DESIGNING COST EFFECTIVE DEMAND MANAGEMENT CONTRACTS USING GAME THEORY

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Abstract

Demand relief from customers can help a utility solve a variety of problems. There exist all sorts of different demand management programs that utilities use. A critical issue is the incentive paid to the customer to participate in demand management programs and provide load relief. The utility has to design cost effective yet attractive demand management contracts. The main goal is to get load relief when needed. If the contracts are designed to be cost effective they can help the utility reduce costs. Customers sign up for programs when the benefits they derive in the form of up front payments and interruption payments exceed their cost of interruption. In order to design such contracts, mechanism design with revelation principle is adopted from Game Theory and applied to the interaction between a utility and its customers.The idea behind mechanism design is to design an incentive structure that encourages customers to sign up for the right contract and reveal their true value of power (and thus, the value of power interruptibility). **Keywords**: Demand management, mechanism design, load interruption, load curtailment, system security.

Introduction

The increased penetration of backup generation [2] and energy management systems opens the door for more creative means for integrating demand management into utility operations. By explicitly examining customer outage costs [4, 2] and analyzing their load behavior it is possible for utilities to design different kinds of demand management programs and attract customers to help in case of emergencies in return for an incentive fee [7]. Because a utility can only estimate the outage costs to a customer, it is difficult for a utility to know how much incentive to offer in order to attract customers to curtail or interrupt their load. The main theme of this paper is to design cost effective demand management programs that do not require the knowledge of customer outage costs, but rather use Game Theory [8] to design optimal curtailment programs. The process of designing contracts that attain this objective is called *mechanism design with revelation principle.* The mechanism (or contract offered by the utility) makes sure that the utility benefit is maximized *and* that customers are compensated sufficiently to participate voluntarily. A new general formulation is developed and illustrated by means of an example. The paper also combines the economic aspects of contracts with power system sensitivity analysis. Sensitivity methods attribute value to power interruptibility at every location in the grid. Thus, contracts can be customized by location.

1 Mechanism Design

Mechanism design and the revelation principle are key concepts used in nonlinear pricing. They are explained in detail in, among other places, [8] and [13]. Mechanism design is a powerful tool that helps a principal (in this case, the utility), with no private information about its customers, decide in an optimal way how much to buy from (or sell to) its customers and at what price. The revelation principle [9, 5, 14] is used to simplify the problem. The mechanism (or contract offer structure) can be designed in a way that customers wishing to maximize their own total benefit are encouraged to reveal their true valuation of power interruptions.

The mechanism has two kinds of output: a decision vector (amounts to buy or sell) and a vector of monetary transfers from the principal to each customer. This theory is applied to the interaction between the utility and its customers. In order to better understand these issues, it is desirable to first understand issues in nonlinear pricing.

1.1 Nonlinear pricing

Consider that a customer values the use of electricity according to a declining marginal benefit as a function of amount of energy consumed (denoted by q). Let the marginal benefit be described by:

$$b(q) = \theta(b_0 - sq) \tag{1}$$

where θ is a parameter that depends on the customer. Figure 1 illustrates the marginal benefit function for $b_0 = 1$, $s = 1$ and two values of θ.

The total benefit B is the integral of this marginal benefit. For the type of benefit function assumed above, the following is the total benefit:

$$B(\theta, q) = \theta b_0 q - \frac{1}{2} s\theta q^2 \tag{2}$$

Figure 2 illustrates the total benefit curves for each of the two customer types.

