

**Draft**

ESTIMATING THE VOLATILITY OF SPOT PRICES IN RESTRUCTURED ELECTRICITY  
MARKETS AND THE IMPLICATIONS FOR OPTION VALUES

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The contingent claims valuation of physical assets and financial derivatives depends critically on the specification and estimation of the stochastic process that describes the price path. Accurate valuation of claims based on competitive electricity prices has proved problematic, as electricity price data are not well represented by traditional commodity price models of Brownian motion. Observed on-peak (high demand period) electricity spot prices are highly volatile and strongly mean reverting, infrequently punctuated by large upward jumps which quickly drop toward the mean price level.<sup>1</sup> Existing commodity price characterizations do not capture this dynamic, though they are often used as there is no established alternative.<sup>2</sup> Based on these stylized facts, continuous time models for real options and financial derivatives where the underlying state variable is the spot price of electricity have been proposed (Ethier 1997, 1999, Barz and Johnson 1998, Deng 1998). To date, these models have not been fit to market data, nor has econometric testing been undertaken.

This paper tests the stylized facts which have evolved with the accumulation of data from competitive electricity prices, estimating a mean reverting price process with stochastic regime switching which allows discontinuous jumps in electricity prices. The non-linear econometric model allows complex state dynamics and the standard models of commodity prices are special cases of the proposed model. Although the model allows complex transition dynamics, it remains tractable for financial applications and requires only observed price data for estimation and

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<sup>1</sup> Markets in Scandinavia and New Zealand are far less volatile but are also not strictly competitive (see Wolak 1996).

<sup>2</sup> See, for example, Pilopovic (1997) and Deng, Johnson and Sogomonian (1998).

forecasting. Thus the problem is a fundamental one: to characterize the marginal distribution of electricity prices. This task is also a logical precursor to the estimation of joint distributions potentially of interest to electricity market participants (e.g. electricity prices and natural gas spot prices).

The model parameters for four existing electricity markets are estimated by maximum likelihood, using the recursive filter of Hamilton (1989, 1994). The Hamilton model allows stochastic jumps between regimes, where each regime is a mean reverting AR(1) process with unique mean and variance. Thus electricity prices are viewed as originating from either a high state or a low state, where the high state would be the ‘jump’ state and the low state would be the ‘normal’ state. Regime switching is controlled by a two state Markov process, with state specific transition probabilities which allow different expected durations for each state.

The model is fit to daily on-peak spot price data from the electricity market in Victoria, Australia and three hubs in the United States (ECAR, PJM East, and SERC). Daily average on-peak spot prices are chosen because they are commonly used for writing hedging instruments, and are regularly reported in financial news sources such as the *Wall Street Journal*. The regime switching character of electricity prices is confirmed using non-parametric econometric tests, both for a simple mean switching model and a mean switching model with different variances. The null hypothesis of a mixture of two distributions (a ‘coin flip’ model of regime switching) is also rejected in favor of a Markov specification. Since states tend to persist under a Markov specification, the model has important implications for practical problems such as generator commitment decisions in the presence of start-up costs.

The estimated model also admits a simple and intuitive option value calculation. Lo and Wang (1995) show how the Black-Scholes option model can be used when asset returns are predictable. One example of their derivation is a mean reverting process. Option values under

our estimated specification are calculated as a linear combination of the Lo and Wang option value in each regime. The incorporation of jumps into the model results in dramatically different option characteristics when compared with a model which ignores jumps, and this has important implications for asset valuation and financial derivatives. High strike prices (or equivalently, the marginal production costs for generation) decrease option values much less in the two-state model than in a one-state model. An implication is that generating assets with high marginal costs are especially likely to be undervalued by traditional models. Given the Markov nature of the regime switching, knowledge of the current state dramatically changes option valuation compared to the one-state model in the short-run, but the value of current state information decays rapidly over time. These results suggest that traditional price processes generally underestimate asset values in competitive electricity markets.

### **Characterizing Competitive Electricity Prices**

Electricity production has qualitatively different characteristics from other commodities. In part this is why electricity production and distribution in the US has been a regulated monopoly for most of the century, and continues to be so in most other countries. The original justifications were the economies of scale in both production and distribution. While those concerns have been mitigated by technology, other characteristics continue to make electricity unique among commodities. They include:

- **Lack of storability.** Electricity is costly to store, resulting in greater price volatility as it is prohibitively costly to arbitrage across time periods (e.g. see Barz and Johnson 1998).
- **Inelastic demand.** Existing electricity demand patterns reflect the fact that few customers respond to short term changes in spot prices for electricity. Most customers still pay fixed prices based on rate schedules set by regulators. Short-run demand curves are nearly vertical. The

yearly residential price elasticity in New York State, for example, has been estimated to be an inelastic 0.042 (Ethier and Mount 1998).

