PREPRINT DECEMBER 1999; SUBMITTED TO IEEE TRANS. ON POWER SYSTEMS

SENSITIVITY OF TRANSFER CAPABILITY MARGINS WITH A FAST FORMULA

Scott Greene **Ian Dobson** **Fernando L. Alvarado**
L.R. Christensen Associates, 4610 University Avenue, Madison WI 53705 USA
Power Systems engineering research center, ECE Department, University of Wisconsin, Madison WI 53706 USA

Abstract: Bulk power transfers in electric power systems are limited by transmission network security. Transfer capability measures the maximum power transfer permissible under certain assumptions. Once a transfer capability has been computed for one set of assumptions, it is useful to quickly estimate the effect on the transfer capability of modifying those assumptions. This paper presents a computationally efficient formula for the first order sensitivity of the transfer capability with respect to the variation of any parameters. The sensitivity formula is very fast to evaluate. The approach is consistent with the current industrial practice of using DC load flow models and significantly generalizes that practice to more detailed AC power system models that include voltage and VAR limits. The computation is illustrated and tested on a 3357 bus power system.

Keywords: power system security, power system control, power transmission planning

INTRODUCTION

Transfer capability indicates how much a particular bulk power transfer can be changed without compromising system security under a specific set of assumptions. The increased attention to the economic value of transfers motivates more accurate and defensible transfer capability computations.

A variety of applications in both planning and operations require the repetitive computation of transfer capabilities. Transfer capabilities must be quickly computed for various assumptions representing possible future system conditions and then recomputed as system conditions change. The usefulness of each computed transfer capability is enhanced if the sensitivity of the transfer capability is also computed [13, 9]. This paper shows how to quickly compute these sensitivities in a general and efficient way. The sensitivities can be used to estimate the effect on the transfer capability of variation in simultaneous transfers, assumed data, and system controls.

While there is general agreement on the overall purpose and outline of transfer capability determination, the precise requirements for such computations vary by region and are evolving. In this paper, we focus on a generic transfer margin computation which is widely applicable.

A GENERIC TRANSFER MARGIN COMPUTATION

1 Establish a secure, solved base case consistent with the study operating horizon.
2 Specify a transfer direction including source, sink, and loss assumptions.
3 Establish a solved transfer-limited case and a binding security limit. The binding security limit can be a limit on line flow, voltage magnitude, voltage collapse or other operating constraint. Further transfer in the specified direction would increase the violation of the binding limit and compromise system security.
4 The transfer margin is the difference between the transfer at the base case and the limiting case.

Calculations of Available Transfer Capability (ATC), Capacity Benefit Margin (CBM), and Transfer Reliability Margin (TRM) typically require that this generic transfer margin computation be repeated for multiple combinations of transfer directions, base case conditions, and contingencies [12, 13].

The generic transfer margin computation can be implemented with a range of power system models. One convenient and standard practice is to use a DC power flow model to establish transfer capability limited by line flow limits. The limiting cases are then checked with further AC load flow analysis to detect possibly more limiting voltage constraints.

Alternatively, a detailed AC power system model can be used throughout and the transfer margin determined by successive AC load flow calculations [9] or continuation methods [2, 3, 1, 14]. A related approach [e.g., EPRI's TRACE] uses an optimal power flow where the optimization adjusts controls such as tap and switching variables to maximize the specified transfer subject to the power flow equilibrium and limit constraints. The formulations in [9] and [15] show the close connection between optimization and continuation or successive load flow computation for transfer capability determination. Methods based on AC power system models are slower than methods using DC load flow models but do allow for consideration of additional system limits and more accurate accounting of the operation guides and control actions that accompany the increasing transfers. Under highly stressed conditions the effects of tap changing, capacitor switching, and generator reactive power limits become significant. A combination of DC and AC methods may be needed to achieve the correct tradeoff between speed and accuracy. The methods in this paper account directly for any limits which can be deduced from equilibrium equations such as DC or AC load flow equations or enhanced AC equilibrium models.

