

# Exotic Electricity Options and the Valuation of Electricity Generation and Transmission Assets

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## Abstract

This paper presents and applies a methodology for valuing electricity derivatives by constructing replicating portfolios from electricity futures and the risk free asset. Futures based replication is argued to be made necessary by the non-storable nature of electricity, which rules out the traditional spot market, storage-based method of valuing commodity derivatives. Using the futures based approach, valuation formulae are derived for both spark and locational spread options for both geometric Brownian motion and mean reverting price processes. These valuation results are in turn used to construct real options based valuation formulae for generation and transmission assets. Finally, the valuation formula derived for generation assets is used to value a sample of

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assets that have been recently sold, and the theoretical values calculated are compared to the observed sales prices of the assets.

# 1 Introduction

With deregulation sweeping through the US electric power industry and a fully competitive marketplace for electricity taking shape, electric utilities and their customers accustomed to a cost-recovery pricing structure for electricity must adapt to market based pricing. The risk management needs this transition will generate are likely to make electricity derivatives one of the fastest growing derivatives markets in the years to come, as financial institutions and other energy market participants work to provide the tools necessary to manage the price and investment risks associated with competitive markets. While many of the risk management tools and methods now well established in other markets can be readily transferred to the electricity markets, the unique characteristics of electricity and electricity markets also present new challenges to the risk management discipline. The most important of these are the challenges that the non-storable nature of electricity presents to the traditional methods of modeling price processes and valuing derivatives. Specifically, due to the non-storable nature of electricity, the traditional storage based, no-arbitrage methods of valuing commodity derivatives are unavailable. As a consequence, an entirely new methodology is required to value even the simplest electricity derivatives, such as electricity futures contracts. Also as a consequence of non-storability, electricity prices can (and do) demonstrate properties such as strong mean reversion over short time horizons that would be inconsistent with an efficient market for a storable good. A second risk management challenge that electricity markets present is the need to value a range of cross commodity transactions, such as spark and locational spreads.

In this paper we present tools for addressing both of these unique properties of electricity and electricity derivatives. We first present a method for valuing electricity derivatives by replicating them with futures contracts rather than by attempting to store or borrow electricity in the spot market. The method thus allows traditional no-

arbitrage based methods of derivatives valuation to proceed without the implausible assumption that electricity can be stored. We then present closed form expressions for the value of a range of cross-commodity derivatives, including spark and locational spread options. The valuation formulae are provided both for the case in which the underlying price processes are geometric Brownian motion processes, and for the more plausible case in which they are mean reverting. Finally, we demonstrate how these results can be used to value both generation and transmission assets, and present a preliminary comparison between the values these models generate and the actual prices at which assets of this kind have been recently sold.

The remainder of the paper is organized as follows. In section 2 we introduce the set of cross commodity derivatives we will consider in the paper, and identify some of their basic characteristics. In section 3 we describe how these derivatives can be replicated (and thus valued by arbitrage) using futures contracts, and present the principal valuation results of the paper. In section 4 we use these results to develop a real options based methodology for valuing generation and transmission assets, and present the results of our preliminary empirical evaluation of the effectiveness of the methodology.

## **2 Cross Commodity Electricity Derivatives**

There are two principal categories of cross-commodity electricity derivatives; spark spread, or heat rate linked derivatives, and locational spread derivatives. We consider each below.

## 2.1 Spark Spread, or Heat Rate Linked Derivatives

The primary cross-commodity transaction in electricity markets is the spark spread, which is based on the difference between the price of electricity and the price of a particular fuel used to generate it. The spread between the price of electricity and a fuel that can be used to generate it is of interest since it is this spread that determines the economic value of generation assets that can be used to transform the fuel into electricity<sup>1</sup>. The amount of fuel that a particular generation asset requires to generate a given amount of electricity will of course depend on the asset's efficiency. This efficiency is summarized by the asset's **heat rate**, which is defined as the number of British thermal units (Btus) of the input fuel (measured in millions) required to generate one megawatt hour (MWh) of electricity. Thus the lower the heat rate, the more efficient the facility. The **spark spread** associated with a particular heat rate is defined as the current price of electricity less the product of the heat rate and the current fuel price. Thus the lower the heat rate, the lower the fuel price, and the higher the electricity price, the larger the spark spread.