- Restrictive transportation networks. Electricity supply in a given region is restricted by the transmission network, meaning that outside suppliers are often unable to respond to price signals even if they have available generating capacity. This is exacerbated by problems of market power (Zimmerman, Bernard, Thomas, and Schulze 1998, Rudkevich, Duckworth, and Rosen 1998).
- Kinked supply curve. Existing generation plant produces a supply stack which, while flat for much of the stack, becomes steeply sloped as maximum generating capacity is approached. When coupled with inelastic demand, this produces large price swings for small changes in demand when system generators approach maximum capacity. This phenomenon is exacerbated by market power (Wolak and Patrick 1997, and Mount 1999).
- Load is highly weather dependent. Since load is closely related to weather, extreme weather often produces extreme load. Since day to day weather is highly correlated, extreme load and extreme prices are positively correlated. Weather dependence also implies difficulty in making accurate medium or short term forecasts.

The electricity market in Victoria, Australia was probably the most competitive market in the world when it opened in 1995. The state government made a concerted effort to limit market power among generators by selling each state-owned power plant to a different company. Consequently, the problems associated with market power that are still chronic in the United Kingdom were avoided. Even where the state-owned generators in New South Wales were integrated into the market in May 1997, the market remained competitive with average spot prices less than one third of the regulated level. However, these prices are very volatile with

many random price spikes. The situation is very different in the UK where the price spikes caused by exploiting market power exhibit a regular daily pattern (see Wolak and Patrick 1997). The pattern of price spikes in Victoria can be seen in a graph of daily electricity spot prices shown in Figure 1.

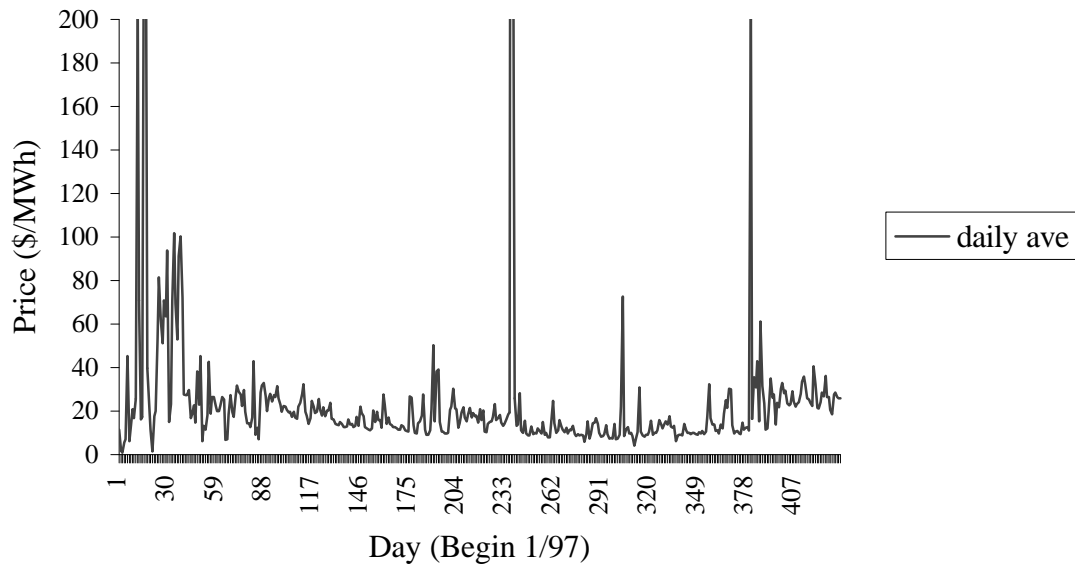


Figure 1. Victoria On-Peak Daily Average Electricity Price (Load Weighted)<sup>3</sup>

Prices generally stay in a narrow, low range, but occasionally spike dramatically upward. When a price spike ends, prices revert to the previous narrow range. This qualitative assessment can be quantitatively strengthened by examining the sample descriptive statistics:

Table 1. Descriptive Statistics From Victoria Spot Price Data

<u>Mean</u>	<u>Median</u>	<u>Standard</u>	<u>Kurtosis</u>	<u>Skewness</u>	<u>Min</u>	<u>Max</u>
		<u>Deviation</u>			<u>Value</u>	<u>Value</u>
23.80	15.98	42.84	150.52	10.85	0.82	697.98

<sup>3</sup> Data from Victoria, Australia ISO. Available at <http://electricity.net.au/vpx.html>.

