Designing Cost Effective Demand Management Contracts using Game Theory

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Abstract

Demand relief from customers can help a utility solve a variety of problems. There exist all sorts of different demand management programs that utilities use. A critical issue is the incentive paid to the customer to participate in demand management programs and provide load relief. The utility has to design cost effective yet attractive demand management contracts. The main goal is to get load relief when needed. If the contracts are designed to be cost effective they can help the utility reduce costs. Customers sign up for programs when the benefits they derive in the form of up front payments and interruption payments exceeds their cost of interruption. In order to design such contracts, mechanism design with revelation principle is adopted from Game Theory and applied to the interaction between a utility and its customers. The idea behind mechanism design is to design a program incentive structure that encourages customers to reveal the value of power (and thus, the value of power interruptibility) without the need to explicitly have customers declare the value. This economic analysis is combined with power system sensitivity analysis to help determine the value of interruptibility for each system location.

Keywords: Demand management, mechanism design, load interruption, load curtailment, system security.

Introduction

The increased penetration of backup generation [2] and energy management systems opens the door for more creative means for integrating demand management into utility operations. By explicitly examining customer outage costs [4, 2] and analyzing their load behavior it is possible for utilities to design different kinds of demand management programs and attract customers to help in case of emergencies in return for an incentive fee [7]. Because a utility can only estimate the outage costs to a customer, it is difficult for a utility to know how much incentive to offer in order to attract customers to curtail or interrupt their load. The main theme of this report is to design cost effective demand management programs that do not require the knowledge of customer outage costs, but rather use Game Theory [8] to design optimal curtailment programs. The process of designing contracts that attain this objective is called *mechanism design with revelation principle*. The mechanism (or contract offered by the utility) makes sure that the utility benefit is maximized *and* that customers are compensated sufficiently to participate voluntarily. A new general formulation is developed and illustrated by means of an example. The report also combines the economic aspects of contracts with power system sensitivity analysis. Sensitivity methods attribute value to power interruptibility at every location in the grid. Thus, contracts can be customized by location.

Chapter 1

Mechanism Design

Mechanism design and the revelation principle are key concepts from nonlinear pricing. They are explained in detail in, among other places, [8] and [13]. Mechanism design is a powerful tool that helps a principal (in this case, the utility), with no private information about its customers, decide in an optimal way how much to buy from (or sell to) its customers and at what price. The revelation principle [9, 5, 14] is used to simplify the problem. The mechanism (or contract offer structure) can be designed in a way that customers wishing to maximize their own total benefit are forced to reveal their true valuation of power interruptions.

Ordinary markets work by posting prices per unit of a commodity. In contrast, mechanism design leads to the specification of both price and quantity. There is no requirement that the price per unit be the same for all offers. In fact, it is efficient not to make the the same. Thus, a mechanism has two kinds of output: a decision vector (amounts to buy or sell) and a vector of monetary transfers from the principal to each customer.

In this report, the theory of mechanism design is applied to the interaction between a utility and its customers. In order to better understand issues of mechanism design, it is desirable to first understand issues in nonlinear pricing.

Chapter 2

Nonlinear pricing

Consider that a customer values the use of electricity according to a declining marginal benefit as a function of amount of energy consumed (denoted by q). Let the marginal benefit be described by:

$$b(q) = \theta(b_0 - sq) \tag{2.1}$$

where θ is a parameter that depends on the customer. Figure 2.1 illustrates the marginal benefit function for $b_0 = 1$, $s = 1$ and two values of θ.

The total benefit B is the integral of this marginal benefit. For the type of benefit function assumed above, the following is the total benefit:

$$B(\theta, q) = \theta b_0 q - \frac{1}{2} s\theta q^2 \tag{2.2}$$

Figure 2.2 illustrates the total benefit curves for each of the two customer types.

The cost per hour of producing electricity under a specific set of conditions is c. Assume that the utility elects to consider only two types of customers it wishes to sell to, a small customer to which it wishes to sell quantity $\underline{q}$ and a large customer to which it wishes to sell a quantity $\bar{q}$, with $\underline{q}$ and $\bar{q}$ yet to be determined ($\underline{q} < \bar{q}$). The cost to produce $\underline{q}$ is $c\underline{q}$. Likewise, the cost to produce $\bar{q}$ is $c\bar{q}$. The straight line defining these (and any other) production costs is also illustrated in Figure 2.2. The utility wishing to sell at a profit selects price/quantity points that lie at or above this line. Let $\underline{C}$ be the price chosen for quantity $\underline{q}$ and $\bar{C}$ be the selected sale price for quantity $\bar{q}$ (shown as C_1 and C_2 in Figure 2.2). Clearly, a utility can hope to sell to the small customer only if $B(\underline{\theta}, \underline{q}) \geq \underline{C}$ and it can hope to sell to the large customer only if $B(\bar{\theta}, \bar{q}) \geq \bar{C}$. This is, in fact, the case in this figure. This condition is called the *rationality constraint*.

A more subtle constraint exists: if the utility were to always charge prices close to $B(\bar{\theta}, q)$, the small consumer would be unable to use power, since this would be

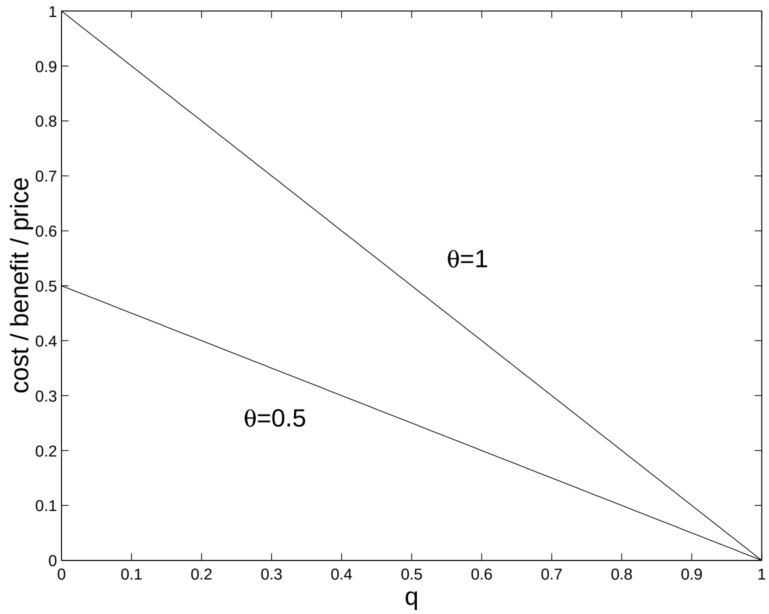

Figure 2.1: Marginal benefit for two customer types.