In a deregulated market, presumably only assets that have a positive spark spreads under prevailing market conditions will be operated. This leads naturally to the definition of the prevailing **market implied heat rate H** as:

$$\begin{aligned} H &= \text{number of } 10^{-3}MMBtu \text{ needed for a marginal} \\ &\quad \text{generating plant to generate one } MWh \text{ of electricity} \quad (2.1) \\ &= \frac{S_E}{S_G} \times 10^{-3}MMBtu/MWh \end{aligned}$$

where  $S_E$  is the spot price of electricity per Megawatt hour (MWh) and  $S_G$  is the spot price of the generating fuel per  $MMBtu$ .

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<sup>1</sup>This idea is analogous to the concept of the "crack spread" used in the oil/refining industry. In that example, we look at the spread between crude oil and refined products like diesel or gasoline (see [8]).

With the notion of a heat rate established, we define European spark spread based put and call options.

**Definition 1** (Spark Spread): *An European spark spread call option* written on fuel  $G$  at a fixed (or “strike”) heat rate  $K_H$  gives the option holder the right but not the obligation to pay  $K_H$  times the unit price of fuel  $G$  at the option’s maturity  $T$  and receive the price of one unit of electricity. Let  $S_E^T$  and  $S_G^T$  be the unit spot prices of electricity and fuel at time  $T$ , respectively. Denote the value of the option at time  $t$  by  $C_1(S_E^t, S_G^t, t)$ . Then the payoff of the option at maturity time  $T$  is:

$$C_1(S_E^T, S_G^T, T) = \max(S_E^T - K_H S_G^T, 0)$$

**Definition 2** *An European spark spread put option* written on fuel  $G$  at a fixed heat rate  $K_H$  gives the option holder the right but not the obligation to pay the price of one unit of electricity and receive  $K_H$  times the unit price of fuel  $G$  at maturity time  $T$ . Denote the value of the option at time  $t$  by  $P_1(S_E^t, S_G^t, t)$ . Then the payoff of the option at time  $T$  is:

$$P_1(S_E^T, S_G^T, T) = \max(K_H S_G^T - S_E^T, 0)$$

The following example provides a simple illustration of how spark spread options can be used to manage electricity price risk.

- A power marketer in a region where the marginal generating fuel is natural gas would like to buy power at time  $T$  at a market implied heat rate not to exceed  $K_H$ . An agreement providing such a heat rate cap would ensure the marketer power at time  $T$  at a price given by  $S_G \times \min(H, K_H)$ . Assuming the marketer sells the power into the spot market at time  $T$ , his payoff will be  $(S_E^T - S_G^T \times \min(H^T, K_H))$ , which is equal to  $S_G^T \times \max(H^T - K_H, 0)$ . Bringing

$S_G^T$  inside the brackets, the payoff is  $\max(S_E^T - K_H S_G^T, 0)$ , which is exactly the same as that of a European spark spread call option with strike heat rate  $K_H$ . The power marketer can therefore achieve his goal by purchasing this spark spread call.

Throughout the remainder of the article, we make the following assumptions:

**Assumption 1** A complete set of futures contracts for electricity and for the relevant generating fuels are traded.

**Assumption 2** The risk-free interest rate  $r$  is constant.

Next we provide a put-call parity relationship between the spark spread put and call options, as well as upper and lower bounds on their values. We delay making specific assumptions about the price processes that electricity and the generating fuels follow until section 3.

**Proposition 1** (*Put-Call Parity*) Let  $F_E^t$  and  $F_G^t$  denote the futures prices of electricity and generating fuel, respectively. The following parity relationship holds for an European spark spread put and call options with the same fixed heat rate  $K_H$  and expiration date  $t$ .