The data is clearly skewed leftward, with a median well below the mean and a very high skewness statistic. This is typical of electricity spot prices in a competitive market.

### **Alternative Stochastic Specifications of Price Paths**

Much of the new financial literature on valuing derivatives is based on continuous time specifications of price paths. Typical continuous time characterizations are Geometric Brownian Motion (GBM) and mean reversion (e.g. the Ornstein-Uhlenbeck process). For a discussion see, for example, Dixit and Pindyck (1994), Pilipovic (1997), and Baker, Mayfield and Parsons (1998). There also exist models which combine GBM with a jump process (for example Dixit and Pindyck 1994, or Ball and Torous 1983). None of these specifications have theoretical characteristics appropriate for observed electricity prices in competitive markets.

GBM is an unsuitable price process for electricity for a variety of reasons. Future values of price depend only on the current price, with no relationship to a long-run mean value. The variance of GBM increases linearly with time, whereas electricity prices exhibit mean reversion and hence bounded variance. And GBM can not exhibit the dramatic price spikes observed in electricity prices. Qualitatively, GBM does not exhibit the skewness and kurtosis evident in electricity prices.

Mean reversion is a better choice for electricity spot prices, as noted in Pilipovic (1997), Tseng and Barz (1998), and Deng, Johnson, and Sogomonian (1998) and for commodity markets in general as in Schwartz (1997), Baker, Mayfield, and Parsons (1998). It incorporates information about a long run mean value, and exhibits bounded long term variance, which is consistent with observed electricity prices. However, it does not exhibit the non-linear deviations present in electricity prices. That is, it does not allow highly skewed distributions. Note that because

electricity is essentially non-storable, mean reversion in this context does not imply that it is only optimal to sell when observed prices are high (see Laughton 1998 for a brief discussion of this related to oil).

Jump diffusion processes do allow discontinuities in the price path, but there is no mean reverting component. Thus the long run variance is again unbounded and jump diffusion is an inappropriate model for electricity prices. Combining a jump process with mean reversion can capture the salient features of daily electricity spot prices. This has been suggested in Ethier (1997, 1998), Dorris and Ethier (1998), Violette, King and Ethier (1998), Barz and Johnson (1998), and Deng (1998). In theory, parameters of these continuous time models might be estimated by calculating appropriate sample moments. In practice this is a non-trivial task that has yet to be undertaken in a formal way. As a sensible alternative, an econometric approach based on discrete time is taken here.

### **The Econometric Model**

As discussed by Dixit and Pindyck (1994) and Lo and Wang (1995), a continuous time mean reverting process can be modeled in discrete time as an AR(1), with the mean reverting parameters easily derived from the intercept and autoregressive coefficient. If the AR(1) model is written:

$$y_t - y_{t-1} = a + by_{t-1} + \mathbf{e}_t$$

Dixit and Pindyck (1994, pp.76-77) show that the parameters of the Ornstein-Uhlenbeck mean reverting process in continuous time can be calculated as:

$$\hat{h} = -\ln(1 + \hat{b})$$

$$\bar{P} = \frac{-\hat{a}}{\hat{b}}$$

$$\hat{S} = \hat{S}_e \sqrt{\frac{\ln(1 + \hat{b})}{(1 + \hat{b})^2 - 1}}$$

where the O-U model is

$$dP = \mathbf{h}(\bar{P} - P)dt + \mathbf{s}dz$$

and  $dz$  is a unit of the standard Wiener process. With that in mind, a two-state AR(1) model is estimated using a variant of the Markov regime switching model of Hamilton (Hamilton 1989, 1994). Versions of this model have previously been applied to GNP growth (Hamilton 1989) exchange rates (Engel and Hamilton 1990), and stock returns (Cecchetti, Lam, and Mark 1990), among other applications. The version presented here allows two states which means that electricity prices can jump discontinuously between states, with state probabilities, means, and autoregressive parameter estimated by maximum likelihood. See Hamilton (1994) for additional derivations and generalizations. The two-state switching model can be written:

$$y_t - \mu_{s(t)} = \phi(y_{t-1} - \mu_{s(t-1)}) + \varepsilon_t$$

where  $y_t$  is the natural log of the daily spot price of electricity with  $\varepsilon_t \sim N(0, \sigma_{s(t)}^2)$  and  $s_t = 1, 2$ .<sup>4</sup> In this specification, the intercepts  $\mu_{s(t)}$  are the mean value of  $y$  in each state. Estimating a Markov model in means if the true process is Markov is preferred to a two-intercept specification because information about the probability of being in a previous state is combined with information about the previous state (in the form of the mean) and included in the current state estimation process. That is, if you have knowledge that you were in state two last period with

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<sup>4</sup> Versions with a common variance across states, as well as different variances, are estimated.

certainty, but are now in state one, incorporating that information will improve your forecast for the future state.