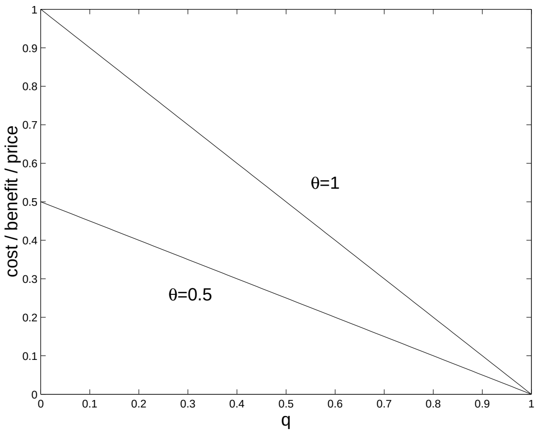

Fig. 1*: Marginal benefit for two customer types.*

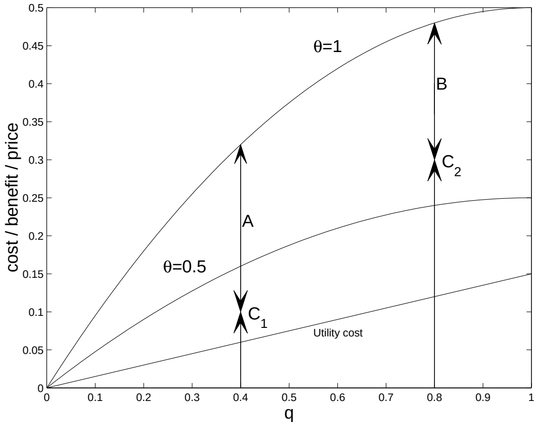

Fig. 2*: Total benefit, cost to producer and consumption levels for two customer types.*

The cost (per hour) of producing electricity under a specific set of conditions is c. Assume that the utility elects to consider only two types of customers it wishes to sell to, a small customer to which it wishes to sell quantity $\underline{q}$ and a large customer to which it wishes to sell a quantity $\bar{q}$, with $\underline{q}$ and $\bar{q}$ yet to be determined ($\underline{q} < \bar{q}$). The cost to produce $\underline{q}$ is $c\underline{q}$. Likewise, the cost to produce $\bar{q}$ is $c\bar{q}$. The straight line defining these (and any other) production costs is also illustrated in Figure 2. The utility wishing to sell at a profit selects price/quantity points that lie at or above this line. Let C_1 be the price chosen for quantity $\underline{q}$ and C_2 be the selected sale price for quantity $\bar{q}$ (as shown in Figure 2). Clearly, a utility can hope to sell to the small customer only if $B(\underline{\theta}, \underline{q}) \geq C_1$ and it can hope to sell to the large customer only if $B(\bar{\theta}, \bar{q}) \geq C_2$. This is, in fact, the case in this figure. This condition is called the *rationality constraint.*

A more subtle constraint exists: if the utility were to always charge prices close to $B(\bar{\theta}, q)$, the small consumer would be unable to use power, since this would be done at a loss. Assume that there is at least one price/quantity offering equal to or below curve $B(\underline{\theta}, q)$ (but above cq). This is, indeed, the case of the $(C_1, \underline{q})$ offering. Now, if the large consumer were to choose a small amount of consumption ($\underline{q}$) its total benefit is the segment illustrated as A in the diagram. If, on the other hand, it were to consume the large amount ($\bar{q}$), its net benefit is illustrated by segment B. It seems reasonable to think that if the large customer can derive more benefit by consuming less (that is, if A>B), it is going to consume less. This is almost never desirable to the utility, as it results in highly suboptimal conditions. Thus, we require that pricing be such that A≤B for the larger customer. This condition is called the *incentive compatibility* condition. Figure 2 illustrates a case that violates this condition, and thus encourages the customer to “lie.” It can be shown mathematically that optimality requires that the lower consumption/price point is determined by the rationality condition, and that the upper price be determined by a binding incentive compatibility condition.

If only the large customer existed, optimality would be attained when $\dfrac{dB(\bar{\theta}, q)}{dq} = c$. If only the small customer existed, it would be optimal to select the situation where $\dfrac{dB(\underline{\theta}, q)}{dq} = c$. It is the role of mechanism design to design pricing structures so that optimality is attained where there is a mix of customers, and when there is uncertainty about the mix.