done at a loss. Assume that there is at least one price/quantity offering equal to or below curve $B(\underline{\theta}, q)$ (but above cq). This is, indeed, the case of the $(\underline{C}, \underline{q})$ offering. Now, if the large consumer were to choose a small amount of consumption $(\underline{q})$ its total benefit is the segment illustrated as A in the diagram. If, on the other hand, it were to consume the large amount $(\bar{q})$, its net benefit is illustrated by segment B. It seems reasonable to think that if the large customer can derive more benefit by consuming less (that is, if A>B), it is going to consume less. This is almost never desirable to the utility, as it results in highly suboptimal conditions in terms of utility benefit. Thus, we require that pricing be such that A≤B for the larger customer. This condition is called the *incentive compatibility* condition. Figure 2.2 illustrates a case that violates this condition, and thus encourages the customer to "lie." It can be shown mathematically that in order to optimize utility benefit the lower consumption/price point is determined by a binding rationality condition, and that the upper price be determined by a binding incentive compatibility condition [8]. In the case of more than two customers the incentive compatibility constraint is binding for everybody, but trivially for the customer with the highest cost.

If only the large customer existed, optimality would be attained when $\dfrac{dB(\bar{\theta}, q)}{dq} = c$. If only the small customer existed, it would be optimal to select the situation where $\dfrac{dB(\underline{\theta}, q)}{dq} = c$. It is the role of mechanism design to design pricing structures

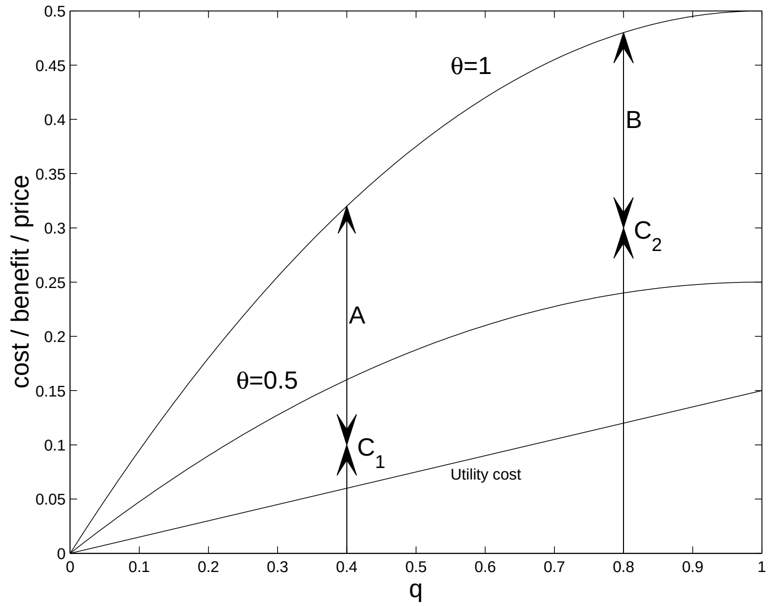

Figure 2.2: Total benefit, cost to producer and consumption levels for two customer types ($\theta = 0.5$ and $\theta = 1$). Contract C_1 is attractive to both customers and to the utility. Contract C_2 is only attractive to the $\theta = 1$ customer and the utility. Unfortunately, in the case depicted here contract C_1 is more attractive to customer $\theta = 2$ than contract C_2. This is suboptimal.

so that optimality is attained where there is a mix of customers, and when there is uncertainty about the mix.

Chapter 3

Contract Design

The cost of power curtailment to a customer depends on both the customer and the amount interrupted. We assume, at least initially, that the cost $c(\theta, x)$ to a customer of type θ of curtailing x MW is:

$$c(\theta, x) = K_1 x^2 + K_2(x - \theta x). \tag{3.1}$$

Here θ is a continuous variable describing the customer type. It can also be called the customer preference parameter. For the sake of simplicity, we assume that K_1 and K_2 are known to be 1/2 and 1 respectively. These assumptions do not affect the fundamental concepts to be considered here, since they amount to simple scaling. It is clear that customers of different types value interruptions differently. Although equation 3.1 gives and expression of the cost of an interruption to a customer, the parameter θ in this equation is not known to the utility.

Another assumption concerns the distribution of θ. Two such possibilities are:

- We can assume that the complete set of customer types can be characterized by allowing θ to vary from 0 to 1. Furthermore, we can assume that there is an equal probability that the customer will be of any of these types (that is, θ is a random variable with a uniform distribution in the interval $[0, 1]$).

- We can assume that θ can take discrete values, each with a presumed probability. Two discrete values of θ represent the simplest such scenario.

The probability distributions associated with these values of θ are subjective probabilities. The utility need not know which, if any, of the distributions is correct. The value of θ is private information of the customer and is unknown to the utility. Having a subjective estimate of the customer types it is dealing with, the utility develops an incentive function $y(x)$ to indicate how much it is willing to pay someone for a given amount of curtailment.

Customers self-select the amount of curtailment they wish to be subjected to, based on an inspection of the incentive function offered to them. They do so rationally, by making the amount of compensation they receive from participation match the monetary incentive offered by the utility minus the actual net loss of benefit that results from the curtailment (from equation 3.1). Clearly, customers will not choose to be curtailed unless they see a net positive benefit. A customer's benefit function is:

$$V_1(\theta, x, y) = y - \frac{1}{2}x^2 - x + \theta x. \tag{3.2}$$

In the absence of an initial sign-up incentive[1], in order for a customer to elect to participate in a program, it is necessary that $V_1 \geq 0$, that is they must see a benefit to the curtailment.

Although customer benefit functions can be quite arbitrary, only benefit functions that satisfy a so-called "single crossing property" lead to the results described in this report. The function in equation (3.2) satisfies this property.