SENSITIVITY COMPUTATION

System Modeling

Assume a general power system equilibrium model written as n equations:

$$0 = f(x, \lambda, p)$$

where

x is an n dimensional state vector that includes voltage magnitudes, angles, branch flows, and generator MW and MVAR outputs.

λ is a vector of generator MW output set points and/or scheduled net area exports.

p is a parameter vector including regulated voltage set points, generator load sharing factors, load and load model parameters and tap settings.

The limits on line flows, voltage magnitudes, or generator VAR outputs are modeled by inequalities in the states:

$$x_i^{\min} \leq x_i \leq x_i^{\max} \qquad , i = 1, \ldots, n.$$

Due to the modeling of operator actions and generator limits, the equilibrium equations and the physical quantities represented by the x and p vectors can change under varying conditions and transfer levels. For example, when a non-slack generator is operated within its reactive power limits, the reactive power output and angle at the generator bus are components of x and the regulated bus voltage and real power output are components of p. However, when the same generator is at a reactive power limit, the generator bus voltage and angle are components of x and the real and reactive power output are components of p.

Base case: The base case specifies the nominal value λ_0 of the generator outputs and net area exports.

Transfer specification: The transfer is specified by changes to the vector λ. The transfer direction describes how λ changes as the transfer increases so that

$$\lambda = \lambda_0 + k\,t$$

where t is the transfer amount and k is a unit vector describing the transfer direction. For the simple case of net exports increasing from one area matched by reduction in net export from another area, the transfer direction k is a column vector with 1 in the row corresponding to the source area export equation and -1 in the row corresponding to the sink area export equation. For transfers specified by changes in individual bus injections, k is a column vector with positive entries at the source buses and negative entries at the sink buses.

Transfer-limited case: Identification of a solved transfer-limited case yields an equilibrium solution (x_*, λ_*, p_*) and an additional constraint referred to as the *binding limit*. The equilibrium equations that model the power system *at the binding limit* are written

$$0 = F(x, \lambda, p) \tag{1}$$

When a limit is encountered, one of the limit equations $x_i = x_i^{\min}$ or $x_i = x_i^{\max}$ holds for some i. We write the applicable equation for the binding limit in the general form

$$0 = E(x, \lambda, p) \tag{2}$$

The form (2) also encompasses more general limits. At the binding limit

$$\begin{aligned} F(x_*, \lambda_*, p_*) &= 0 \\ E(x_*, \lambda_*, p_*) &= 0 \end{aligned}$$

Transfer margin: The transfer margin is the change in the transfer between the base case and the transfer-limited case. Since $\lambda_* = \lambda_0 + kT$, the transfer margin is T.

Sensitivity Formula

Once the binding limit and the corresponding transfer-limited solved case have been found, the sensitivity of the transfer margin T can be evaluated. The sensitivity of T to the parameter p, often written as $\frac{\partial T}{\partial p}$ and here written as T_p, is computed using a formula derived in Appendix A:

$$T_p = \frac{-w \begin{pmatrix} F_p \\ E_p \end{pmatrix}\Bigg|_{(x_*, \lambda_*, p_*)}}{w \begin{pmatrix} F_\lambda k \\ E_\lambda k \end{pmatrix}\Bigg|_{(x_*, \lambda_*, p_*)}} \tag{3}$$

where

- F_p and E_p are the derivatives of the equilibrium and limit equations with respect to p.
- $F_\lambda k$ and $E_\lambda k$ are the derivatives of the equilibrium and limit equations with respect to the amount of transfer t.
- w is a nonzero row vector orthogonal to the range of the Jacobian matrix J of the equilibrium and limit equations, where

$$J = \begin{pmatrix} F_x \\ E_x \end{pmatrix}\Bigg|_{(x_*, \lambda_*, p_*)}$$

The row vector w is found by solving the linear system

$$wJ = 0 \tag{4}$$

Since J has one more row than column, there is always a nonzero vector w that solves (4). J generically has full column rank, so that w is unique up to a scalar multiple. The sensitivity T_p computed from (3) is independent of the scalar multiple.