1.2 Contract Design

The cost of power curtailment to a customer depends on both the customer and the amount interrupted. We assume, at least initially, that the cost $c(\theta, x)$ to a customer of type θ of curtailing x MW is:

$$c(\theta, x) = K_1 x^2 + K_2 x - K_2 x \theta. \tag{3}$$

Here θ is a continuous variable describing the customer type. It can also be called the customer preference parameter. The “$-K_2 x\theta$” term is included so that different values of θ lead to different values of $\partial c/\partial x$ (marginal cost for the customer). Notice that, as θ increases the marginal cost decreases. That is, θ has effectively been used to “sort” the customers from “least willing” to “most willing” to shed load. This form of the cost function suggests that the customer with the lowest θ will have the highest marginal cost and hence the lowest marginal benefit. This provides a good way of modeling the *willingness* of each customer to shed load by way of θ.

For the sake of simplicity, we assume that K_1 and K_2 are known to be 1/2 and 1 respectively. These assumptions do not affect the fundamental concepts to be considered here, since they amount to simple scaling. It is clear that customers of different types value interruptions differently. Although equation (3) gives an expression of the cost of an interruption to a customer, the parameter θ in this equation is not known to the utility.

Another assumption concerns the probability distribution of θ (denoted by $f(\theta)$). Two such possibilities are:

- We can assume that the complete set of customer types can be characterized by allowing θ to vary from

0 to 1. Furthermore, we can assume that there is an equal probability that the customer will be of any of these types (that is, θ is a random variable with a uniform distribution in the interval $[0, 1]$).

- We can assume that θ can take discrete values, each with a presumed probability. Two discrete values of θ represent the simplest such scenario.

The probability distributions associated with these values of θ are subjective probabilities. The utility need not know which, if any, of the distributions is correct. The value of θ is private information of the customer and is unknown to the utility. Having a subjective estimate of the customer types it is dealing with, the utility develops an incentive function $y(x)$ to indicate how much it is willing to pay someone for a given amount of curtailment.

Customers self-select the amount of curtailment they wish to be subjected to, based on an inspection of the incentive function offered to them. They do so rationally, by making the amount of compensation they receive from participation match the monetary incentive offered by the utility minus the actual net loss of benefit that results from the curtailment (from equation (3)). Clearly, customers will not choose to be curtailed unless they see a net positive benefit. A customer's benefit function is:

$$V_1(\theta, x, y) = y - \frac{1}{2}x^2 - x + \theta x. \tag{4}$$

In the absence of an initial sign-up incentive[1], in order for a customer to elect to participate in a program, it is necessary that $V_1 \geq 0$, that is they must see a benefit to the curtailment.

Although customer benefit functions can be quite arbitrary, only benefit functions that satisfy a so-called "single crossing property" lead to the results described in this paper. The function in equation (4) satisfies this property.

Under stressed conditions it is expensive for the utility to deliver power to certain locations. The utility can compute the value of *not* delivering power to a certain customer. This value of "power interruptibility" is parameterized in λ. The value of λ can be computed using existing efficient optimal power flow routines [6, 12]. Knowing λ enables the utility to define their own benefit function for a curtailment at a specific location:

$$V_2(\theta, x, \lambda) = \lambda x(\theta) - y(\theta) \tag{5}$$

where λ is in dollars per MW not delivered to a customer. The objective of the utility is to maximize the utility benefit function.

$$\max_{x,y} \int_0^1 [\lambda x(\theta) - y(\theta)] f(\theta) d\theta \tag{6}$$

such that,

$$y(\theta) - \frac{1}{2}x^2(\theta) - x(\theta) + \theta x(\theta) \geq 0 \tag{7}$$

$$\begin{aligned} y(\theta) - \frac{1}{2}x^2(\theta) - x(\theta) + \theta x(\theta) \geq \\ y(\hat{\theta}) - \frac{1}{2}x^2(\hat{\theta}) - x(\hat{\theta}) + \theta x(\hat{\theta}) \end{aligned} \tag{8}$$

where $\hat{\theta}$ is the preference parameter of a customer if they were to report it incorrectly. Constraint (7) is the *individual rationality constraint* which makes sure every customer is encouraged to participate, and constraint (8) is the *incentive compatibility constraint* which encourages the customers to tell the truth about their θ. This maximization problem could be solved by using the theory on mechanism design and revelation principle in [8]. The results are shown below:

