Min-max Transfer Capability: A New Concept

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Abstract - In this paper we discuss a new concept of an interval for transfer capability and present an algorithm for computing its lower bound which we term the min-max transfer capability. The algorithm is a Bisection Search algorithm. We compare it to a branch-and-bound algorithm, which is standard for min-max problems. We find that the Bisection Search algorithm is efficient and simple to implement. We describe the conceptual analysis and the algorithm using a DC load flow setting. A generalization of the algorithm to problems using an AC load flow is briefly discussed. We then demonstrate the new algorithm using the IEEE 118-bus system.

Keywords: Power System, Optimization, Transfer Capability

Introduction

The notion of the transfer capability of a transmission interface is often used by operators for monitoring transmission system security. Traditionally, the maximum transfer capability of a line or an interface is determined based on a single dispatch used to meet a given load. That is, given a load, maximum transfer can be described as the maximum flow that can occur across an interface to meet that load. If more than one load level is involved, the transfer capabilities are calculated based on repeated simulations using single-level optimization tools [1-7]. A single-level optimization tool yields a result that is generally optimistic because there is a single generation pattern needed to achieve the maximum transfer. To remedy the problem of repeated simulations, one often assumes that generation follows certain pattern (for example, a change in proportion to their capabilities). We explore the notion more fully in other recent work [8].

Our thesis is that that transfer capability is best described by an interval, denoted [$P_{min-max}$, P_{max}] rather than a single value, P_{max}. The lower bound of this interval is termed the min-max transfer capability. This number is the maximum transfer that can be achieved by any generation pattern that respects the generation limits of individual machines. This number is important, especially in a market setting, because as long as the actual transfer is below this number, any generator knows they can offer the difference between $P_{min-max}$ and the current transfer into the market and, if accepted, it will not cause P_{max} to be exceeded. The generation pattern associated with P_{max} is never revealed so, given the current generation pattern, one does not know how much of their generation can actually be accepted into the market. As discussed in [8], computing the min-max transfer capability involves solving a bi-level optimization problem. Branch-and-bound algorithm appears to be the state-of-the-art for solving such a problem [9].

Although min-max transfer capability is described as a bi-level optimization problem, it can be solved as if it were a continuous problem. This is true because it has a special structure: The upper-level objective function is a single variable (the transfer capability) and a simple Bi-sectioning Search algorithm is ideal for such problems. In this paper, we describe this algorithm around a DC load flow model though extension to a AC load flow model is straightforward.

Motivation

Suppose one is interested in finding the transfer capability of a transmission interface composed of line 1-3 and 2-3, as illustrated in Fig. 1. For simplicity, we do not consider line outages.

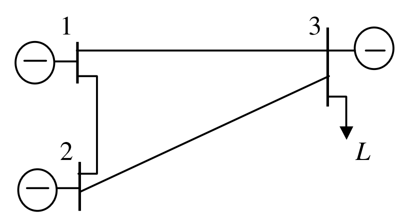

Fig. 1. The single-line diagram of an example power system

An optimization algorithm can be formulated to solve the transfer capability problem as follows:

$$
\begin{aligned}
\underset{\mathbf{P}}{Max} \quad & P_1 + P_2 \\
S.T. \quad & T_{11}P_1 + T_{12}P_2 \le \overline{F}_1 \\
& T_{21}P_1 + T_{22}P_2 \le \overline{F}_2 \\
& T_{31}P_1 + T_{32}P_2 \le \overline{F}_3 \\
& 0 \le P_1 \le \overline{P}_1
\end{aligned}
$$

$$0 \le P_2 \le \overline{P}_2$$
$$0 \le P_3 = L - P_1 - P_2 \le \overline{P}_3$$

In this example, T denotes distribution factors [10], P denotes generator output, $\overline{P}$ is the maximum generator capability, and L is the load. The solution of this problem is the point "*max*" shown in Fig. 2. This point is an optimistic estimate of the transfer capability because operating points in the shaded area are not considered. It is obvious also that the point "*min-max*" in Figure 2 yields a conservative estimate of transfer capability. In practical applications, however, one often needs to find this "min-max" point.