The first order estimate of the change in transfer margin corresponding to the change in p of Δp is

$$\Delta T = T_p \, \Delta p \quad (5)$$

If the binding limit is an immediate voltage collapse due to a reactive power limit [5], then the analysis of this paper applies with the limit equation (2) becoming $Q_i = Q_i^{\max}$. If the binding limit is voltage collapse due to a fold bifurcation, the sensitivity formula of [7] applies.

Computational Efficiency

Once the transfer-limited solution is obtained, the margin estimates corresponding to varying a large number of different parameters can be obtained for little more computational effort than solving the sparse linear equations (4) for w. Solving (4) is roughly equivalent to one Newton iteration of a load flow solution. Note that w need only be computed once but can be used to find the sensitivity with respect to any number of parameters. If a sequential LP is used to determine the transfer margin as part of an optimization program, then w is found from the Lagrange multipliers obtained at the last LP solution. The remaining computations (3) and (5) needed for the estimates require only sparse matrix-vector multiplications.

The Jacobian matrix J in (4) is available, often in factored form, from the computation of the transfer-limited solution by Newton based methods. The matrix F_p in (3) is different for each parameter p but its construction is a simple sparse index operation, especially when the parameters appear linearly.

The sensitivity of the transfer capability with respect to thousands of changes in load, generation, interarea transfers, or voltage set points can be obtained in less time than a single AC load flow solution.

3357 BUS EXAMPLE

The application of sensitivity formula (3) is illustrated using a 3357 bus model of a portion of the North American eastern interconnect. The model contains a detailed representation of the network operated by the New York independent system operator and an equivalent representation of more distant portions of the network. From a base case representative of a severely stressed power system, small increases in transfer between Ontario Hydro and New York City lead to low voltages, cascading generator reactive power limits, and finally voltage instability. The sensitivity formulas are used to identify effective control action to avoid low voltage and VAR limit conditions, and to estimate the effects of variation in transfers and loading on the security of the system.

Base case: The base case is motivated by a scenario identified as problematic in the New York Power Pool summer 1999 operating study. The loss of two 345 KV lines, Kintigh-Rochester and Rochester-Pannell Road during high west to east transfer leads to low voltage conditions at the Rochester 345 kV bus. At the base case solution, the voltage at the Rochester 345 kV bus is 333 kV, slightly above the 328 kV low voltage rating.

Limiting events: From the base case, a sequence of AC load flow solutions are obtained for increasing levels of export from Ontario Hydro and increasing demand in the New York City zone. A 100 MW increase in this transfer results in the voltage at the 345 kV Rochester bus reaching its low voltage rating of 328 kV. Additional transfer leads to several low voltages and nine additional generating units reaching maximum VAR limits. Finally, for transfer of 140 MW beyond that corresponding to the Rochester voltage limit, a reactive power limit at one of the Danksammer generating units leads to immediate voltage instability [5]. (System behavior under the stressed conditions is unstable without voltage regulation at Danksammer.)

Table 1: Net zone exports in MW at base case, the initial voltage limit at the Rochester 345KV bus, and the final reactive power limit at Danksammer.

ZONE	net export base case	net export voltage limit	net export VAR limit
NYC	–4806	–4906	–5046
OH	4080	4180	4320
HQ	976	976	976
PJM	–3422	–3422	–3422
ISO-NE	–28	–28	–28

Table 1 shows the net exports for five of the zones at the base case and at two different limits. The transfer margin to the voltage limit is 100 MW and the transfer margin to the critical VAR limit is 240 MW. Since it is of interest how avoiding the low voltage limit also improves the margin to voltage instability, we compute the sensitivities of both these margins.