Under stressed conditions it is expensive for the utility to deliver power to certain locations. The utility can compute the value of *not* delivering power to a certain customer. This value of "power interruptibility" is parameterized in λ. The value of λ can be computed using existing efficient optimal power flow routines [6, 12]. Knowing λ enables the utility to define their own benefit function for a curtailment at a specific location:

$$V_2(\theta, x, \lambda) = \lambda x(\theta) - y(\theta) \tag{3.3}$$

where λ is in dollars per MW not delivered to a customer. The objective of the utility is to maximize the utility benefit function.

$$\max_{x,y} \int_0^1 [\lambda x(\theta) - y(\theta)] f(\theta) d\theta \tag{3.4}$$

such that,

$$y(\theta) - \frac{1}{2}x^2(\theta) - x(\theta) + \theta x(\theta) \geq 0 \tag{3.5}$$

$$\begin{aligned} y(\theta) - \frac{1}{2}x^2(\theta) - x(\theta) + \theta x(\theta) \geq \\ y(\hat{\theta}) - \frac{1}{2}x^2(\hat{\theta}) - x(\hat{\theta}) + \theta x(\hat{\theta}) \end{aligned} \tag{3.6}$$

[1]A fixed one-time sign up incentive can be a part of the overall compensation, but it is not considered here. It would have the effect of modifying the perceived net customer benefit.

where $\hat{\theta}$ is the preference parameter of a customer if they were to lie about it. First constraint is the *individual rationality constraint* which makes sure every customer is tempted to participate, and the second constraint is the *incentive compatibility constraint* which forces the customers to tell the truth about their θ. This maximization problem could be solved by using the theory on mechanism design and revelation principle in [8]. The results are shown below:

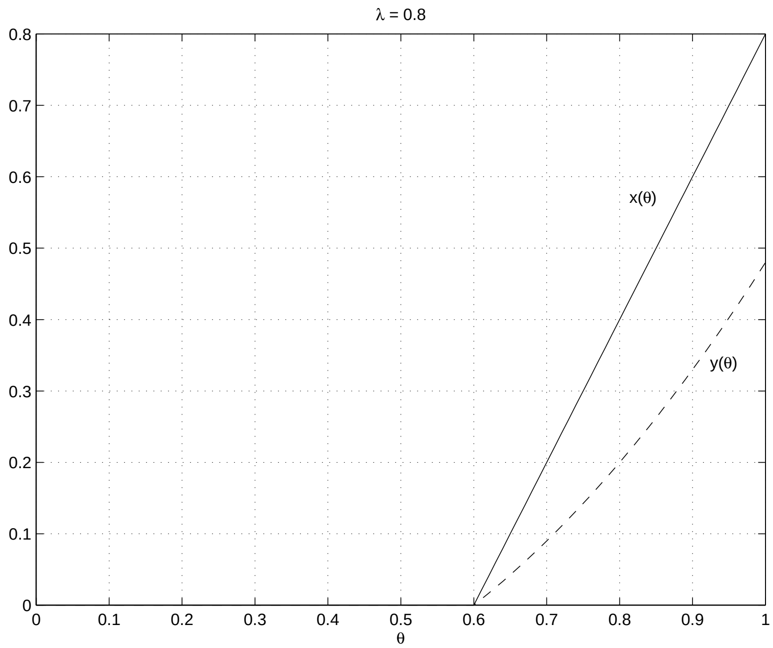

Figure 3.1: Monetary Incentive and amount curtailed as a function of θ.

$$x(\theta) = \begin{cases} 0 & \text{if } 0 \leq \theta < 1 - \frac{\lambda}{2} \\ 2\theta + \lambda - 2 & \text{if } 1 - \frac{\lambda}{2} \leq \theta \leq 1 \end{cases} \tag{3.7}$$

$$y(\theta) = \begin{cases} 0 & \text{if } 0 \leq \theta < 1 - \frac{\lambda}{2} \\ \theta^2 - 2\theta + 2\theta\lambda & \\ \quad + \frac{3}{4}\lambda^2 - 2\lambda + 1 & \text{if } 1 - \frac{\lambda}{2} \leq \theta \leq 1 \end{cases} \tag{3.8}$$

Equations 3.7 and 3.8 define the contracts to be offered to different types of customers. Figure 3.1 depicts the plots of equations 3.7 and 3.8 for a fixed value λ. As seen in Figure 3.1 contracts are only useful for the customers with type $\theta \geq 0.6$ when $\lambda = 0.8$. A family of incentive functions as λ varies is shown in Figure 3.2. Figure

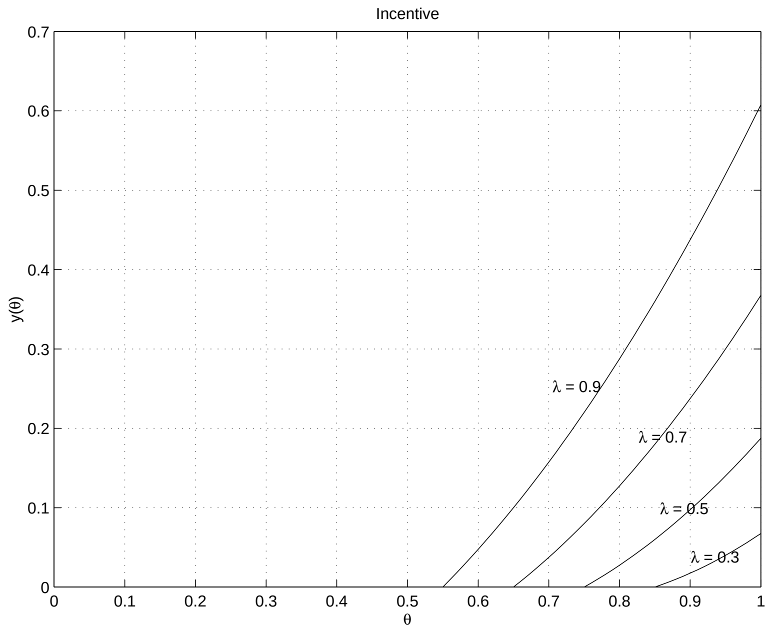

Figure 3.2: Normalized incentive function vs θ.

3.2 shows that the value of λ plays a key role in determining the incentive to be paid to each customer. The role of θ is obvious since it determines the type of customer (which in turn determines their cost of a curtailment or interruption), however the role of λ can be subtle. λ is the parameter that brings engineering into this economic analysis, it shows that location of the customer is one of the most important aspects in this analysis.. Some locations will be more costly to deliver power than others and it only makes sense that the utility will want to have curtailment contracts with it's customers who are at expensive locations.