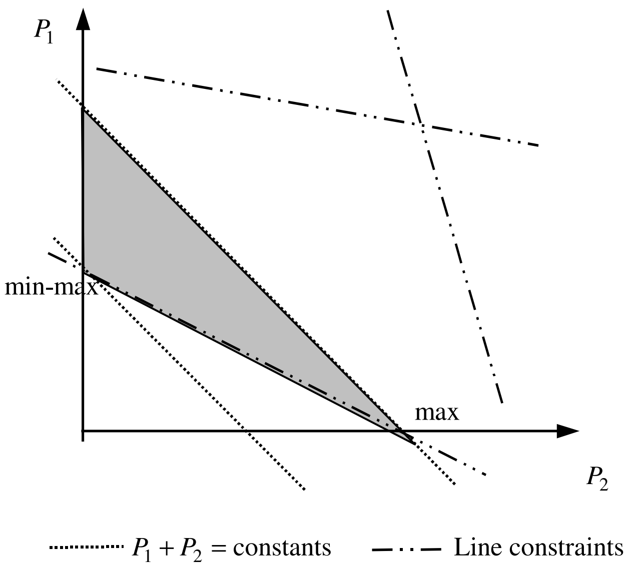

Fig. 2. Graphical solutions to the example problem

Usually the min-max transfer capability can be found approximately by an exhaustive search in generation-load space using a load flow program. However, this kind of exhaustive search is not desired for obvious reasons. The following sections describe a method that finds the exact min-max transfer capability without exhaustive search.

Formulation

The idea of min-max transfer capability is that, regardless of the generation pattern choosen, the flow on each line is within limits. A bi-level optimization model is therefore appropriate for such calculation: first level optimization solves "maximum transfer" while a second-level optimization checks to see if maximum flows on individual lines exceed their limit. Consider the following notation

$\mathbf{P}'$ - a fictitious dispatch that yields maximum transfer if the constraints present in the lower-level optimization problem are not binding
$\mathbf{P}''$ - a fictitious dispatch representing the worst-case combination of generation outputs.
NLI - number of lines in an interface.
NGen - number of generators.
T_{ij} - distribution factor of line *i* with respect to generator *j*.
L_j - active power load at *j*-th node.
$\overline{F}_i$ - thermal limit of *i*-th transmission line.
NTranC - number of transmission constraints.
K - contains the indices of line flow constraints associated with the interface under study.

The formulation of the min-max transfer capability has one first level optimization, and a number of second level optimizations as follows:

$$\underset{\mathbf{P}',\mathbf{P}''}{Max} \quad \sum_{i=1}^{NLI} \sum_{j=1}^{NGen} T_{ij} \cdot (P'_j - L_j) \qquad (1\text{-}1)$$

$$S.t. \quad \sum_j P'_j = \sum_j L_j \qquad (1\text{-}2)$$

$$\sum_j T_{ij} \cdot (P'_j - L_j) \le \overline{F}_i \qquad (i = 1,2,\ldots,NTransC) \qquad (1\text{-}3)$$

$$\underline{P}_j \le P'_j \le \overline{P}_j \qquad (j = 1,2,\ldots,NGen) \qquad (1\text{-}4)$$

$$\left\{ \begin{array}{ll} \sum_j T_{kj} \cdot (P''^k_j - L_j) \le \overline{F}_k & (2\text{-}0) \\ \underset{\mathbf{P}''^k}{Max} \quad \sum_j T_{kj} \cdot (P''^k_j - L_j) & (2\text{-}1) \\ S.t. \quad \sum_j P''^k_j = \sum_j L_j & (2\text{-}2) \\ \underline{P}_j \le P''^k_j \le \overline{P}_j \qquad (j = 1,2,\ldots,NGen) & (2\text{-}3) \\ \sum_{i=1}^{NLI} \sum_{j=1}^{NGen} T_{ij} \cdot (P''_j - L_j) \le \sum_{i=1}^{NLI} \sum_{j=1}^{NGen} T_{ij} \cdot (P'_j - L_j) & (2\text{-}4) \end{array} \right\} \quad (k \in K)$$

In equations (1-3), we include both normal-state limits as well as post-contingency limits. The number k in (2-1) to (2-4) is calculated as follows: Suppose that an interface consists of 4 lines, then the number of lower-level problems, taking into account *n-1* contingencies, is equal to 4 + 4 * 3 = 16.