Sensitivity to regulated voltage set points

The sensitivity of the transfer margins to the Rochester voltage limit and the Danskammer VAR limit with respect to all parameters is obtained using formula (3). Ranking of all the NY ISO generator buses according to the sensitivity of the transfer margins with respect to regulated generator voltages indicates that the regulated voltage with the greatest effect on the transfer margin to the Rochester voltage limit and the second greatest effect on the margin to the Danskammer VAR limit is the Hydro facility in Niagara.

Fig. 1 shows the linear estimate for the change in transfer margin to the voltage limit as a function of the voltage set point at the Niagara generator. The estimates are compared with actual values computed by AC loadflow analysis represented by the circles in Fig. 1. Fig. 2 compares the linear estimate with actual values computed by AC loadflow analysis for the change in the transfer margin to the Danksammer VAR limit as a function of the Niagara voltage set point. Figs. 1 and 2 show that the estimates are accurate for a $\pm 5\%$ variation in the regulated output voltage of the Niagara unit. Note that for both limits, setting the voltage set point greater than 1.07 pu does not improve the margin as predicted because at that voltage the generator reactive power output reaches its maximum before the transfer limit is encountered.

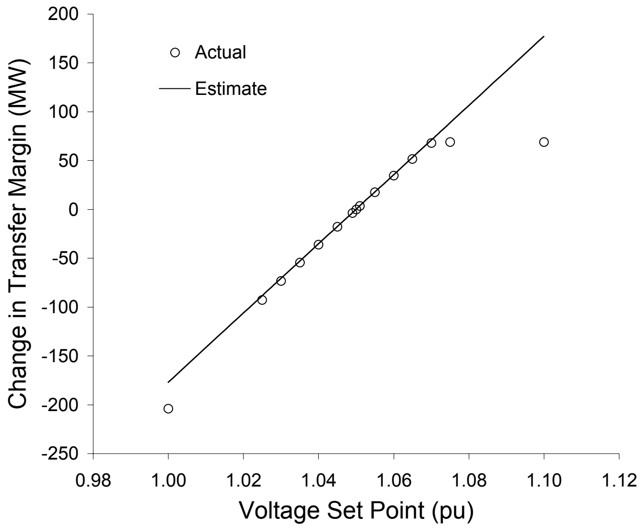

Fig. 1*: Effect of regulated output voltage on margin to voltage limit.*

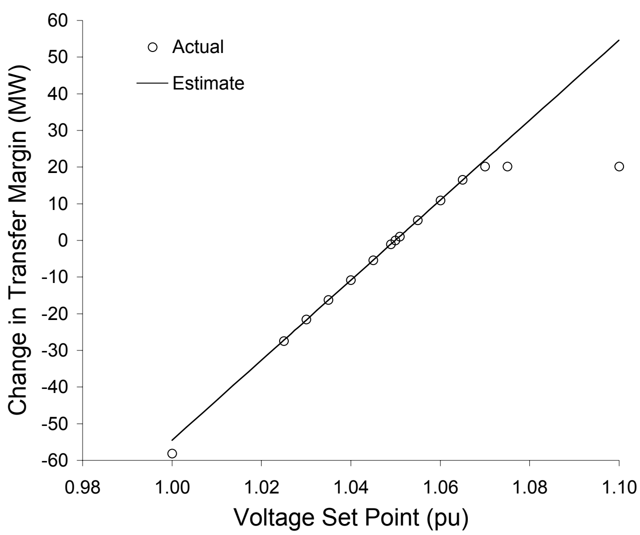

Fig. 2*: Effect of regulated output voltage on margin to critical VAR limit.*

Sensitivity to Simultaneous Transfers

One concern is the effect of simultaneous transfers on the computed transfer margins. Figs. 3 and 4 show the effects on the voltage and VAR limited transfer margins of a simultaneous Hydro Quebec to PJM transfer. The simultaneous transfer affects the VAR limit more than the voltage limit, and the sensitivity based estimates are accurate for a ± 200 MW transfer variation, which is a 20% variation in export from Hydro Quebec.