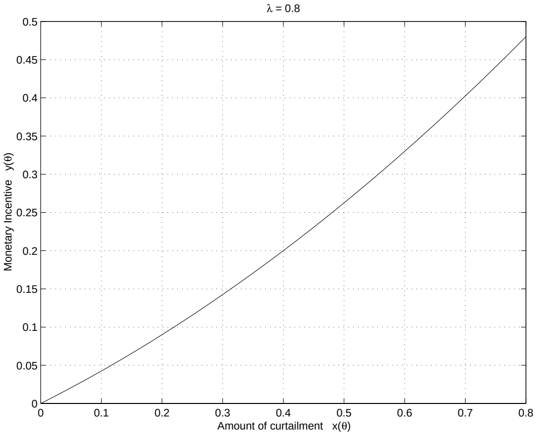

Fig. 3*: Designed Contracts*

$$x(\theta) = \begin{cases} 0 & \text{if } 0 \leq \theta < 1 - \frac{\lambda}{2} \\ 2\theta + \lambda - 2 & \text{if } 1 - \frac{\lambda}{2} \leq \theta \leq 1 \end{cases} \tag{9}$$

$$y(\theta) = \begin{cases} 0 & \text{if } 0 \leq \theta < 1 - \frac{\lambda}{2} \\ \theta^2 - 2\theta + 2\theta\lambda & \\ +\frac{3}{4}\lambda^2 - 2\lambda + 1 & \text{if } 1 - \frac{\lambda}{2} \leq \theta \leq 1 \end{cases} \tag{10}$$

Equations (9) and (10) define the contracts to be offered to different types of customers. Figure 3 depicts the plot of the monetary incentive offered as a function of the curtailed amount for a given value of λ. A family of incentive functions as λ varies is shown in Figure 4. It should be noted that the number of participating customers will change as the value of λ changes. Figure 4 shows that the value of λ plays a key role in determining the incentive to be paid to each customer. The role of θ is obvious since it determines the type of customer (which in turn determines their cost of a curtailment or interruption), however the role of λ can be subtle. λ is the parameter that brings engineering into this economic analysis, it shows that location of the customer is one of the most important aspects in this analysis.. Some locations will be more costly to deliver power than others and it only makes sense that the utility will want to have curtailment contracts with it's customers who are at expensive locations.

[1] A fixed one-time sign up incentive can be a part of the overall compensation, but it is not considered here. It would have the effect of modifying the perceived net customer benefit.

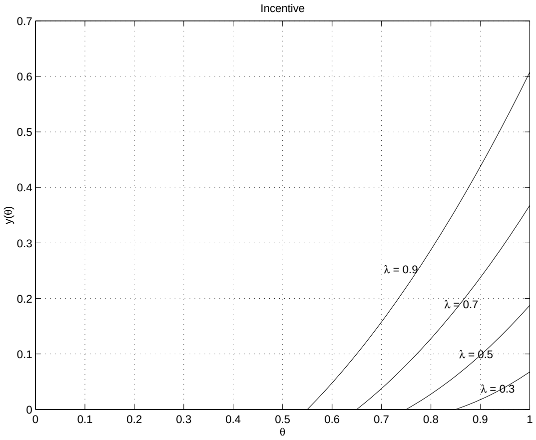

Fig. 4*: Normalized incentive function vs* θ*.*

2 Sensitivity Analysis

Sensitivity analysis can be used to determine the value of interruptible power for the utility. In [10], the authors compute the sensitivity of the loading margin of a system with respect to arbitrary parameters. If loads are the parameters, then sensitivity of the loading margin can be computed with respect to each load. Let:

$$f(x, \gamma, p) = 0 \tag{11}$$

where x is the vector of state variables, γ is the vector of real and reactive load powers, and p is the vector of loads. If a pattern of load increase is specified with a unit vector k, the point of collapse method [3] can be applied to yield the left eigenvector w. The sensitivity of the loading margin to a change in any load is:

$$\frac{\Delta L}{\Delta p} = L_p = \frac{-\omega f_p}{\omega f_\gamma k} \tag{12}$$