The model allows two states which can persist, rather than isolated and independent jumps that are typical for continuous time models. This is important because jumps in electricity prices are often driven by extreme weather or plant outages, which tend to persist for more than one period. Specifying a Markov switching process requires a matrix of transition probabilities, rather than the single probability specified for a continuous time jump model. The matrix of conditional jump probabilities is written:

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

where  $p_{11}$  is the probability of entering state 1 in time  $t$  conditional on being in state 1 in time  $t-1$ . Thus  $p_{21}$  is the probability of entering state 2 in time  $t$  conditional on being in state 1 in time  $t-1$ , and  $p_{21} = 1 - p_{11}$ . Since only two states are considered in the current context, it is easier to use the following simplified notation:

$$P = \begin{bmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{bmatrix} = \begin{bmatrix} p & 1 - q \\ 1 - p & q \end{bmatrix}$$

The  $p, q$  notation will be used for the remaining discussion. While this specification is more complex than a simple mixture of distributions, it allows the potentially important characteristic of persistent jumps. Note that if jumps do not persist, then the model collapses to a mixture of distributions (or ‘coin-flip’) specification where  $p = 1 - q$  and the matrix of transition probabilities becomes:

$$P = \begin{bmatrix} p & p \\ 1-p & 1-p \end{bmatrix}$$

Note that for any given day during the forecast period, the unconditional state probabilities (Ergodic probabilities) can be calculated as:

$$p = \frac{1-q}{(1-p+1-q)}$$

where  $\pi$  is the unconditional probability of being in state one. The unconditional probability of being in state two is then  $1 - \pi$ . Clearly if the process is not Markov, and  $p = 1 - q$ ,  $\pi = p$  and the conditional and unconditional probabilities are equal. Conditional on being in state one, state one is expected to persist for  $(1 - p)^{-1}$  periods, while conditional on being in state two, state two is expected to persist for  $(1 - q)^{-1}$  periods.

The Hamilton model is a recursive filter which produces a probabilistic inference about the unobserved state  $s_t$  given observations on values of  $y_t$ . The sample likelihood is produced as a byproduct of the recursive filter, allowing standard numerical maximization. The following discussion illustrates the one-lag case, but readily generalizes to include higher order lag terms as discussed in Hamilton (1989, 1994).

The filter in each iteration uses the following conditional probability:

$$P[S_{t-1} = 1 | y_{t-1}, y_{t-2}, \dots, y_0]$$

Producing the revised probability for a new observation  $y_t$ :

$$P[S_t = 1 | y_t, y_{t-1}, \dots, y_0]$$

along with the conditional likelihood of  $y_t$  used for maximization which can be written:

$$f(y_t | y_{t-1}, y_{t-2}, \dots, y_0).$$

The first step in the iterative filter uses the current period full-information transition probabilities which can be written as:

$$\begin{aligned} P[S_t = s_t, S_{t-1} = s_{t-1} | y_{t-1}, y_{t-2}, \dots, y_0] &= P[S_t = s_t | S_{t-1} = s_{t-1}] \\ &\times P[S_{t-1} = s_{t-1} | y_{t-1}, y_{t-2}, \dots, y_0] \end{aligned}$$

$P[S_t = s_t | S_{t-1} = s_{t-1}]$  is  $p$  if  $s_t = s_{t-1} = 1$ ,  $q$  if  $s_t = s_{t-1} = 2$ .  $P[S_{t-1} = s_{t-1} | y_{t-1}, y_{t-2}, \dots, y_0]$  is the previous filtered probability of being in state  $s_{t-1}$ . In words,  $P[S_t = s_t, S_{t-1} = s_{t-1} | y_{t-1}, y_{t-2}, \dots, y_0]$  is the joint probability that last period was state  $s_{t-1}$  and this period is state  $s_t$ , given all previous values of  $y$ .