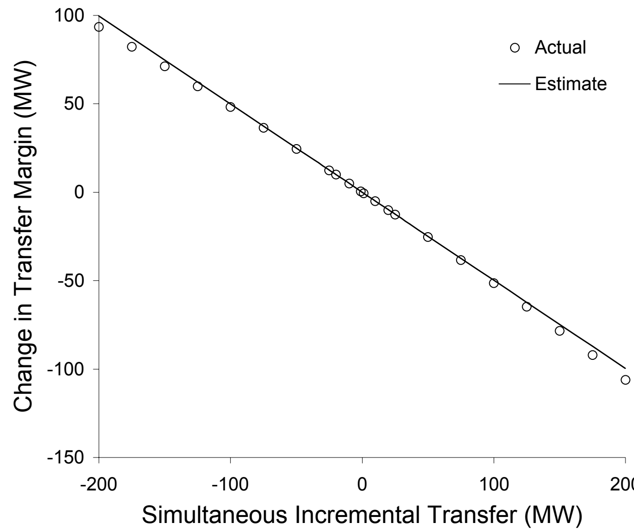

Fig. 3*: Effect of simultaneous transfer on margin to voltage limit.*

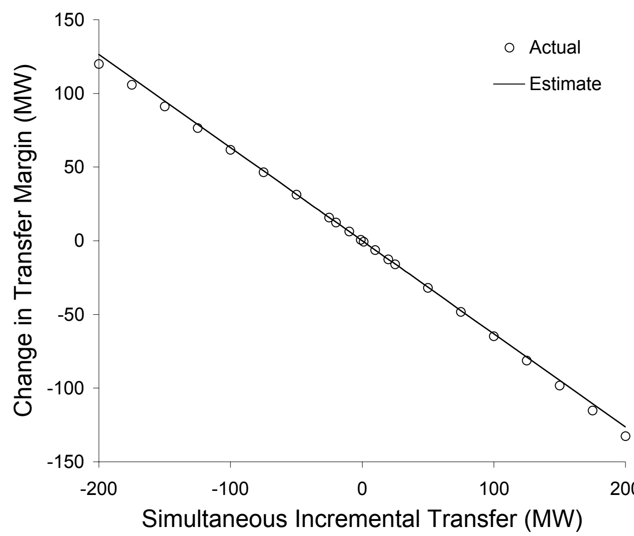

Fig. 4*: Effect of simultaneous transfer on margin to critical VAR limit.*

Sensitivity to Load Variation

Another concern is load forecast error. For example, consider the effect of load variation in the Albany region on the transfer margins. The real and reactive power loads in Albany are changed keeping constant power factor. The estimates are compared with the actual values computed directly from AC loadflow analysis in Figs. 5 and 6. The results are very accurate for ± 200 MW total load variation, but less accurate for ± 400 MW. The base case Albany zone load is 2000 MW.