Chapter 4

Sensitivity Analysis

Sensitivity analysis can be used to determine λ. In [10], the authors compute the sensitivity of the loading margin of a system with respect to arbitrary parameters. If loads are the parameters, then sensitivity of the loading margin can be computed with respect to each load. Let:

$$f(x, \lambda, p) = 0 \tag{4.1}$$

where x is the vector of state variables, λ is the vector of real and reactive load powers, and p is the vector of loads. If a pattern of load increase is specified with a unit vector k, the point of collapse method, [3] can be applied to yield the left eigenvector w. The sensitivity of the loading margin to a change in any load is:

$$\frac{\Delta L}{\Delta p} = L_p = \frac{-\omega f_p}{\omega f_\lambda k} \tag{4.2}$$

Once we have the sensitivity of the loading margin to a change in any load, we use it to rank loads. Let L be the loading margin of the system. The above formula lets us construct an expression relating changes in individual loads (Δp_1, Δp_2, etc) to changes in the security margin:

$$\Delta L = L_{p_1}\Delta p_1 + L_{p_2}\Delta p_2 + ... + L_{p_m}\Delta p_m \tag{4.3}$$

where m is the number of loads of interest. As equation 4.3 suggests, the load with the highest sensitivity would help increase the loading margin the most. By using these sensitivities and the dollar per kW figures from the designed contracts, the utility can estimate how much it would cost to increase the security of the system:

$$\frac{\Delta L}{\Delta \$} = \frac{\Delta L}{\Delta p}\frac{\Delta p}{\Delta \$} \tag{4.4}$$

where $\Delta\$$ is the amount the utility will spend.

Equation 4.4 helps determine how much it would cost to increase the loading margin by curtailing one of the loads signed up for a demand management contract. Other kinds of sensitivities could also be computed and combined with the economic analysis done in the previous section to give the utility a dollar figure in solving their problems.

Chapter 5

A Comprehensive Example

Before any demand management contracts are offered to customers, the utilities need to go through a planning stage. The first step is to analyze their electric power system and identify which load locations (customers) would be most helpful in case of emergencies or anticipated problems (voltage collapse, line overloads, insufficient generation, etc.). A sensitivity analysis needs to be performed on the system to determine the most valuable loads for each problem. This analysis involves load forecasting and consideration of multiple scenarios and time periods. The contracts will vary by location, class and type of customer. The following example will be developed in three main stages:

1. Sensitivity analysis to determine the most valuable loads to increase loading margin to voltage collapse.
2. Game Theory analysis to determine the optimal demand management contracts.
3. Comparison of different scenarios of demand management contracts.

An example that uses an 8 bus system (see fig. 5.1) with 2 generators and 6 loads is analyzed. The generator at bus 1 is designated to be the slack generator. Of concern is the loading margin to voltage collapse. If the load is increased equally on each load bus and only the slack generator picks up the extra load the sensitivity of the loading margin to voltage collapse with respect to a change in each load is shown in Table 5.1. In this example the most valuable loads are 7 and 8. They have the highest sensitivity. If the system gets close to a voltage bifurcation point [1, 11] the utility may want to curtail a guaranteed amount of load, hence the contracts would be designed for specific amounts of load curtailment. It is important to determine the ranking and quantitative impact of loads before the utility offers curtailment contracts. After the contracts are signed, an algorithm can be developed to check

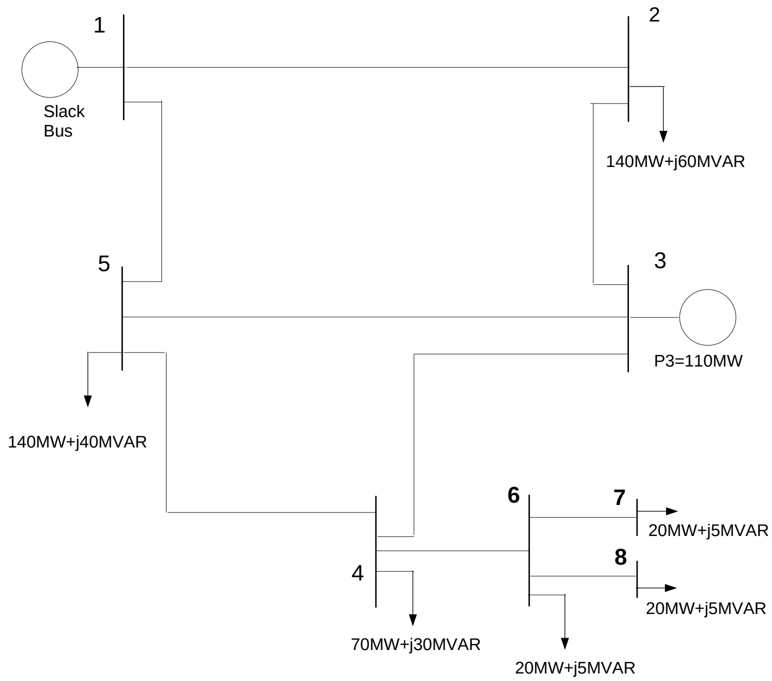

Figure 5.1: Example 8-bus system

for distance to collapse and suggest the optimum curtailment order of loads when required.

After the relative value of all load locations is determined, customer attributes, designed demand management programs and the game theory formulation are used to design optimal demand management contracts[1]. Table 5.2 shows the customer attributes. The first attribute is the "value of power interruptibility" and it varies by location and system conditions. Customer private information[2] θ is what the utility does not know about the customer, this value is revealed by the customer when they choose a contract.

Customers at buses 4, 6, 7 and 8 are at locations where λ is 0.9. Using mechanism design, the utility offers them the contract range (as a function of the private information θ) shown in Figure 5.2. Customers can lie about their θ and try to deceive the utility, however the contracts are designed to discourage the customers from misinforming the utility. Figure 5.3 shows the benefit function for the customers at bus 4 and 6 for all possible values of θ. Their benefit is maximized when they tell the

[1]These contracts are "Agreed Relief Program" [7] contracts, i.e. the customer agrees to curtail a certain "guaranteed" amount of load for a monetary incentive in return.