The next step uses the joint conditional density of  $y_t$  and  $(S_t, S_{t-1})$  which can be written:

$$\begin{aligned} f(y_t, S_t = s_t, S_{t-1} = s_{t-1} | y_{t-1}, y_{t-2}, \dots, y_0) \\ = f(y_t | S_t = s_t, S_{t-1} = s_{t-1}, y_{t-1}, y_{t-2}, \dots, y_0) \\ \times P[S_t = s_t, S_{t-1} = s_{t-1} | y_{t-1}, y_{t-2}, \dots, y_0] \end{aligned}$$

Thus the current period joint conditional density is the product of the density in the current period when you know  $S_t = s_t, S_{t-1} = s_{t-1}, y_{t-1}, y_{t-2}, \dots, y_0$  and the filtered probability that  $S_t = s_t, S_{t-1} = s_{t-1}$ , are indeed the realized states. The filtered probabilities sum to one by construction.

The current period density function is:

$$f(y_t | S_t = s_t, S_{t-1} = s_{t-1}, y_{t-1}, y_{t-2}, \dots, y_0) = \frac{1}{\sqrt{2ps}} \exp \left\{ \frac{-[(y_t - \mathbf{m}_{s_t}) - \mathbf{f}(y_{t-1} - \mathbf{m}_{s_{t-1}})]^2}{2s^2} \right\}$$

Note that the standard deviation would also be indexed by  $i$  for a model with different variances between states. For a two state AR(1) model, there are four resulting density functions, one for each possible combination of state transitions. These correspond to the four state transition probabilities. The density of  $y_t$  conditional on previously observed values of  $y$  is thus the sum of the joint conditional densities calculated above:

$$f(y_t | y_{t-1}, y_{t-2}, \dots, y_0) = \sum_{s_t=0}^1 \sum_{s_{t-1}=0}^1 f(y_t, S_t = s_t, S_{t-1} = s_{t-1} | y_{t-1}, y_{t-2}, \dots, y_0)$$

It is this value that is logged and summed over the  $T$  time periods to generate the log likelihood for a particular set of observed prices, and the log likelihood is:

$$L(\theta) = \sum_{t=1}^T \ln (f(y_t | y_{t-1}, y_{t-2}, \dots, y_0))$$

The log likelihood is maximized numerically. The filter continues by updating this period's joint conditional probability for states as:

$$\begin{aligned} P[S_t = s_t, S_{t-1} = s_{t-1} | y_t, y_{t-1}, \dots, y_0] &= f(y_t, S_t = s_t, S_{t-1} = s_{t-1} | y_{t-1}, y_{t-2}, \dots, y_0) \\ &\div f(y_t | y_{t-1}, y_{t-2}, \dots, y_0) \\ &= f(y_t, S_t = s_t, S_{t-1} = s_{t-1} | y_{t-1}, y_{t-2}, \dots, y_0) \\ &\div \sum_{s_t=0}^1 \sum_{s_{t-1}=0}^1 f(y_t, S_t = s_t, S_{t-1} = s_{t-1} | y_{t-1}, y_{t-2}, \dots, y_0) \end{aligned}$$

The output needed to start the filter for the next time period  $t+1$  is the probability of being in state one or state two in the current period  $t$ . This is obtained by summing the transition probabilities over states  $s_{t-1}$ :

$$P[S_t = s_t \mid y_t, y_{t-1}, \dots, y_0] = \sum_{s_{t-1}=0}^1 P[S_t = s_t, S_{t-1} = s_{t-1} \mid y_t, y_{t-1}, \dots, y_0]$$

The filter starts the next period as:

$$P[S_{t+1} = s_{t+1}, S_t = s_t \mid y_t, y_{t-1}, \dots, y_0] = P[S_{t+1} = s_{t+1} \mid S_t = s_t] \\ \times P[S_t = s_t \mid y_t, y_{t-2}, \dots, y_0]$$

The initial value for the filter is set to the limiting probability of the Markov process given by:

$$\pi = (1-q)/(1-p+1-q)$$

where  $\pi$  is the probability of observing state one and correspond to the ergodic probability of state one.

To provide intuition, note that Hamilton (1990) shows that the maximum likelihood estimates of the transition probabilities are given by:

$$\hat{p}_{ij} = \frac{\sum_{t=2}^T P[s_t = j, s_{t-1} = i \mid Y_T; \hat{q}]}{\sum_{t=2}^T P[s_{t-1} = i \mid Y_T; \hat{q}]}$$

where  $\hat{\mathbf{q}}$  denotes the full vector of maximum likelihood estimates. So the cells  $\hat{p}_{ij}$  of the estimated transition probability matrix are simply the estimated number of times states  $i$  follows state  $j$  divided by the estimated number of times the process was in state  $i$ .