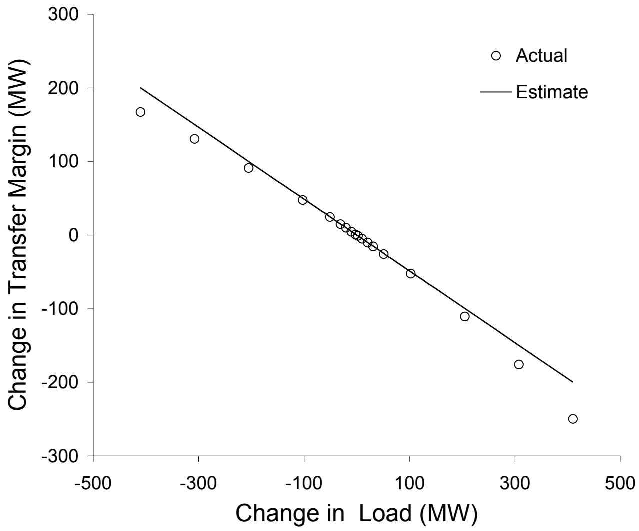

Fig. 5*: Effect of Albany loading on margin to voltage limit.*

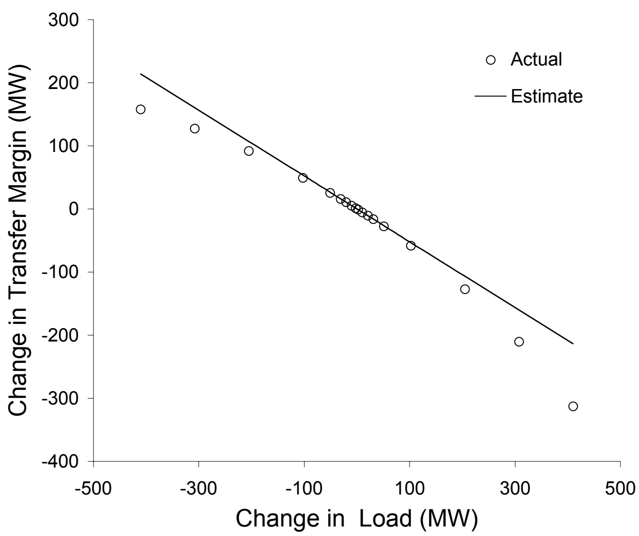

Fig. 6*: Effect of Albany loading on margin to critical VAR limit.*

DISCUSSION AND RELATED WORK

The primary tool used in industry for computing transfer capability margins is the DC loadflow model with PTDF and OTDF computations (e.g., PTI program MUST [11]). It can be shown [6] that the sensitivity formula (3) reduces to PTDFs and OTDFs for the appropriate DC load flow models and this is illustrated in Appendix B. Thus this paper significantly generalizes standard sensitivity methods to encompass more accurate transfer capability calculations on more detailed models. In particular, account can be taken of power system nonlinearity, operator and automatic control actions, and voltage and VAR limits. The detailed models also expand the range of parameters with respect to which transfer capabilities can be computed. For example, the sensitivity of a line flow limit to reactive power injection can be computed.

Gravener and Nwankpa [9] have also nicely demonstrated the use of transfer margin sensitivities; the difference with this paper lies in the way the sensitivities are computed. In [9], the sensitivities are computed numerically by incrementing the parameter and rerunning whereas we suggest a fast analytical formula for the sensitivities. Thus this paper supports the conclusions of [9] by proposing a much faster way to achieve similar benefits.

The overall margin sensitivity approach used in this paper arose in the context of loading margins to voltage collapse caused by fold bifurcation [4, 7]. The sensitivity formula of [7] differs from formula (3) in that w stands for a different vector and that no event equation is used. However, [6] formulates an event equation for the fold bifurcation to obtain a formula of the form (3) which does reduce to the formula of [7]. [10] demonstrates the use of the margin sensitivity methods of [7] for fast contingency screening for voltage collapse limits only. Testing of fast contingency screening using the more general security limits of this paper is future work.

The sensitivity formula (3) was first stated in [8] and then in the PhD thesis [6]. This paper greatly extends the initial work reported in [8] by deriving and testing the formula and applying it to a realistic power system.

CONCLUSIONS

We show how the sensitivity of the transfer capability can be computed very quickly by evaluating an analytic formula at the binding limit. The sensitivities can be used to estimate the effect on the transfer capability of variations in parameters such as those describing other transfers, operating conditions or assumed data. The approach is consistent with current industrial practice using DC load models and significantly generalizes this practice to include more elaborate AC power system models and voltage and VAR limits on power system operation. Once the transfer capability and corresponding binding limit and solved case have been computed, the first order sensitivity of this transfer capability to a wide range of parameters can be quickly computed. These first order sensitivities can contribute to the quick update of transfer capabilities when operating conditions or other transfers change. Moreover, the sensitivities can be used to select operator actions to increase transfer capability.

A simple approach computes the sensitivity of the transfer margin to the single binding limit. In practice, there are often other limits encountered just after the binding limit. Since these limits could become binding for some parameter changes, it is recommended that the sensitivity of these transfer margins also be computed. Then the power system can be steered away from several security limits that may become binding.