$$C_1 = P_1 + e^{-rt}(F_E^t - K_H F_G^t) \quad (2.2)$$

**Proof.** At time  $t$ , the payoff of a long position in one unit of spark spread call option  $C_1$  is  $\max(S_E^t - K_H S_G^t, 0)$ ; the payoff of a short position in one unit of spark spread put option  $P_1$  is  $\max(K_H S_G^t - S_E^t, 0)$ . Consequently, the payoff of  $(C_1 - P_1)$  at maturity time  $t$  is  $(K_H S_G^t - S_E^t)$ . The present value of  $(K_H S_G^t - S_E^t)$  is  $e^{-rt}(F_E^t - K_H F_G^t)$ . Therefore,

$$C_1 - P_1 = e^{-rt}(F_E^t - K_H F_G^t)$$

■

**Proposition 2** (*No-arbitrage lower/upper bounds*) Let  $F_E^t$  and  $F_G^t$  denote the futures prices of electricity and a generating fuel, respectively. Then the value of a spark spread call option,  $C_1$ , has both a lower bound and an upper bound, i.e.

$$e^{-rt} \max(F_E^t - K_H F_G^t, 0) \leq C_1 \leq e^{-rt} F_E^t \quad (2.3)$$

**Proof.** The first inequality is by the Put-Call parity (2.2) and the fact  $P_1 \geq 0$ . The second inequality is due to the fact  $\max(S_E^t - K_H S_G^t, 0) \leq S_E^t$  at time  $t$  and the present values of  $\max(S_E^t - K_H S_G^t, 0)$  and  $S_E^t$  are  $C_1$  and  $e^{-rt} F_E^t$ , respectively. ■

## 2.2 Locational Spread Options

Due to transmission costs and constraints<sup>2</sup>, substantial differences frequently exist between the price of electricity at different locations. We refer to these differences as **locational spreads**, and define call options on them as follows:

**Definition 3** (*Locational Spread*): An European call option on the locational spread between location one and location two with maturity  $T$  gives its holder the right but not the obligation to pay the price of one unit of electricity at location one at time  $T$  and receive the price of one unit of electricity at location two. Let  $S_i^T$  be the unit price of electricity at location  $i$  ( $i = 1, 2$ ) at time  $T$ . Denote the value of the option at time  $t$  by  $C_2(S_1^t, S_2^t, t)$ . Then the payoff of the option at time  $T$  is:

$$C_2(S_1^T, S_2^T, T) = \max(S_1^T - S_2^T, 0)$$

A European locational spread put option can be defined in a similar way.

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<sup>2</sup>Note that the impact of transmission constraints is compounded by the non-storability of electricity, which forces real time delivery.

### **3 Valuation of Electricity Derivatives**

In this section we present a futures based method of replicating electricity derivatives, and illustrate the method by using it to derive explicit expressions for the value of the spark spread and locational spread options defined above. Valuation equations are provided for these instruments for both geometric Brownian motion price processes and mean-reverting price processes. In both cases we explicitly derive only the value of the call options. The value of put options can then be derived using the put-call parity relationship presented in section 2.

#### **3.1 Futures Based Replication of Electricity Derivatives**

As noted above, because electricity is non-storable, the traditional storage-based methods of constructing replicating portfolios for commodity derivatives cannot be used to value electricity derivatives. In place of the storage based methods, we present a method for replicating electricity derivatives by dynamically trading futures contracts of the appropriate maturity. Since at maturity the price of a futures contract must converge to the then current spot price, the methodology permits exact replication. Since the precise nature of the replicating strategy will naturally depend on the specific derivative being replicated, to illustrate the method we use it to derive the replicating strategy for spark and locational spread options. We do so first under the assumption that the relevant price processes follow geometric Brownian motion processes, and then under the more reasonable assumption that they follow mean reverting processes.

## 3.2 Geometric Brownian motion Price Processes

We first consider case in which the futures price processes of electricity and the appropriate generating fuel,  $F_e$  and  $F_g$ , follow geometric Brownian motion processes

$$\begin{aligned} dF_e/F_e &= \mu_e dt + \sigma_e dB^1 \\ dF_g/F_g &= \mu_g dt + \sigma_g dB^2 \end{aligned} \quad (3.1)$$

where  $B^1$  and  $B^2$  are two Wiener processes with instantaneous correlation  $\rho$ .  $\mu_e$ ,  $\mu_g$ ,  $\sigma_e$ , and  $\sigma_g$  are assumed to be constants for the moment. The more general case where they can be functions of time is considered in the mean-reverting model.