Once we have the sensitivity of the loading margin to a change in any load, we use it to rank loads. Let L be the loading margin of the system. The above formula lets us construct an expression relating changes in individual loads (Δp_1, Δp_2, etc) to changes in the security margin:

$$\Delta L = L_{p_1}\Delta p_1 + L_{p_2}\Delta p_2 + ... + L_{p_m}\Delta p_m \tag{13}$$

where m is the number of loads of interest. As equation (13) suggests, the load with the highest sensitivity would help increase the loading margin the most. By using these sensitivities and the dollar per kW figures from the designed contracts, the utility can estimate how much it would cost to increase the security of the system:

$$\frac{\Delta L}{\Delta \$} = \frac{\Delta L}{\Delta p}\frac{\Delta p}{\Delta \$} \tag{14}$$

where $\Delta\$$ is the amount the utility will spend.

Equation (14) helps us determine how much it would cost to increase the loading margin by curtailing one of the loads signed up for a demand management contract. Other kinds of sensitivities could also be computed and combined with the economic analysis done in the previous section to give the utility a dollar figure in solving their problems.

3 A Comprehensive Example

Before any demand management contracts are offered to customers, the utilities need to go through a planning stage. The first step is to analyze their electric power system and identify which load locations (customers) would be most helpful in case of emergencies or anticipated problems (voltage collapse, line overloads, insufficient generation, etc.). A sensitivity analysis needs to be performed on the system to determine the most valuable loads for each problem. This analysis involves load forecasting and consideration of multiple scenarios and time periods. The contracts will vary by location, class and type of customer. The following example will be developed in three main stages:

1 Sensitivity analysis to determine the most valuable loads to increase loading margin to voltage collapse.
2 Game Theory analysis to determine the optimal demand management contracts.
3 Comparison of different scenarios of demand management contracts.

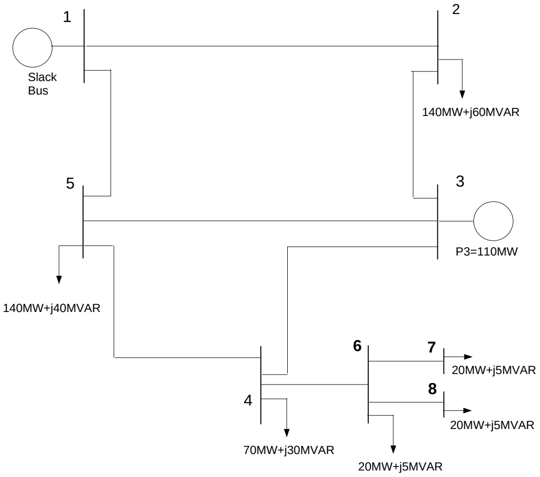

Fig. 5*: Example 8-bus system*

An example that uses an 8 bus system (see fig. 5) with 2 generators and 6 loads is analyzed. The generator at bus 1 is designated to be the slack generator. Of concern is the loading margin to voltage collapse. If the load is increased equally on each load bus and only the slack generator picks up the extra load the sensitivity of the loading margin to voltage collapse with respect to a change in each load is shown in Table 1. In this example the most valuable loads are 7 and 8. They have the highest sensitivity. If the system gets close to

Table 1: Sensitivity of the loading margin to voltage collapse with respect to each load (loading direction chosen as equal increments for each load)

Load Bus	Sensitivity (MW/MW)
2	-0.03
4	-0.89
5	-0.12
6	-1.48
7	-1.73
8	-1.73
Loading Margin = 36.18 MW	

a voltage bifurcation point [1, 11] the utility may want to curtail a guaranteed amount of load, hence the contracts would be designed for specific amounts of load curtailment. It is important to determine the ranking and quantitative impact of loads before the utility offers curtailment contracts. After the contracts are signed, an algorithm can be developed to check for distance to collapse and suggest the optimum curtailment order of loads when required.