[2]This is the preference parameter defines the type and class of a customer, which in turn determines their cost of a curtailment or interruption.

Table 5.1: Sensitivity of the loading margin to voltage collapse with respect to each load (loading direction chosen as equal increments for each load)

Load Bus	Sensitivity (MW/MW)
2	-0.03
4	-0.89
5	-0.12
6	-1.48
7	-1.73
8	-1.73
Loading Margin = 36.18 MW	

Table 5.2: Customer Attributes

Customer	Locational Value (λ)	Customer Private Information (θ)
Bus 2	0.4	0.7
Bus 4	0.9	0.8
Bus 5	0.8	0.7
Bus 6	0.9	0.8
Bus 7	0.9	0.5
Bus 8	0.9	0.9

truth at $\theta = 0.8$. Hence the contract[3] should be (from Figure 5.2):

$$\begin{aligned} x(\theta) &= 5.00\text{MW} \\ y(\theta) &= \$2875.00 \\ \text{Customer Benefit} &= \$625.00 \end{aligned}$$

where $x(\theta)$ is the amount the customer agrees to curtail, and $y(\theta)$ is the incentive they receive in return. However if they want to lie about θ in order to get more incentive, their actual benefit would suffer, e.g. the customer above ($\theta = 0.8$) chooses $\theta = 0.9$ then the contract becomes:

$$\begin{aligned} x(\theta) &= 7.00\text{MW} \\ y(\theta) &= \$4375.00 \\ \text{Customer Benefit} &= \$525.00 \end{aligned}$$

[3]In order to convert from normalized numbers base for the amount of agreed relief of 10 MW and base for the incentive of $10000.00 are used.

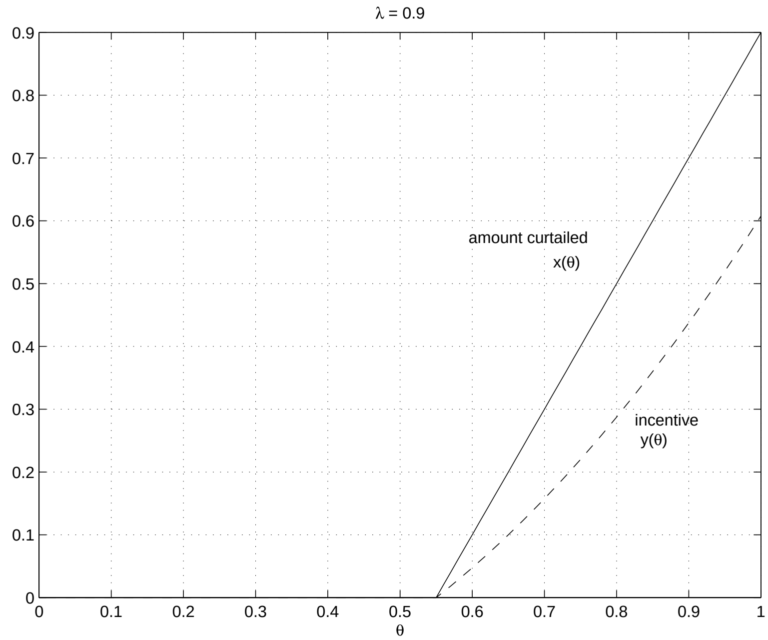

Figure 5.2: Contracts offered to location $\lambda = 0.9$ customers, amount curtailed (solid line), incentive paid (dashed line).

In this case customer benefit is lower, hence each customer is better off if they reveal their true θ.

Customer at bus 2 is offered a different range of contracts due to a different locational value, and Figure 5.4 shows that customer at bus 2 will not make a profit by participating in this program since for all values of θ its profit is zero or negative. After this analysis an optimal portfolio of contracts are shown in Table 5.3.

The value of power interruptibility (λ) and customer preference parameter (private information θ) are the two critical elements of the economic analysis. In order to emphasize the importance of these values, some further tests are performed. In one simulation λ was fixed to be 0.7, i.e. the utility decides that the value of power interruptibility for each customer is the same. As shown in Table 5.4 the number of participating customers increases but the amount of available relief decreased and the result was also a smaller increase in the loading margin. In the other simulation the utility assumes that the costs of an outage is the same to all customers and it fixes $\theta = 0.7$. However it is observed that this also is not an optimal portfolio since the available load relief and the increase in loading margin is lower than the optimal case shown in Table 5.3. Some economic facts are also computed for each portfolio and a comparison is made in Table 5.5. In the scenarios where λ and θ are fixed to a certain value, a non-optimal portfolio of contracts is still obtained,

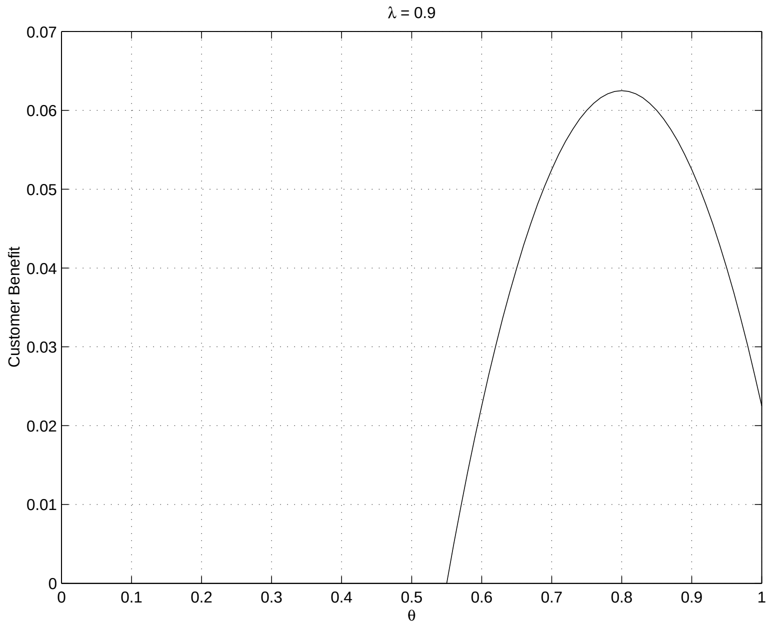

Figure 5.3: Customer benefit function for $\lambda = 0.9$ (customer's true $\theta = 0.8$)

however the results verify that the *optimal* portfolio of contracts help both the utility and the customer in maximizing their profit. More importantly it indicates that the optimal portfolio maximizes the amount of available load relief and the increase in the loading margin. The discussion above is focused on avoiding voltage collapse, however a similar approach can be used to relieve line overloads.