### 3.2.1 Valuation of Spark Spread Options

Denote the time- $t$  value of a spark spread call option which matures at time  $T$  by  $V(x, y, t) \equiv C_1(F_e^{t,T}, F_g^{t,T}, T - t)$ .  $F_*^{t,T}$  is the commodity futures price at time  $t$  with maturity date  $T$ . By constructing an instantaneous risk free portfolio using the electricity and generating fuel futures contracts and the riskless asset, it follows that  $C_1$  must satisfy the partial differential equation (PDE):

$$-V_t + \frac{1}{2}[x^2 V_{xx} \sigma_x^2 + 2\rho \sigma_x \sigma_y xy V_{xy} + y^2 V_{yy} \sigma_y^2] = 0 \quad (3.2)$$

with boundary conditions  $V(x, y, 0) = \max(x - y, 0)$ ,  $V(x, 0, t) = x$ , and  $V(0, y, t) = 0$ .

**Proposition 3** (*Value of a spark spread call option*) *The closed form solution for  $C_1$  is:*

$$C_1(F_e^{t,T}, F_g^{t,T}, T - t) = e^{-r(T-t)} [F_e^{t,T} N(d_1) - K_H F_g^{t,T} N(d_2)] \quad (3.3)$$

where

$$\begin{aligned} d_1 &= \frac{\ln(F_e^{t,T}/(K_H F_g^{t,T})) + v^2(T-t)/2}{v\sqrt{T-t}} \\ d_2 &= d_1 - v\sqrt{T-t} \\ v^2 &= \sigma_e^2 - 2\rho\sigma_e\sigma_g + \sigma_g^2 \end{aligned}$$

**Proof.** Verify  $V = C_1(F_e^{t,T}, F_g^{t,T}, T - t)$  solves PDE (3.2) with the corresponding boundary conditions. ■

### 3.2.2 Valuation of Locational Spread Options

The value of the locational spread call option can be derived in exactly the same way the value of the spark spread call option was derived above. Specifically, defining  $F_{e,1}$  and  $F_{e,2}$  to be the geometric Brownian motion price processes that the futures prices of electricity at locations 1 and 2 follow, we have:

**Proposition 4** (*Value of a locational spread call option*) The value of  $C_2$  is given by

$$C_2(F_{e,1}^{t,T}, F_{e,2}^{t,T}, T - t) = e^{-r(T-t)} [F_{e,1}^{t,T} N(d_1) - F_{e,2}^{t,T} N(d_2)] \quad (3.4)$$

where

$$\begin{aligned} d_1 &= \frac{\ln(F_{e,1}^{t,T}/F_{e,2}^{t,T}) + v^2(T-t)/2}{v\sqrt{T-t}} \\ d_2 &= d_1 - v\sqrt{T-t} \\ v^2 &= \sigma_{e,1}^2 - 2\rho\sigma_{e,1}\sigma_{e,2} + \sigma_{e,2}^2 \end{aligned}$$

### 3.3 Mean-reverting price processes

In this section we assume that the futures price processes of electricity  $F_e$  and of the relevant generating fuel  $F_g$  follow the mean-reverting processes

$$\begin{aligned} dF_e &= \kappa_e(\mu_e - \ln F_e)F_e dt + \sigma_e(t)F_e dB^1 \\ dF_g &= \kappa_g(\mu_g - \ln F_g)F_g dt + \sigma_g(t)F_g dB^2 \end{aligned} \quad (3.5)$$

where  $\sigma_e(t)$  and  $\sigma_g(t)$  are functions of time  $t$ ,  $\mu_e$  and  $\mu_g$  are the long-term means,  $\kappa_e$  and  $\kappa_g$  are the mean-reverting coefficients, and  $B^1$  and  $B^2$  are, as above, two Wiener processes with instantaneous correlation  $\rho$ .

### 3.3.1 Valuation of Spark Spread options

Denote the time- $t$  value of a spark spread call option which matures at time  $T$  by  $V(x, y, t) \equiv C_1(F_e^{t,T}, F_g^{t,T}, T - t)$ . Applying the same replication arguments applied above, it follows that  $C_1$  must satisfy the partial differential equation:

$$-V_t + \frac{1}{2}[x^2 V_{xx} \sigma_x^2(t) + 2\rho \sigma_x(t) \sigma_y(t) xy V_{xy} + y^2 V_{yy} \sigma_y^2(t)] = 0 \quad (3.6)$$

with boundary conditions  $V(x, y, 0) = \max(x - y, 0)$ ,  $V(x, 0, t) = x$ , and  $V(0, y, t) = 0$ .