The output of the Hamilton model is a matrix of estimated transition probabilities and estimated parameters for each state. The algorithm used here is the same as that used by Hamilton (1989), running on Gauss using the Davidon-Fletcher-Powell routine in the Optimum library.

### **The Data**

Daily on-peak spot price data were used for estimating model parameters. On-peak data corresponds to a typical 1x16 daily electricity price contract during weekdays. Such data are reported in the Dow Jones Electricity Price Index printed daily in the *Wall Street Journal*.<sup>5</sup> Data are available for a number of hubs in the United States. Data were gathered for the Victoria market in Australia, and SERC, ECAR, and PJM East hubs in the United States.

Note that the data are generated by fundamentally different market mechanisms. The US data are from hubs, where loosely coordinated traders buy and sell bulk power across utilities and regional grids. The Australia data are produced by a centralized, single-sided auction market, which is more structured than the current US hub markets.

The Victoria market data were obtained directly from the Victoria pool ISO (<http://electricity.net.au/vpx.html>). The spot price data are in half hourly observations with associated load levels. For each of the five business days of the week, a load weighted average price for the 16 peak hours was constructed, resulting in five price observations per week. Data

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<sup>5</sup> See, for example, the DJ Electricity Price Indexes, *The Wall Street Journal*, Thursday, October 15, 1998, p.C17.

were available from 1995 to the present. Only data until May 11, 1997 were used because on that day the Victoria pool was integrated with the New South Wales pool, substantially changing the market character and effectively increasing available generation.

The on-peak data for US hubs were obtained from the newsletter "Power Markets Week." A sample copy with details on the calculations is available at <http://www.mhenergy.com/demos/index.html>. Data from ECAR, PJM East and SERC Florida-Georgia Border hubs are used. The ECAR data begin 1/8/96. The PJM East data begin 4/9/96. The SERC Fla-Ga data begin 7/1/96. All US data run through 8/31/98.

To account for seasonality, the data for each market have been divided into three month seasons. Winter is defined as December through February, with the remaining seasons following appropriately. An obvious extension of the model would be to incorporate seasonal fluctuations in the parameters on a continuous, rather than discrete, basis. However, this is likely to complicate the maximum likelihood estimation problem considerably. Additionally, it is not clear at this point exactly which parameters should fluctuate, and how. Discrete seasonal estimation can provide valuable information about the appropriateness of the model in general, and also about which parameters should be the focus for estimating a model with time varying parameters. Extending the current model this way is thus left for future research.

Note that weekends were not included in the data, as they are 'off-peak'. This means that the model views Monday as following immediately after Friday. This is not strictly true because weekends allow a large period of time for an unobserved regime change (e.g. extreme weather changes). There is no clear solution to this problem. Since Friday's state will not necessarily be the state on Sunday, the weekend transition might be more appropriately modeled as a coin flip rather than a Markov process. Thus any results supporting the existence of a Markov process occur in spite of the Friday-Monday gap and not because of it.

## Results

Seasonal models using the log of the daily on-peak electricity spot price were estimated for each of the four datasets. Models were estimated with a common variance and with different variances for each state. The maximum likelihood estimation generally was robust to choice of starting values, with few local maxima encountered. For some starting values, numeric over or underflows occurred. In these cases, estimation was started over at different start values. Without prior information about parameter values, a wide grid search must be conducted to assure that a global maximum has been achieved. For the models estimated in this paper, there exists prior information for the AR(1) parameters.<sup>6</sup> Thus start values were set in the range suggested by prior experience to avoid numeric difficulties. Start values in these bands around this range produced the same likelihood and parameter estimates. Start values outside these bands caused numerical under and overflow problems.<sup>7</sup>

### Single Variance Models

For the US data, the means of the two states were higher in the summer than in the other three seasons. They also diverged from each other by a larger amount during summer than the other seasons. The variance behaved similarly, with a high level in the summer, and similar levels for the other seasons. The ergodic probabilities show that prices are much more likely to be in the low state than in the high state on any given day during a season. These results are what one would expect from the discussion of electricity price formation in the previous section for a system that has the highest load in the summer. Estimates of the autoregressive parameter  $\phi$

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<sup>6</sup> Note that models were estimated which included higher order AR processes. The extra AR parameters were close to zero ( $<.1$ ) and would not pass a significance test at the 10% level. It was concluded that including higher order AR terms was unnecessary.