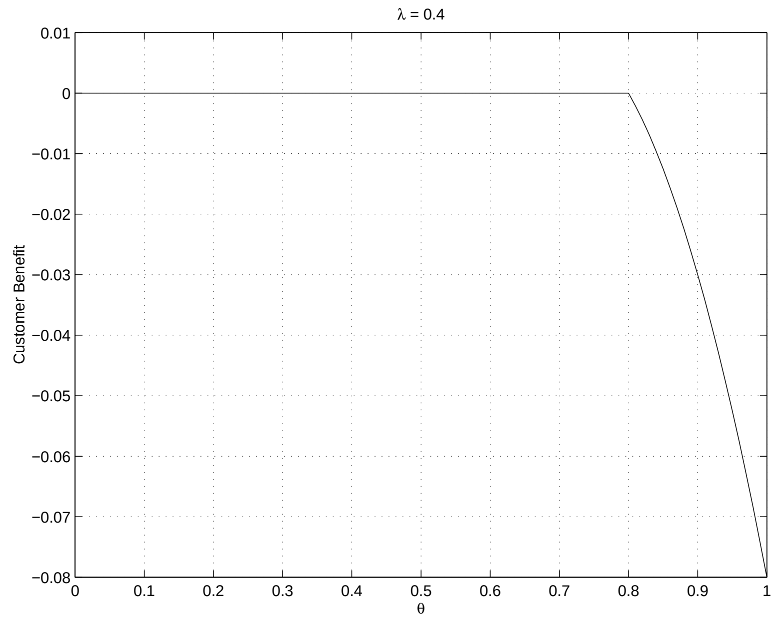

Figure 5.4: Customer benefit function for $\lambda = 0.4$ (customer's true $\theta = 0.7$)

Table 5.3: Optimal Portfolio of Demand Management Contracts

Customer	Amount Curtailed $x(\theta)$	Incentive Offered $y(\theta)$
Bus 2	0.00 MW	\$0.00
Bus 4	5.00 MW	\$2875.00
Bus 5	2.00 MW	\$900.00
Bus 6	5.00 MW	\$2875.00
Bus 7	0.00 MW	\$0.00
Bus 8	7.00 MW	\$4375.00
Available Relief = 19 MW		
Increase in Margin = 24.34 MW		

Table 5.4: Non-Optimal Portfolio of Demand Management Contracts with Fixed $\lambda = 0.7$

Customer	Amount Curtailed $x(\theta)$	Incentive Offered $y(\theta)$
Bus 2	1.00 MW	$375.00
Bus 4	3.00 MW	$1275.00
Bus 5	1.00 MW	$375.00
Bus 6	3.00 MW	$1275.00
Bus 7	0.00 MW	$0.00
Bus 8	5.00 MW	$2375.00
Available Relief = 13 MW		
Increase in Margin = 15.91 MW		

Table 5.5: Different Scenarios for Contracts

Scenario	Relief (MW)	$\uparrow$ LM (MW)	U. Profit ($)	C. Profit ($)
Optimal	19	24.34	5875.00	2575.00
$\lambda = 0.7$	13	15.91	3425.00	1125.00
$\theta = 0.7$	14	17.73	5200.00	1000.00

Chapter 6

Conclusions

Nonlinear pricing can be used as a means for extracting maximum value from demand management contracts. By using mechanism design, optimal contracts can be designed that encourage customers to voluntarily sign up for the contract that best suits their needs. These contracts maximize the benefit a utility can get for a given amount of relief. It is not necessary for a utility to know in advance the type of customer it faces when designing such programs. The report also illustrates the importance of location, and describes a method for incorporating location into the process. Some locations in the grid are more valuable than others. This report shows which ones and by how much.

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Appendix A

General Formulation for Contract Design

The theory of mechanism design (or contract offer structure design) can be applied to the interaction between the electric utility and its customers. The utility does not know the willingness of its customers to shed power if offered an incentive. Each customer would value the interrupted power differently depending on how much it would cost them to shed load. The utility could simply ask the customers how much their interruption costs are, but they will not report it correctly unless they are given an incentive to do so. The utility can design an incentive scheme that determines the monetary transfer received by each customer as a function of the amount of power they are willing to curtail. The customer's willingness to curtail is modeled by a variable $\theta \in [0, 1]$ called the customer's *type*.

Assume a utility is buying x MW of contracted curtailable power from its customers. We characterize customers' preference for curtailment probabilistically, through a random variable θ as described above . This "preference parameter" θ can possess any probability distribution $f(\theta)$ over [0,1], and let $F(\theta) = \int^{\theta} f(\tilde{\theta})d\tilde{\theta}$. The value of θ is private information of the customer, and is unknown to the utility. $V_1(x, \theta)$ is the assumed cost of curtailing x MW for a customer with preference parameter θ. $V_1(x, \theta)$ needs to be positive for all θ and x, and $V_1(x, \theta)$ needs to be increasing in x. The utility is paying y amount of money to the customer willing to shed x amount of power. Hence, the customer benefit function is:

$$u_1(x, y, \theta) = y - V_1(x, \theta). \tag{A.1}$$

Under stressed conditions it is expensive for the utility to deliver power to certain locations. The utility can compute the value $V_0(x, \lambda)$ of *not* delivering power to a certain customer, where λ is the incremental benefit of not delivering power to a certain location in the network. Then the utility profit from this power curtailment of a customer is:

$$u_0(x, y, \lambda) = V_0(x, \lambda) - y. \tag{A.2}$$

Having a subjective estimate of the customer types it is dealing with, the utility develops an incentive function $Y(x)$ to indicate how much it is willing to pay someone for a given x amount of curtailment, and develops a function $X(\theta, \lambda)$ for how much it thinks a customer of type θ at location λ should curtail.