**Proposition 5** (*Value of a spark spread call option*) *The close-form solution for  $C_1$  is:*

$$C_1(F_e^{t,T}, F_g^{t,T}, T - t) = e^{-r(T-t)} [F_e^{t,T} N(d_1) - K_H F_g^{t,T} N(d_2)] \quad (3.7)$$

where

$$\begin{aligned} d_1 &= \frac{\ln(F_e^{t,T} / (K_H F_g^{t,T})) + v^2(T - t)/2}{v\sqrt{T - t}} \\ d_2 &= d_1 - v\sqrt{T - t} \\ v^2 &= \frac{\int_t^T [\sigma_e^2(s) - 2\rho\sigma_e(s)\sigma_g(s) + \sigma_g^2(s)] ds}{T - t} \end{aligned}$$

**Proof.** Verify  $V = C_1(F_e^{t,T}, F_g^{t,T}, T - t)$  solves PDE (3.6) with the corresponding boundary conditions. ■

Several comparative static properties of the spark spread call option value can be derived by investigating the sign of the partial derivatives of  $C_1$  with respect to its parameters.

### Proposition 6

$$\begin{aligned} \text{As } F_e^{t,T} \nearrow (\text{increases}), \text{ or } F_g^{t,T} \searrow (\text{decreases}) &\Rightarrow C_1 \nearrow (\text{increases}) \\ F_g^{t,T} \nearrow \text{ and } \frac{F_e^{t,T}}{F_g^{t,T}} \nearrow &\Rightarrow C_1 \nearrow \\ \rho \searrow \text{ or } r \searrow &\Rightarrow C_1 \nearrow \end{aligned}$$

### 3.3.2 Valuation of locational spread options

Defining  $F_{e,1}$  and  $F_{e,2}$  to be the mean-reverting price processes that govern the futures prices of electricity at locations 1 and 2 and following the derivation above, we have:

**Proposition 7** (*Value of a locational spread option*) *The value of  $C_2$  is given by*

$$C_2(F_{e,1}^{t,T}, F_{e,2}^{t,T}, T - t) = e^{-r(T-t)} [F_{e,1}^{t,T} N(d_1) - F_{e,2}^{t,T} N(d_2)] \quad (3.8)$$

where

$$\begin{aligned} d_1 &= \frac{\ln(F_{e,1}^{t,T} / F_{e,2}^{t,T}) + v^2(T - t)/2}{v\sqrt{T - t}} \\ d_2 &= d_1 - v\sqrt{T - t} \\ v^2 &= \frac{\int_t^T [\sigma_{e,1}^2(s) - 2\rho\sigma_{e,1}(s)\sigma_{e,2}(s) + \sigma_{e,2}^2(s)] ds}{T - t} \end{aligned}$$

## 4 Real Options Valuation of Generation and Transmission Assets

The right to operate a generation asset with heat rate  $H$  that uses generating fuel  $g$  is clearly given by the value of a spark spread option with “strike” heat rate  $H$  written on generating fuel  $g$ . Similarly, the value of a transmission asset that connects location 1 to location 2 is equal to the sum of the value of the locational spread option to buy electricity at location 1 and sell it at location 2 and the value of the option to buy electricity at location 2 and sell it a location 1 (in both cases, less the appropriate transmission cost). This equivalence between the value of appropriately defined spark and locational spread options and the right to operate a generation or a transmission asset can be easily used to value such assets. In this section we illustrate this approach by developing a simple spark spread based model of the value of a gas-fired generation asset. Once established, we fit the model and use it to generate estimates of the value

of several gas-fired plants that have recently been sold. The accuracy of the model is then evaluated by comparing the estimates constructed to the prices at which the assets were actually sold.