After the relative value of all load locations is determined, customer attributes, designed demand management programs and the game theory formulation are used to design optimal demand management contracts[2] (similar to the contracts shown in Fig. 3). If the assumptions hold the customers will sign up for the contracts as shown in Table 2. Notice that the customers at location bus 7 and 8 signed up for different contracts even though they are at equally sensitive locations. This is due to the fact that the customer at bus 7 has a higher marginal cost for shedding load than the customer at bus 8. The developed contract design formulation captures both locational and cost attributes of the customers.

Table 2: Optimal Portfolio of Demand Management Contracts

Customer	Amount Curtailed x	Incentive Offered y
Bus 2	1.00 MW	$225.00
Bus 4	3.50 MW	$1620.00
Bus 5	3.00 MW	$975.00
Bus 6	4.00 MW	$2000.00
Bus 7	5.00 MW	$2875.00
Bus 8	7.00 MW	$4375.00
Available Relief = 24 MW		
Increase in Margin = 30 MW		

Table 3: Non-Optimal Portfolio of Demand Management Contracts with Fixed $\lambda = 0.7$ (average value)

Customer	Amount Curtailed x	Incentive Offered y
Bus 2	1.00 MW	$375.00
Bus 4	3.00 MW	$1275.00
Bus 5	5.00 MW	$2375.00
Bus 6	3.00 MW	$1275.00
Bus 7	3.00 MW	$1275.00
Bus 8	5.00 MW	$2375.00
Available Relief = 20 MW		
Increase in Margin = 22 MW		

Table 4: Different Scenarios for Contracts

Scenario	Relief (MW)	$\uparrow$ LM (MW)	U. Profit ($)	C. Profit ($)
Optimal	24	30	6456.00	2806.00
$\theta = 0.8$	19	26	5681.00	1981.00
$\lambda = 0.7$	20	22	5050.00	1950.00

The value of power interruptibility (λ) and customer preference parameter (private information θ that helps the utility estimate customer cost) are the two critical elements of the economic analysis. In order to emphasize the importance of these values, some further tests are performed. In one simulation λ was fixed to be 0.7 (its average value), i.e. the utility decides that the value of interrupting power at each location is the same. As shown in Table 3 the amount of available relief decreased and the result was also a smaller increase in the loading margin. In another simulation the utility assumes that the cost of shedding load is the same to all customers (i.e. it fixes θ at its average value ($\theta = 0.8$)). However it is observed that this also is not an optimal portfolio since the available load relief and the increase in loading margin is lower than the optimal case shown in Table 2. Some economic facts are also computed for each portfolio and a comparison is made in Table 4. In the scenarios where λ and θ are fixed to a certain value, a non-optimal portfolio of contracts is still obtained, however the results verify that the *optimal* portfolio of contracts help both the utility and the customer in maximizing their profit. More importantly it indicates that the optimal portfolio maximizes the amount of available load relief and the increase in the loading margin. The discussion above is focused on avoiding voltage collapse, however a similar approach can be used to relieve line overloads.

4 Conclusion

Nonlinear pricing can be used as a means for extracting maximum value from demand management contracts. By using mechanism design, optimal contracts can be designed that encourage customers to voluntarily sign up for the contract that best suits their needs. It is not necessary for a utility to know in advance the type of customer it faces when designing such programs. Available cost effective load relief can be a substitute for building extra generation.The paper also illustrates the importance of location, and describes a method for incorporating location into the process.

Acknowledgement

The support of Energy Center of Wisconsin is greatly appreciated.

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