $\lambda_0^* = \sigma_0 - j\omega_0$ . We are interested in the eigenstructure of  $J(\alpha)$  for  $\alpha$  near to  $\alpha_0$ . Since the eigenvalues of  $J(\alpha)$  are continuous functions of  $\alpha$ , by shrinking the neighborhood  $U$  as necessary, the eigenvalues can be expressed as functions  $\lambda_1(\alpha)$ ,  $\lambda_2(\alpha)$ ,  $\lambda_1^*(\alpha)$ ,  $\lambda_2^*(\alpha)$  for  $\alpha \in U$  with  $\lambda_1(\alpha_0) = \lambda_2(\alpha_0) = \lambda_0$ . Here  $U$  is shrunk so that  $\lambda_1(\alpha)$  and  $\lambda_2(\alpha)$  lie inside a disk centered on  $\lambda_0$  for  $\alpha \in U$  and that there are, counting algebraic multiplicity, exactly two eigenvalues in the disk for  $\alpha \in U$ .

Now we reduce the Jacobian  $J$  to a  $4 \times 4$  matrix  $M$  which has the eigenstructure corresponding to the four eigenvalues of interest. The projection  $P(\alpha)$  onto the four dimensional right eigenspace spanned by the generalized right eigenvectors corresponding to  $\lambda_1(\alpha)$ ,  $\lambda_2(\alpha)$ ,  $\lambda_1^*(\alpha)$ ,  $\lambda_2^*(\alpha)$  is an analytic function of  $\alpha$  [3]. Also the projection  $Q(\alpha)$  onto the corresponding four dimensional left eigenspace is an analytic function of  $\alpha$ . Define  $M = Q^T J P$ .  $M(\alpha)$  is an analytic  $4 \times 4$  matrix valued function of the parameters  $\alpha$  for  $\alpha \in U \subset \mathbb{R}^p$ .  $M(\alpha)$  has exactly the eigenstructure corresponding to the four eigenvalues of  $J(\alpha)$  of interest. In particular,  $M(\alpha_0)$  has two complex eigenvalues coinciding at  $\lambda_0 = \sigma_0 + j\omega_0$ .

There are now two cases depending on whether  $M(\alpha_0)$  is diagonalizable or not. In the diagonalizable, weak resonance case  $M(\alpha_0)$  is similar to the matrix

$$\begin{pmatrix} \lambda_0 & 0 & 0 & 0 \\ 0 & \lambda_0 & 0 & 0 \\ 0 & 0 & \lambda_0^* & 0 \\ 0 & 0 & 0 & \lambda_0^* \end{pmatrix}$$

in Jordan canonical form. Arnold [2], section 6.30E shows that weak resonance is codimension 6 in real parameter space.

#### A.1 Strong resonance.

In the strong resonance case  $M(\alpha_0)$  is similar to the matrix  $M_{R0}$  in real Jordan canonical form

$$M_{R0} = \begin{pmatrix} \sigma_0 & -\omega_0 & 1 & 0 \\ \omega_0 & \sigma_0 & 0 & 1 \\ 0 & 0 & \sigma_0 & -\omega_0 \\ 0 & 0 & \omega_0 & \sigma_0 \end{pmatrix}$$

A miniversal deformation of  $M_{R0}$  is  $M_R : \mathbf{R}^4 \rightarrow \mathbf{R}^{16}$  given by

$$M_R(\sigma, \omega, \mu_r, \mu_i) = \begin{pmatrix} \sigma & -\omega & 1 & 0 \\ \omega & \sigma & 0 & 1 \\ \mu_r & -\mu_i & \sigma & -\omega \\ \mu_i & \mu_r & \omega & \sigma \end{pmatrix} \quad (12)$$