We conclude that after each computation of a transfer capability, it is so quick and easy to compute sensitivities of that transfer capability that this should be done routinely to extract the maximum amount of engineering value from each computation. In the case of predicting the effect of large parameter changes on transfer margins, even if more than first order accuracy is ultimately required, it is still desirable to first estimate the effects with first order sensitivities.

Support in part from NSF grant EEC-9815325 is gratefully acknowledged.

REFERENCES

[1] V. Ajjarapu, C. Christy, The continuation power flow: a tool for steady state voltage stability analysis, *IEEE Trans. Power Systems*, vol.7, no.1, Feb.1992, pp.416-423.

[2] C.A. Cañizares, F.L. Alvarado, Point of collapse and continuation methods for large AC/DC systems, *IEEE Trans. Power Systems*, vol.7, no.1, Feb.1993, pp.1-8.

[3] H.-D. Chiang, A. Flueck, K.S. Shah, N. Balu, CPFLOW: A practical tool for tracing power system steady-state stationary behavior due to load and generation variations, *IEEE Trans. Power Systems*, vol. 10, no. 2, May 1995, pp. 623-634.

[4] I. Dobson, L. Lu, Computing an optimum direction in control space to avoid saddle node bifurcation and voltage collapse in electric power systems, *IEEE Trans. Automatic Control*, vol 37, no. 10, Oct. 1992, pp. 1616-1620.

[5] I. Dobson, L. Lu, Voltage collapse precipitated by the immediate change in stability when generator reactive power limits are encountered, *IEEE Trans. Circuits and Systems, Part 1*, vol. 39, no. 9, Sept. 1992, pp. 762-766.

[6] S. Greene, Margin and sensitivity methods for security analysis of electric power systems, PhD thesis, ECE dept., University of Wisconsin, Madison WI USA, 1998.

[7] S. Greene, I. Dobson, F.L. Alvarado, Sensitivity of the loading margin to voltage collapse with respect to arbitrary parameters, *IEEE Trans. Power Systems*, vol. 12, no. 1, February 1997, pp. 262-272.

[8] S. Greene, I. Dobson, F.L. Alvarado, P.W. Sauer, Initial concepts for applying sensitivity to transfer capability, NSF workshop on available transfer capability, Urbana IL USA, June 1997.

[9] M.H. Gravener, C. Nwankpa, Available transfer capability and first order sensitivity, *IEEE Trans. Power Systems*, vol. 14, no. 2, May 1999, pp. 512-518.

[10] S. Greene, I. Dobson, F.L. Alvarado, Contingency analysis for voltage collapse via sensitivities from a single nose curve, *IEEE Trans. Power Systems*, vol. 14, no. 1, Feb. 1999, pp. 232-240.

[11] G. Heydt, *Computer analysis methods for power systems*, Macmillan, New York 1987.

[12] Transmission Transfer Capability Task Force, Available transmission capability definitions and determination, North American Reliability Council, Princeton, New Jersey, June 1996.

[13] P.W. Sauer, S. Grijalva, Error analysis in electric power system available transfer capability computation, *Decision Support Systems*, vol. 24, 1999, pp. 321-330.

[14] T. Van Cutsem, C. Vournas, *Voltage stability of electric power systems*, ISBN 0-7923-8139-4, Kluwer Academic Publishers, Boston, 1998.

[15] T. Wu, R. Fischl, An algorithm for detecting the contingencies which limit the inter-area megawatt transfer, Proceedings 1993 North American Power Symposium, Washington D.C., October 1993, pp.222-227.