In the analysis we make the following simplifying assumptions about the operating characteristics of the generation assets under consideration:

**Assumption 3** Ramp-ups and ramp-downs of the facility can be done with very little advance notice.

**Assumption 4** The facility's operation (e.g. start-up/shutdown costs) and maintenance costs are constant.

These assumptions are reasonable, since for a typical gas turbine combined cycle cogeneration plant the response time (ramp up/down) is several hours and the variable costs (e.g. operation and maintenance) are generally stable over time.

To construct a spark spread based estimate of the value of a generation asset, we estimate the value of the right to operate the asset over its remaining useful life. This value can be found by integrating the value of the spark spread option to operate the facility over its remaining useful life. Specifically,

**Definition 4** *Let one unit of the **time- $t$  capacity right** of a natural gas fired electric power plant represent the right to convert  $K_H$  units of natural gas into one unit of electricity by using the plant at time  $t$ , where  $K_H$  is the plant's heat rate.*

The payoff of one unit of time- $t$  capacity right is  $\max(S_E^t - K_H S_G^t, 0)$ , where  $S_E^t$  and  $S_G^t$  are the spot prices of electricity and natural gas at time  $t$ , respectively. Denote the value of one unit of the time- $t$  capacity right by  $u(t)$ .

**Definition 5** Denote the **virtual value** of one unit of capacity of a gas fired power plant by  $V$ . Then  $V$  is equal to one unit of the plant's time- $t$  capacity right over the remaining life  $[0, T]$  of the power plant, i.e.  $V = \int_0^T u(t)dt$ .

Without making any distributional assumptions about the price processes that  $S_E^t$  and  $S_G^t$  follow, we have the following proposition:

**Proposition 8** The value of one unit of capacity  $V$  of a plant that has a useful life of  $T$  has both a lower bound and an upper bound:

$$\int_0^T e^{-rt} \max(F_e^t - K_H F_g^t, 0)dt \leq V \leq \int_0^T e^{-rt} F_e^t dt$$

**Proof.** By definition and **Proposition 2**. ■

If we further assume that the price processes of electricity and natural gas spot and futures prices follow the mean-reverting processes as given by (3.5), then we have  $u(t) = C_1(t)$  where  $C_1(t)$  is given by **Proposition 5**. The value of a gas-fired power plant with life time  $T$  is therefore

$$V_{gen} = \int_0^T C_1(t)dt \tag{4.1}$$

Similarly, if we assume that the price processes of the electricity futures prices at two different locations follow mean-reverting processes, the value of a transmission line connecting the two locations in a radial network is

$$V_{tran} = \int_0^T C_2(t)dt \tag{4.2}$$

Equations (4.1) and (4.2) are two fundamental valuation formulae we propose for the valuation of generation and transmission assets in a competitive electricity market.

## 4.1 Application of the Model to Recent Generation Asset Sales

To evaluate the accuracy of (4.1), we fit the model and use it to construct estimates of the value of several generation assets that have been recently sold. For purposes of comparison, we also estimate the value of each asset using a standard discounted cash flow calculation.

In order to fit the model, we first estimate the volatilities of the price processes of the relevant futures contracts. Let  $f_n^t$  be the price of the futures contract that matures in  $n$  months, and assume that  $f_n^t$  follows a mean-reverting process of the kind considered above. Let  $R_n^t \equiv \ln f_n^t$ , then

$$dR_n = \kappa_1(\hat{\mu}_n - R_n)dt + \sigma_n dB$$

We estimate  $\sigma_n$  using the New York Mercantile Exchange (NYMEX) electricity futures historical price data. The natural gas volatility term structure and the gas-to-electricity price correlation are also estimated using historical data on the NYMEX natural gas (Henry Hub) futures contracts. Once estimated, these parameters are inserted in the valuation formulae derived in the previous section to construct real options based estimates of the value of the assets in question. To calculate the discounted cash flow value of the assets we use the expected future electricity and natural gas prices implied by the growth-adjusted means of the relevant forward curves.