<sup>7</sup> Hamilton (1990) suggests the use of the EM algorithm as a solution to convergence problems in estimating the model.

vary from .7 to over .9, with no discernable pattern across seasons. The autoregressive parameter is always significantly different from zero, and most of the means and variances are also significant using a standard t-test.

The Victoria results are similar to the US markets in that the means are highest in the northern summer (the peak load occurs in their cold season). They differ in that the two means are closest together when they are both at their highest values. The ergodic probabilities also show an interesting characteristic: in the northern winter, spring, and fall, it is the high state which is most likely to occur, not the low state. This implies downward ‘jumps’ and not upward jumps that occur in the US. Estimates of the autoregressive parameter vary from .67 to over .89 and they are always significant. All but one of the other parameters (one of the transitional probabilities) are significant.

All the estimated parameters and standard errors for each of the markets are summarized in Tables 2 – 5, and graphs of the estimated parameters by season are summarized in an appendix.

### **Two Variance Models**

The US data produces means for the two states that are closer together under the two-variance model than under the one variance model. The means also vary less across seasons, though they generally are highest in the summer months, with the SERC data being the one exception. The high variance is always associated with the high mean for each season. The high variance is also generally much higher than the low variance. In every case but the SERC winter data, the high variance is at least an order of magnitude greater than the low variance, and often much more. The ergodic probabilities are generally more similar across seasons under the two variance model than under the one variance model. In two cases, SERC fall and ECAR fall, the high

variance/mean state is more likely than the low state. The autoregressive parameter varies from .69 to over .9 and is always significant.

The comparisons between models in Victoria are similar to those in the United States data: the means are closer together and exhibit less variation by season. The differences in regime are captured by the differences in the variance terms. In Victoria, the less frequent state (in three of four cases) exhibits the higher variance, but in two of four seasons, the high variance/low frequency state has the low mean. The autoregressive parameter varies from .63 to .97 and is always significant.

Table 2. SERC Parameter Estimates with Standard Errors

	<u>Mean 1</u>	<u>Mean 2</u>	<u>Phi</u>	<u>Sigma 1</u>	<u>Sigma 2</u>	<u>P(1,1)</u>	<u>P(2,2)</u>
SERC_wint	3.0052 (0.034)	3.5189 (0.097)	0.7303 (0.063)	0.0104 (0.001)		0.9823 (0.013)	0.3375 (0.269)
SERC_spr	3.0228 (0.067)	4.3691 (0.144)	0.7913 (0.060)	0.0232 (0.003)		0.9933 (0.007)	0.9720 (0.049)
SERC_sm	3.7352 (0.183)	5.5148 (0.255)	0.8454 (0.047)	0.1018 (0.013)		0.9840 (0.011)	0.3387 (0.276)
SERC_fall	3.1118 (0.051)	3.4792 (0.062)	0.8288 (0.051)	0.0094 (0.001)		0.9574 (0.019)	0.4776 (0.183)
SERC_wnt	2.9853 (0.028)	3.4694 (0.093)	0.6945 (0.066)	0.0082 (0.001)	0.0558 (0.029)	0.9766 (0.015)	0.6889 (0.167)
SERC_sprl	2.9883 (0.026)	3.3019 (0.091)	0.7194 (0.066)	0.0050 (0.001)	0.2155 (0.074)	0.9733 (0.016)	0.8769 (0.090)
SERC_sm	3.1881 (0.097)	3.2118 (0.128)	0.8353 (0.028)	0.0089 (0.002)	0.4625 (0.100)	0.7487 (0.063)	0.6325 (0.088)
SERC_fall	3.0287 (0.048)	3.1875 (0.067)	0.8561 (0.050)	0.0024 (0.001)	0.0301 (0.006)	0.9497 (0.031)	0.9621 (0.029)