Customers self-select the amount of curtailment they wish to be subjected to, based on an inspection of the incentive function offered to them. They are assumed to do so rationally, by making the amount of compensation they receive from participation match the monetary incentive offered by the utility minus the actual net loss of benefit that result from the curtailment. Customers will not choose to be curtailed unless they see a net positive benefit. Thus it is necessary that $u_1(X(\theta, \lambda), Y(X(\theta, \lambda)), \theta) \geq 0$.

The customer benefit function (A.1) needs to satisfy the necessary single crossing property; $\dfrac{\partial u_1}{\partial y} > 0$ and $\dfrac{-\partial u_1/\partial x}{\partial u_1/\partial y}$ should be decreasing in θ. In the case of function (A.1) since $\dfrac{\partial u_1}{\partial y} = 1$ the single crossing property reduces to $\dfrac{\partial V_1}{\partial x}$ being decreasing in θ.

The goal is to maximize expected profit for the utility.

$$\max_{X(.),Y(.)} E_\theta u_0(X(\theta, \lambda), Y(X(\theta, \lambda)), \lambda) \tag{A.3}$$

subject to,

$$u_1(X(\theta, \lambda), Y(X(\theta, \lambda)), \theta) \geq 0 \tag{A.4}$$

$$u_1(X(\theta, \lambda), Y(X(\theta, \lambda)), \theta) \geq u_1(X(\hat{\theta}, \lambda), Y(X(\hat{\theta}, \lambda)), \theta) \tag{A.5}$$

where $\hat{\theta}$ is the preference parameter of a customer if they were to report it incorrectly. Constraint (A.4) is the *individual rationality constraint* which makes sure every customer is encouraged to participate, and constraint (A.5) is the *incentive compatibility constraint* which encourages the customers to tell the truth about their θ.
The individual rationality constraint binds at the lowest participating $\theta = \underline{\theta}$, i.e.

$$u_1(X(\underline{\theta}, \lambda), Y(X(\underline{\theta}, \lambda)), \underline{\theta}) = 0 \tag{A.6}$$

Let

$$U_1(\theta,\lambda) \equiv \max_{\hat{\theta}} u_1(X(\hat{\theta},\lambda), Y(X(\hat{\theta},\lambda)),\theta). \tag{A.7}$$

Then when incentive compatibility constraint is binding for all θ and λ:

$$U_1(\theta,\lambda) = u_1(X(\theta,\lambda), Y(X(\theta,\lambda)),\theta). \tag{A.8}$$

Using the chain rule,

$$\frac{dU_1}{d\theta} = \frac{\partial u_1}{\partial x}\frac{dX}{\partial\theta} + \frac{\partial u_1}{\partial y}\frac{\partial Y}{\partial x}\frac{\partial X}{\partial\theta} + \frac{\partial u_1}{\partial\theta}. \tag{A.9}$$

The customer benefit once the utility chooses the scheme $Y(x)$ is $u_1(x, Y(x), \theta)$. Since the customers will pick the optimal x to maximize their profit
$\frac{\partial u_1}{\partial x} + \frac{\partial u_1}{\partial y}\frac{\partial Y}{\partial x} = 0$. Then

$$\frac{dU_1}{d\theta} = \frac{\partial u_1}{\partial\theta} = -\frac{\partial V_1}{\partial\theta} \tag{A.10}$$

$$\Rightarrow U_1(\theta,\lambda) = -u_1(X(\underline{\theta},\lambda), Y(X(\underline{\theta},\lambda)),\underline{\theta}) - \int_{\underline{\theta}}^{\theta} \frac{\partial V_1}{\partial\theta}(X(\tilde{\theta},\lambda),\tilde{\theta})d\tilde{\theta} \tag{A.11}$$

Furthermore, using (A.1), (A.2) and (A.8) we get:

$$u_0 = V_0 - V_1 - U_1. \tag{A.12}$$

Substitute (A.12) and (A.11) in (A.3) to obtain:

$$\max_{X(.)} \int_0^1 \left[V_0(X(\theta,\lambda),\lambda) - V_1(X(\theta,\lambda),\theta) + \int_{\underline{\theta}}^{\theta} \frac{\partial V_1}{\partial\theta}(X(\tilde{\theta},\lambda),\tilde{\theta})d\tilde{\theta} \right] f(\theta)d\theta. \tag{A.13}$$

After an integration by parts the utility's optimization program is:

$$\max_{X(.)} \int_0^1 \left[V_0(X(\theta,\lambda),\lambda) - V_1(X(\theta,\lambda),\theta) + (\frac{1-F(\theta)}{f(\theta)})\frac{\partial V_1}{\partial\theta}(X(\theta,\lambda),\theta) \right] f(\theta)d\theta. \tag{A.14}$$

Once the solution $X(.)$ to the optimization is obtained, the customer's profit can be computed from (A.11):

$$U_1(\theta) = -\int_{\underline{\theta}}^{\theta} \frac{\partial V_1}{\partial\theta}(X(\tilde{\theta},\lambda),\tilde{\theta})d\tilde{\theta}. \tag{A.15}$$

and the monetary incentive is,

$$Y(X(\theta,\lambda)) = U_1(\theta,\lambda) + V_1(X(\theta,\lambda),\theta). \tag{A.16}$$

Application of mechanism design theory to design utility demand management contracts hinges on the development and *calibration* of a predicted customer cost function for shedding power. Hence it is of great value to calibrate and validate the cost function using real data. An accurate cost function will lead to more efficient contracts.

A.1 Preliminary Design and Calibration of the Customer Cost Function

The first assumption we make in designing the cost function is that it costs the customer progressively more to shed more load. Figure A.1 shows the predicted marginal benefit of a customer. We can see as the customer sheds power its loss of surplus is quadratic. The predicted shape of the total cost function for this customer is shown in Figure A.2.

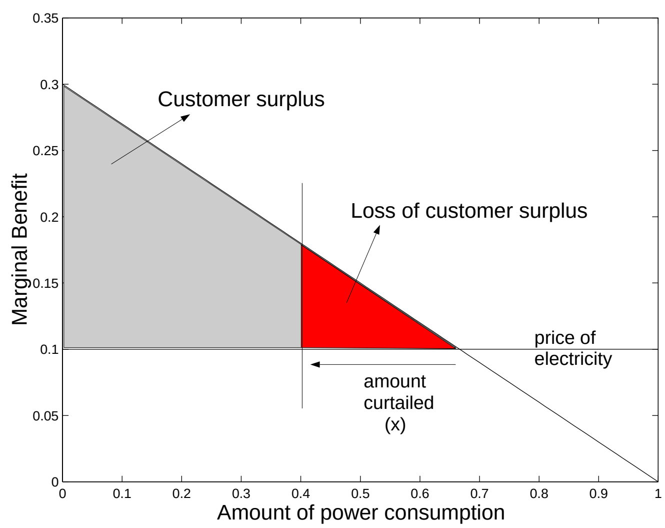

Figure A.1: Marginal benefit for a customer. Areas in this figure denote total surplus.