In other words the two-level optimization problem finds the transfer capability under two classes of constraints: regular operating constraints (1-2) ~ (1-4), and constraints under a worst-case generation dispatch scenario (2-0) ~ (2-4).

The familiar maximum transfer capability satisfies only the first class of constraints.

The min-max problem can be solved using a standard branch-and-bound algorithm [9]. The optimality conditions can also be found in [9] or in our previous work [8]. An undesired characteristic of this algorithm is that it requires an exceedingly large CPU time for large-scale problems because of the famous "combinatorial explosion" phenomena. The following section describes an algorithm that is free of this problem.

First, we study the structure of the bi-level optimization problem (1-1) to (2-4). Note that the solution of the upper level problem (1-1) to (1-4) is the upper bound of min-max transfer capability P_{max}. If none of the lower level constraints (2-0) are binding, the max transfer capability equals the min-max transfer capability. A lower bound of the min-max transfer capability is zero. Let:

$$x = \sum_{i=1}^{NLI} \sum_{j=1}^{NGen} T_{ij} \cdot (P'_j \ - L_j)$$

It follows that:

$$0 \le x \le P_{Max}$$

The *k*-th lower level optimization problem can be re-stated as follows:

$$\underset{\mathbf{P}''^k}{Max} \quad F_k \ = \sum_j T_{kj} \cdot (P''^k_j - L_j) \qquad (3\text{-}1)$$

$$S.t. \quad \sum_j P''^k_j = \sum_j L_j \qquad (3\text{-}2)$$

$$\underline{P}_j \le P''^k_j \le \overline{P}_j \qquad (j = 1,2,...,NGen) \qquad (3\text{-}3)$$

$$\sum_{i=1}^{NLI} \sum_{j=1}^{NGen} T_{ij} \cdot (P''_j \ - L_j) \le x \qquad (3\text{-}4)$$

It is obvious that there is a correspondence between the solution to the above problem, F_k^*, and x. Denote this correspondence by $F_k^* = F_k(x)$. Note that it has three properties. First, it is a continuous, single-valued, function. Second, it is non-decreasing. In other words, as x decreases, the maximum flow on each line, F_k^*, can only decrease or remain the same.

Third, the set $\Gamma_k = \{ \ x : F_k(x) \le \overline{F}_k \ \}$ is convex. To see this is true suppose x_1 and x_2 belong to Γ_k. Then any point within the interval $[x_1, x_2]$ also belongs to Γ_k. Note that the intersection of the convex sets Γ_k, $k \in K$, is convex.

Now let us re-formulate the (1-1) ~ (2-4) as the following one-dimensional optimization problem:

$$\underset{x}{Max} \quad x \qquad (4\text{-}1)$$

$$S.T. \quad F_k(x) \le \overline{F}_k, \quad k \in K \qquad (4\text{-}2)$$

$$0 \le x \le \max\_TC \qquad (4\text{-}3)$$

The above optimization problem has a convex feasibility region and therefore any solution is unique. If there is a solution, it can be found by the following bi-sectioning algorithm. Let $x = P_{Max}$, solve (3-1)~(3-4) and check constraints (4-2) for $k \in K$. If constraints (8-2) are satisfied, x is the solution. If not, decrease the value of x, solve (3-1)~(3-4) and check (4-2) again. This procedure is repeated until the solution difference between two iterations is small. This algorithm, illustrated in Figure 3, can be replaced by the more efficient Golden Search algorithm.