This key result can be deduced from [24, 2]. The consequence of the miniversal deformation is that there exist real analytic functions written, with some abuse of notation, as  $\sigma(\alpha)$ ,  $\omega(\alpha)$ ,  $\mu_r(\alpha)$ ,  $\mu_i(\alpha)$  and a  $4 \times 4$  real matrix valued coordinate transformation  $T_R(\alpha)$  analytic in  $\alpha$  such that

$$M(\alpha) = T_R(\alpha) M_R(\sigma(\alpha), \omega(\alpha), \mu_r(\alpha), \mu_i(\alpha)) (T_R(\alpha))^{-1}$$

for  $\alpha$  in some neighborhood  $U_1 \subset U$  of  $\alpha_0$ . That is, a matrix similar to  $M(\alpha)$  can be analytically parameterized via the four parameters  $\sigma$ ,  $\omega$ ,  $\mu_r$ , and  $\mu_i$ . Also  $\sigma(\alpha_0) = \sigma_0$ ,  $\omega(\alpha_0) = \omega_0$ ,  $\mu_r(\alpha_0) = 0$ , and  $\mu_i(\alpha_0) = 0$  so that, in particular,

$$M(\alpha_0) = T_R(\alpha_0) M_R(\sigma_0, \omega_0, 0, 0) (T_R(\alpha_0))^{-1} \quad (13)$$

The eigenvalues of  $M_R(\sigma, \omega, \mu_r, \mu_i)$  are  $\sigma + j\omega \pm \sqrt{\mu_r + j\mu_i}$ . It is convenient to shrink  $U_1$  if necessary to ensure that the eigenvalues of  $M_R(\sigma(\alpha), \omega(\alpha), \mu_r(\alpha), \mu_i(\alpha))$  for  $\alpha \in U_1$  are never real. Then it follows, for  $\alpha \in U_1$ , that the eigenvalues of  $M_R(\sigma(\alpha), \omega(\alpha), \mu_r(\alpha), \mu_i(\alpha))$  coincide iff  $\mu_r(\alpha) = \mu_i(\alpha) = 0$ .

It is convenient to also express this result in terms of a  $2 \times 2$  complex matrix describing the two eigenvalues with positive frequency. Permuting the second and third basis elements yields a matrix  $M'_R$  similar to  $M_R$ :

$$M'_R(\sigma, \omega, \mu_r, \mu_i) = \begin{pmatrix} \sigma & 1 & -\omega & 0 \\ \mu_r & \sigma & -\mu_i & -\omega \\ \omega & 0 & \sigma & 1 \\ \mu_i & \omega & \mu_r & \sigma \end{pmatrix} = \begin{pmatrix} A_r & -A_i \\ A_i & A_r \end{pmatrix}$$

Applying a complex coordinate change to  $M'_R$  gives a  $4 \times 4$  complex matrix

$$M'_C = \begin{pmatrix} I_2 & jI_2 \\ I_2 & -jI_2 \end{pmatrix} M'_R \begin{pmatrix} I_2 & jI_2 \\ I_2 & -jI_2 \end{pmatrix}^{-1} = \begin{pmatrix} M_C & 0 \\ 0 & M_C^* \end{pmatrix}$$

where

$$M_C(\lambda, \mu) = \begin{pmatrix} \lambda & 1 \\ \mu & \lambda \end{pmatrix}$$

and

$$\begin{aligned} \lambda &= \sigma + j\omega \\ \mu &= \mu_r + j\mu_i \end{aligned}$$

The  $2 \times 2$  complex matrix  $M_C = A_r + jA_i$  is called the complexification of  $M'_R$ . The two eigenvalues of  $M_C$  are the two eigenvalues of  $M_R$  with positive frequency. Note that, setting  $\mu = 0$ ,  $M_C(\lambda, 0)$  is in Jordan canonical form and that for  $\mu = 0$  and  $\lambda = \lambda_0$ ,  $M'_C$  is the Jordan canonical form of  $M(\alpha_0)$ .

In terms of  $M_C(\lambda, \mu)$ , the consequence of the miniversal deformation is that there exist complex analytic functions written, with some abuse of notation, as  $\lambda(\alpha)$ ,  $\mu(\alpha)$ , and a  $4 \times 4$  complex matrix valued coordinate transformation  $T_C(\alpha)$  analytic in  $\alpha$  such that

$$M(\alpha) = T_C(\alpha) \begin{pmatrix} M_C(\lambda(\alpha), \mu(\alpha)) & 0 \\ 0 & M_C^*(\lambda(\alpha), \mu(\alpha)) \end{pmatrix} (T_C(\alpha))^{-1}$$

for  $\alpha$  in some neighborhood  $U_1 \subset U$  of  $\alpha_0$ . Also  $\lambda(\alpha_0) = \lambda_0$  and  $\mu(\alpha_0) = 0$ .