After the shape of the cost function is predicted to be a quadratic, we have to find the best quadratic form to fit the actual behavior of customers. This is done by extracting information from the real data provided by a utility. The data we received

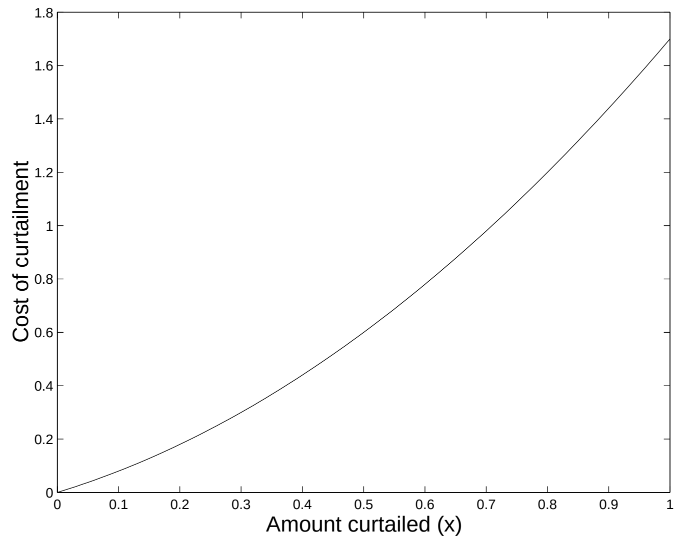

Figure A.2: Predicted cost function for a customer.

from a utility shows that this utility is running three kinds of demand management programs so that the customers are grouped into three types. Each group of customers provide a different amount of load relief for a different amount of payment received in return, as shown in Table A.1. We can use this kind of real data to calibrate the customer cost function. This will be illustrated by means of an example.

Table A.1: Available Utility Data (Extracted Summary)

Customer Group	Provided Relief	Incentive Paid
1	61.54 MW	$49,279.00
2	6.74 MW	$4,182.00
3	6.36 MW	$3,605.00
Total Relief = 74.64 MW		
Total Payment = $57066.00		

A.2 Example

In light of the information shown in Table A.1, let's assume that the utility only has three customers and it is in the process of designing demand management contracts.

The utility does not know which customer it is facing but it knows the probabilities p_1, p_2 and p_3 of having a type θ_1, θ_2 and θ_3. One can think of this as the utility having only one customer that could be any one of the three types θ_1, θ_2 and θ_3. We can assume a general quadratic form for each type of customer:

$$c(\theta, x) = K_1 x^2 + K_2 x - K_2 x\theta \tag{A.17}$$

where x is the amount of power curtailed, K_1 and K_2 (both non-negative) are the coefficients of the general cost function that needs calibrating. The "$-K_2 x\theta$" term is included so that different values of θ lead to different values of $\frac{\partial c}{\partial x}$ (marginal cost for the customer). Notice that, as θ increases the marginal cost decreases. That is, θ has effectively been used to "sort" the customers from "least willing" to "most willing" to shed load. This form of the cost function suggests that the customer with the lowest θ will have the highest marginal cost and hence the lowest marginal benefit. This provides a good way of modeling the *willingness* of each customer to shed load by way of θ.

Using Table A.1 we can see the amount of power each customer curtailed and the amount of monetary incentive they received in return. This incentive could be used as an upper bound on the cost of the customer. In the absence of more extensive data, this provides a reasonable initial bound for the cost.

By substituting the values from Table A.1 into the cost function we can calibrate the values of K_1 and K_2. Since we have two unknowns and three equations, we use the extra degree of freedom to solve for one of the θ values. This is done by picking the highest and the lowest θ and letting the available data determine the middle value.

If the utility pays y_i to customer θ_i to shed x_i, the customer benefit will be:

$$u_i = y_i - (K_1 x_i^2 + K_2 x_i \cdot (1 - \theta_i)) \tag{A.18}$$

for $i = 1, 2, 3$. And if the incremental benefit for not delivering x amount of power is λ, the utility profit will be:

$$u_0 = \lambda x_i - y_i \tag{A.19}$$

The utility wants to maximize their expected profit:

$$\max_{x_1,x_2,x_3,y_1,y_2,y_3} \sum_{i=1}^{3} [\lambda x_i - y_i]\, p_i \tag{A.20}$$

subject to,

$$y_i - (K_1 x_i^2 + K_2 x_i \cdot (1 - \theta_i)) \geq 0 \tag{A.21}$$

$$y_i - (K_1 x_i^2 + K_2 x_i \cdot (1 - \theta_i)) \geq y_j - (K_1 x_j^2 + K_2 x_j \cdot (1 - \theta_i)) \tag{A.22}$$

for $i = 1, 2, 3$ and $j = 1, 2, 3$. The first constraint makes sure each customer is encouraged to participate, and the second constraint encourages the customers to pick the right contract. We can solve this mechanism for any given λ. In order to get a reasonable value for λ the real data was analyzed to show that currently the utility is paying \$800 per MW to its most expensive customer, hence it is reasonable to assume that their benefit of not delivering is greater than \$800 per MW. For this example we assume $\lambda = \$900$ per MW. After this assumption the contracts are designed to be:

$$x_1 = 26.24 \quad MW \quad y_1 = \$19680.00 \tag{A.23}$$

$$x_2 = 89.41 \quad MW \quad y_2 = \$70550.00 \tag{A.24}$$

$$x_3 = 115.70 \quad MW \quad y_3 = \$88210.00 \tag{A.25}$$

In this case the total relief will be 231.35 MW and the total cost will be \$178440.00, and using $\lambda = \$900$ the total profit to the utility will be \$29770.00, before these contracts were designed their total profit was \$10110.00 (still using $\lambda = \$900$).