The sample of generation assets considered consists of four gas-fired power plants which Southern California Edison recently sold to Houston Industries. At present, not all of the individual plant dollar investments have been made public. As a proxy we use the total investment made by Houston Industries (\$237 million to purchase four plants - Coolwater, Ellwood, Etiwanda and Mandalay), divided by the total number of megawatts (MW) of capacity (2172MW) to get approximately \$110,000

## Electricity and Natural Gas Forward Curves

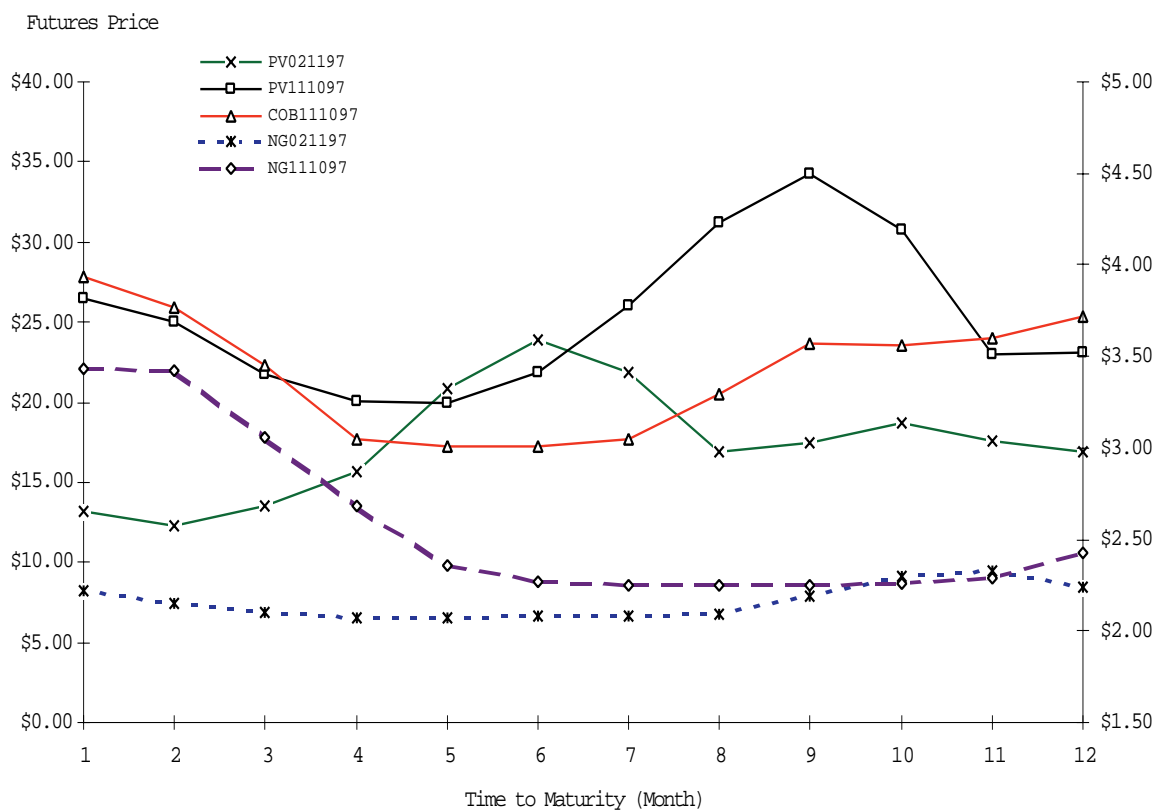


Figure 1: Electricity and Natural Gas Futures Price Curve

per MW (or \$110/kW) of capacity for the entire package of plants. However, the Coolwater Plant<sup>3</sup>, in Daggett, California, is the most efficient (with an average heat rate of 9,500) of the four plants in the package and thus should have a higher value per MW. We therefore assume that the implied market value for Coolwater could

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<sup>3</sup>The Coolwater Plant is made up of four units. Two 256MW Combined Cycle Gas Turbines plus a steam turbine; and two conventional turbines with capacity 65MW and 81MW each. Some re-power work has been done on the larger units.

range from \$110,000 to \$220,000 per MW, or equivalently, \$110/kW to \$220/kW.

Using the NYMEX electricity and natural gas futures price data on 02/11/97 and 11/10/97, (see Figure 1), we compute both the option value and DCF value (using a risk-adjusted discount rate of 10%) of a gas-fired plant with various possible heat rates assuming a remaining operating life of fifteen years for the plant and an availability factor of 90% for peak hours. Figure 2 shows the plot of the option values and the DCF values of a plant of various possible heat rates using forward curves at different times and at different trading hubs.