Thus the study of the eigenstructure of  $M(\alpha)$  reduces to the study of the eigenstructure of  $M_C(\lambda(\alpha), \mu(\alpha))$ . In particular, the eigenvalues of  $M_C(\lambda(\alpha), \mu(\alpha))$  are the eigenvalues of  $M(\alpha)$  with positive frequency and the real and imaginary parts of the generalized eigenvectors of  $M_C(\lambda(\alpha), \mu(\alpha))$  are generalized eigenvectors of  $M'_R(\sigma(\alpha), \omega(\alpha), \mu_r(\alpha), \mu_i(\alpha))$ , which is similar to  $M(\alpha)$ .

## A.2 Structure of matrices near $M(\alpha_0)$ .

The miniversal deformation result above can be applied to determine the structure of all real  $4 \times 4$  matrices near  $M(\alpha_0)$  by a choice of the parameterization  $\alpha$ . Let  $\alpha \in \mathbf{R}^{16}$  be the entries of a real  $4 \times 4$  matrix. That is, we parameterize  $4 \times 4$  matrices by their own entries. Then  $\mu(\alpha) = (\mu_r(\alpha), \mu_i(\alpha))$  may be regarded as an analytic map  $\mu : U_1 \rightarrow \mathbf{R}^2$  where  $U_1 \subset \mathbf{R}^{16}$ . Since  $\mu$  can be computed from the matrix eigenvalues (see (9)) and the eigenvalues of  $M_R(\sigma_0, \omega_0, \mu_r, \mu_i)$  are  $\sigma_0 + j\omega_0 \pm \sqrt{\mu_r + j\mu_i}$ ,

$$\mu(M_R(\sigma_0, \omega_0, \mu_r, \mu_i)) = (\mu_r, \mu_i)$$

and

$$\mu(T_R(\alpha_0) M_R(\sigma_0, \omega_0, \mu_r, \mu_i) (T_R(\alpha_0))^{-1}) = (\mu_r, \mu_i)$$

which implies, using (13), that  $\mu$  is regular near  $M(\alpha_0)$ . Therefore  $\Gamma = \mu^{-1}((0, 0))$  is an analytic codimension 2 submanifold of the real  $4 \times 4$  matrices near  $M(\alpha_0)$ .  $\Gamma$  is the set of real  $4 \times 4$  matrices near  $M(\alpha_0)$  which are similar to  $M_R(\sigma, \omega, 0, 0)$  for some values of  $\sigma$  and  $\omega$ .

Every matrix  $N$  in  $U_1$  is similar to  $M_R(\sigma(N), \omega(N), \mu_r(N), \mu_i(N))$  and the eigenvalues of  $N$  and  $M_R(\sigma(N), \omega(N), \mu_r(N), \mu_i(N))$  are  $\sigma(N) + j\omega(N) \pm \sqrt{\mu_r(N) + j\mu_i(N)}$ . Since  $U_1$  is assumed to be shrunk so that these eigenvalues are never real,  $N$  has coincident eigenvalues iff  $\mu_r(N) = \mu_i(N) = 0$ . Hence  $\Gamma$  is the set of matrices in  $U_1$  which have a coincident complex conjugate pair eigenvalues away from the real axis. Moreover, each matrix in  $\Gamma$  is not diagonalizable. Thus  $\Gamma$  is the matrices in  $U_1$  with strong resonance and is a submanifold of codimension 2. A generic two parameter system of differential equations will have Jacobians which are diagonalizable except at isolated points at which strong resonance occurs (see the first corollary in Arnold [2] chapter 6, section 30E).

## Appendix B: Power system models and parameters

### B.1 3 bus system

The dynamic model for both generators consists of a fourth-order synchronous machine (angle, speed, field flux, one damper winding) with an IEEE type I excitation system (third order), and a first-order model each for the turbines, boilers, and governors. The machine equations are (6.110–6.116), (4.98), (4.99), (6.118) and (6.121) in [17]. The limits on exciter voltage  $V_R$  and the steam valve  $P_{SV}$  are neglected. All data is in per unit except that time constants are in seconds.