Appendix A: Derivation of Sensitivity

Define

$$H(x,\lambda,p) = \begin{pmatrix} F(x,\lambda,p) \\ E(x,\lambda,p) \end{pmatrix}$$

$H(x_*, \lambda_0 + kt_*, p) = 0$. Assume that H is smooth and assume the generic transversality condition that

$$\left(H_x \;\; H_\lambda k \right)\Big|_* \quad \text{has rank } n+1. \tag{6}$$

Then the implicit function theorem implies that there are smooth functions $X(p)$, $T(p)$ defined near p_* with $X(p_*) = x_*$ and $T(p_*) = t_*$ such that

$$H(X(p), \lambda_0 + kT(p), p) = 0 \tag{7}$$

Differentiating (7) yields

$$\left(H_x \;\; H_\lambda k \right)\Big|_* \begin{pmatrix} X_p \\ T_p \end{pmatrix} = -H_p|_* \tag{8}$$

There is a nonzero row vector w such that $wH_x|_* = 0$. w is unique up to a scalar multiple when $H_x|_*$ has full rank, which is implied by condition (6). Pre-multiplying (8) by w yields

$$wH_\lambda k|_* \, T_p = -wH_p|_* \tag{9}$$

Condition (6) implies that $wH_\lambda k|_*$ is not zero and hence (9) can be solved to obtain (3). The geometric interpretation of the quantities in (3) is that $(wH_\lambda k, -wH_p)$ is the normal vector to the hypersurface in (t,p) space corresponding to the binding limit. In an optimization formulation ([14, chap. 7],[15]) which maximizes the transfer subject to the power system limits, these normal vectors become Lagrange multipliers.

The sensitivity X_p of the states at the binding limit is often useful and this can be obtained by solving (8). For example, $X_p \Delta p$ can be used to screen for cases where new limits would be violated (e.g., $X_p \Delta p[i] \geq x_i^{\max}$) [6].

Appendix B: DC Load Flow Example

We show how the general formula (3) applies in a simple DC load flow example with 6 buses. The slack bus is numbered 0. For the non-slack buses, write $\theta = (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5)^T$ for the angles and $\lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)^T$ for the power injections. The DC load flow equations are $F(\theta, \lambda) = X\lambda - \theta$. The transfer is from bus 3 to bus 4 so that $k = (0,0,1,-1,0)^T$. The limit on the transfer is overload on line 1–2 so that the limit equation is $E(\theta,\lambda) = b_{12}(\theta_1 - \theta_2) - \lambda_{12\max}$. The parameter is λ_5^0, the base case power injection at bus 5. $F_\theta = -I$ and $E_\theta = (b_{12}, -b_{12}, 0, 0, 0)$ and hence $w = (b_{12}, -b_{12}, 0, 0, 0, 1)$. $F_\lambda = X$ and $E_\lambda = 0$. $F_{\lambda_5^0} = X\,(0,0,0,0,1)^T$ and $E_{\lambda_5^0} = 0$. The transfer margin T is the increase in transfer from bus 3 to bus 4 which causes the flow limit on line 1–2. Substitution in (3) gives the sensitivity of T with respect to injection at bus 5:

$$T_{\lambda_5^0} = \frac{X_{15} - X_{25}}{X_{13} - X_{23} - X_{14} + X_{24}} = \frac{\rho_{12,5}}{\rho_{12,3} - \rho_{12,4}}$$

where $\rho_{12,m} = b_{12}(X_{1m} - X_{2m})$ is the well known sensitivity of the flow on line 1–2 with respect to power injection at bus m.

Scott Greene (M) received the PhD in Electrical Engineering from the University of Wisconsin-Madison in 1998 and is now a senior engineer with L.R. Christensen Associates, Madison WI. He is a registered professional engineer in Wisconsin.

Ian Dobson (SM) received the PhD in electrical engineering from Cornell University in 1989 and is now professor of electrical engineering at the University of Wisconsin-Madison. His interests are applications of bifurcations and nonlinear dynamics, electric power system stability and power electronics.

Fernando Alvarado (F) obtained the PhD from the University of Michigan in 1972 and is now professor at the University of Wisconsin-Madison in the department of electrical and computer engineering. His main areas of interest are computer applications to power systems and large scale problems.