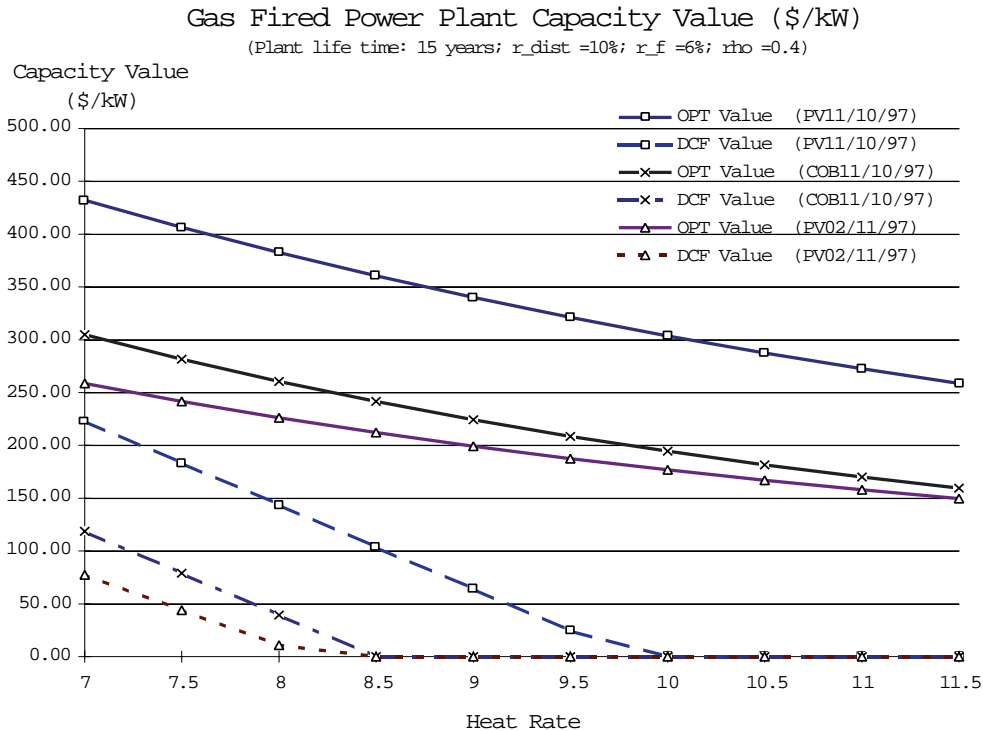


Figure 2: Capacity Value of a Gas-Fired Plant

We can see that the option values of capacity are significantly higher than the DCF values. For heat rate higher than 8500 Btu/kWh, the DCF values of capacity

are close to zero. At the heat rate level of 9500 and using the electricity forward curves at Palo Verde, the theoretical option-based capacity value of a plant comparable to the Coolwater Plant<sup>4</sup> ranges from \$188/kW to \$321/kW.

## 5 Conclusions

This article has presented a methodology for valuing electricity derivatives by constructing replicating portfolios with futures contracts and the risk free asset. Futures based replication is made necessary by the non-storable nature of electricity, which rules out the traditional spot market, storage-based method of valuing commodity derivatives. Once developed, the methodology was used to derive valuation formulae for both spark and locational spread options when the prices of the underlying assets follow either geometric Brownian motion or mean reverting processes. These valuation results were in turn used to construct real options based valuation formulae for generation and transmission assets. Application of the generation asset valuation formula to a sample of recent asset sales suggests that the spark spread analysis generates reasonable estimates of the actual market value of the assets, and certainly much more accurate estimates than those which traditional discounted cash flow methods provide. In addition, the estimates generated could almost certainly be improved by incorporating a greater level detail about the plants and their sites in the analysis. Doing so presents a nature avenue for future research.

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<sup>4</sup>The natural trading hub for Coolwater to sell into is the Mead hub. However, due the liquidity of the Palo Verde financial futures contracts we use the PV futures contracts as a proxy for the electricity price information for Coolwater. In addition, the basis differential associated with PV/Mead is typically not large.

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