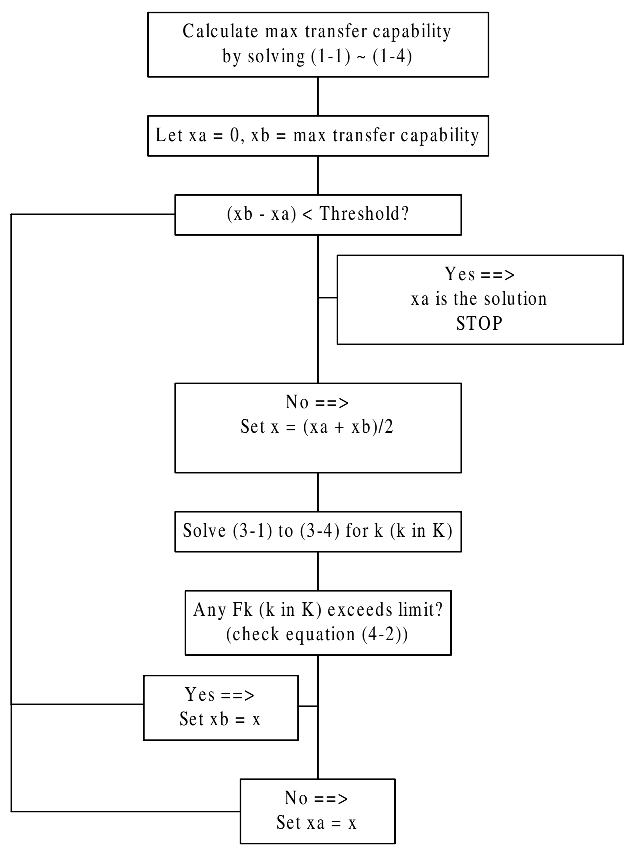

Figure 3. Bi-sectioning Search Algorithm for Min-Max Transfer Capability

To further illustrate the algorithm, consider a comparison between it and an algorithm described in [2]. Both the method presented here and the method in [2] are based on a certain line search algorithm such as bi-section search. The difference between our method and that in [2] is that in our method a worst-case generation dispatch scenario is sought at each step by solving a second level optimization (3-1) ~ (3-4) whereas in the method described in [2], a specific generator is designated to pick up load unbalances. In some commercial-grade computer program, all generators participate in picking up load unbalances proportional to their capabilities.

Some observations about the performance of the proposed algorithm are in order. First, the algorithm requires a computational effort that is only about ten times greater than what a standard OPF requires. The computational effort needed does not increase exponentially with the size of the problem as is typical of these kind of algorithms. Second, the special properties of the min-max transfer capability are independent of the model. Whether a DC load flow model or an AC load flow is used does not matter. Therefore the search algorithm can be easily extended to solve problems that require AC load flow constraints.

Numerical Example

In this section we report results of tests on an aggregated 118-bus system. The original single-line diagram for the system can be found at http://www.powerworld.com. The interface we choose to study consists of the transmission lines 62-67, 62-66, 64-65, 59-54, 59-55, 59-56, 59-56. In the study, contingencies are not considered for simplicity. We choose the starting point for the Bi-sectioning search to be the value of the maximum transfer capability, which is 2500 MW as can be seen from the iteration behavior illustrated in Figure 4. The search algorithm yields the min-max transfer capability of 980 MW after nine iterations, while the exact solution of min-max transfer capability is 982 MW.

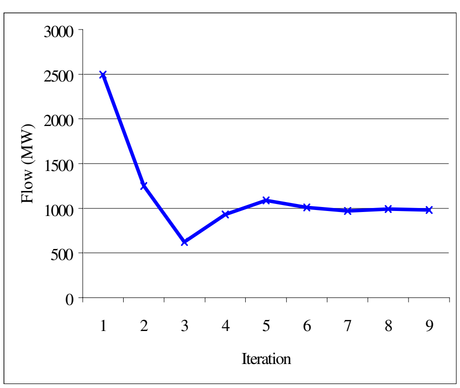

Figure 4. Iteration Behavior of the Search Algorithm

Conclusion

The formulation of min-max transfer capability involves combinatorics which implies that the problem would require extra-high computational efforts using a standard branch-and-bound algorithm. A closer look at the problem suggests that, as demonstrated in the paper, it is possible to solve min-max transfer capability using bi-sectioning search algorithm which is much more efficient than branch-and-bound algorithm. The major computational burden is in solving about tens of OPF problems. Given the existing state-of-the-art computer technology and OPF software, the proposed search algorithm should be able to solve, off-line, real world problems in AC load flow setting.

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Biographies

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