#### Three bus power system data

Generator	Exciter	Gov/Turbine
$T'_{do} = 5.33$	$K_A = 50.0$	$T_{RH} = 10.0$
$T''_{go} = 0.593$	$T_A = 0.02$	$K_{HP} = 0.26$
$H = 2.832$	$K_E = 1.0$	$T_{CH} = 0.5$
$T_{FW} = 0$	$T_E = 0.78$	$T_{SV} = 0.2$
$X_d = 2.442$	$K_F = 0.01$	$R_d = 0.05$
$X_q = 2.421$	$T_F = 1.2$	$\omega_s = 120\pi$ rad/s
$X'_d = 0.830$	$S_E(E_{fd}) = 0.397 e^{0.09E_{fd}}$	
$X'_q = 1.007$		
$R_s = 0.003$		
Load	Line 1-2	Line 2-3
$P_L = 1.0$	$R = 0.042$	$R = 0.031$
$Q_L = 0.3$	$X = 0.168$	$X = 0.126$
	$B = 2 \times 0.01$	$B = 2 \times 0.008$

The generator dispatch is controlled by a parameter  $\alpha$  which specifies the proportion of power specified at the governors at buses 1 and 3:

$$P_{c1} = \alpha P_{ctotal}$$

$$P_{c3} = (1 - \alpha) P_{ctotal}$$

( $P_{ctotal}$  is determined when the equilibrium equations are solved.) The base case has  $\alpha = 0.5$  and the results are produced by decreasing  $\alpha$  to 0.1 in steps of  $-0.1$ .

### B.2 9 bus system

The overall form of the 9 bus model is that of the western North American power system shown in Figure 7.4 of [17], except that PQ loads are added at buses one and two. The generators are round rotor with IEEE Type 1 exciters. The generator dynamic equations are consistent with (6.173) to (6.181) of [17]. The generator algebraic equations are consistent with (6.186), (6.187) and (6.188) of [17]. The saturation function relations are consistent with (6.189) to (6.193) of [17], with

$$S_{smi}(|\psi''|) = \begin{cases} 0 & \text{if } |\psi''| \leq S_{GA} \\ S_{GB}(|\psi''| - S_{GA})^2 & \text{if } |\psi''| > S_{GA} \end{cases}$$

$$S_E(E_{fd}) = \begin{cases} 0 & \text{if } E_{fd} \leq S_{EA} \\ S_{EB}(E_{fd} - S_{EA})^2/E_{fd} & \text{if } E_{fd} > S_{EA} \end{cases}$$

The network data is given in Table 7.2 of [17]; other parameters are as follows. All data is in per unit except that time constants are in seconds.

#### Machine Data

Parameter	bus1	bus2	bus3
$T'_{do}$	8.96	8.5	3.27
$T''_{go}$	0.31	1.24	0.31
$T''_{do}$	0.05	0.037	0.032
$T''_{go}$	0.05	0.074	0.079
$H$	22.64	6.47	5.047
$T_{FW}$	0	0	0
$X_d$	0.146	1.75	2.201
$X_q$	0.0969	1.72	2.112
$X'_d$	0.0608	0.427	0.556
$X'_q$	0.0608	0.65	0.773
$X''_d = X''_q$	0.05	0.275	0.327
$X_l$	0.026	0.22	0.246
$S_{GA}$	0.898	0.911	0.825
$S_{GB}$	9.610	8.248	2.847

#### Exciter Data

Parameter	bus1,2,3
$T_R$	0
$K_A$	20.0
$T_A$	0.2
$K_E$	1.0
$T_E$	0.314
$K_F$	0.063
$T_F$	0.35
$S_{EA}$	2.5484
$S_{EB}$	0.5884

#### Load Data

Parameters	bus1	bus2	bus5	bus6	bus8
$P_L$	1.80	0.50	0.25	0.25	1.0
$Q_L$	0.265	0	0.075	0.075	0.35

Bus 1 has a constant power load. The loads on buses 2,5,6,8 have real power loading of 40% constant current and 60% constant admittance and reactive power loads of 50% constant current and 50% constant admittance. Base MVA is 100 and the system frequency is 60 Hz. Bus voltage settings are  $v1 = 1.02, v2 = 0.99, v3 = 1.005$ .

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