Table 3. PJM East Parameter Estimates with Standard Errors

	<u>Mean 1</u>	<u>Mean 2</u>	<u>Phi</u>	<u>Sigma 1</u>	<u>Sigma 2</u>	<u>P(1,1)</u>	<u>P(2,2)</u>
pjmE_wint	3.1256 (0.059)	3.4543 (0.073)	0.8796 (0.045)	0.0061 (0.001)		0.9836 (0.012)	0.2959 (0.286)
pjmE_spr	3.1390 (0.096)	3.4309 (0.099)	0.9261 (0.040)	0.0059 (0.001)		0.9589 (0.020)	0.7956 (0.115)
pjmE_sm	3.4328 (0.098)	4.1517 (0.185)	0.7892 (0.058)	0.0498 (0.009)		0.9620 (0.029)	0.6608 (0.144)
pjmE_fall	3.2052 (0.060)	3.4608 (0.065)	0.8929 (0.042)	0.0050 (0.001)		0.9532 (0.021)	0.3589 (0.175)
pjmE_wint	3.0803 (0.063)	3.2468 (0.087)	0.9006 (0.054)	0.0035 (0.001)	0.0298 (0.011)	0.9570 (0.030)	0.7420 (0.147)
pjmE_spr	3.0958 (0.021)	3.2130 (0.063)	0.7283 (0.072)	0.0032 (0.001)	0.0419 (0.015)	0.9637 (0.024)	0.8703 (0.085)
PJME_sm	3.2089 (0.043)	3.6490 (0.074)	0.7300 (0.055)	0.0091 (0.002)	0.1177 (0.025)	0.8915 (0.040)	0.8402 (0.067)
pjmE_fall	3.2537 (0.049)	3.4355 (0.061)	0.8789 (0.035)	0.0028 (0.001)	0.0298 (0.010)	0.9249 (0.027)	0.6416 (0.163)

Table 4. ECAR Parameter Estimates with Standard Errors

	<u>Mean 1</u>	<u>Mean 2</u>	<u>Phi</u>	<u>Sigma 1</u>	<u>Sigma 2</u>	<u>P(1,1)</u>	<u>P(2,2)</u>
ECAR_wnt	2.8847 (0.041)	3.3868 (0.054)	0.7662 (0.059)	0.0118 (0.001)		0.9722 (0.014)	0.8213 (0.094)
ECAR_spr	3.0485 (0.082)	3.6405 (0.088)	0.8898 (0.036)	0.0154 (0.002)		0.9618 (0.014)	0.4024 (0.155)
ECAR_sm	3.4157 (0.110)	4.9988 (0.189)	0.7877 (0.047)	0.1006 (0.011)		0.9839 (0.010)	0.7710 (0.119)
ECAR_fall	2.9905 (0.089)	3.3328 (0.091)	0.9077 (0.040)	0.0079 (0.001)		0.9198 (0.029)	0.6105 (0.163)
ECAR_wint	2.8962 (0.038)	3.1911 (0.067)	0.8219 (0.045)	0.0057 (0.001)	0.0854 (0.024)	0.9643 (0.016)	0.8186 (0.077)
ECAR_spr	2.9591 (0.021)	3.3857 (0.062)	0.6921 (0.050)	0.0065 (0.001)	0.1673 (0.049)	0.9641 (0.015)	0.7958 (0.095)
ECAR_sm	2.9860 (0.077)	3.4113 (0.093)	0.8585 (0.031)	0.0092 (0.002)	0.3151 (0.056)	0.8759 (0.034)	0.8054 (0.055)
ECAR_fall	2.8573 (0.011)	3.0014 (0.027)	0.6543 (0.040)	0.0005 (0.000)	0.0339 (0.006)	0.9231 (0.041)	0.9662 (0.021)

Table 5. Victoria Parameter Estimates with Standard Errors

	<u>Mean 1</u>	<u>Mean 2</u>	<u>Phi</u>	<u>Sigma 1</u>	<u>Sigma 2</u>	<u>P(1,1)</u>	<u>P(2,2)</u>
vic_wint	1.9834 (0.191)	3.7851 (0.176)	0.7600 (0.058)	0.2204 (0.026)		0.8008 (0.067)	0.9262 (0.025)
vic_spr	2.1208 (0.108)	3.4726 (0.079)	0.6703 (0.057)	0.1045 (0.011)		0.8596 (0.052)	0.9572 (0.017)
vic_sm	3.6529 (0.156)	4.5525 (0.157)	0.8948 (0.043)	0.0316 (0.004)		0.9497 (0.022)	0.8699 (0.067)
vic_fall	1.3810 (0.221)	3.3305 (0.158)	0.7957 (0.053)	0.1288 (0.017)		0.2541 (0.224)	0.9700 (0.016)
vic_wint	3.0315 (0.172)	3.9407 (0.082)	0.6353 (0.055)	0.8051 (0.125)	0.0569 (0.013)	0.9231 (0.033)	0.9034 (0.036)
vic_spr	2.1344 (0.118)	3.4776 (0.081)	0.6739 (0.058)	0.1132 (0.027)	0.1016 (0.013)	0.8613 (0.052)	0.9568 (0.018)
vic_sm	3.4120 (0.634)	3.5125 (0.659)	0.9650 (0.032)	0.0168 (0.003)	0.4038 (0.140)	0.9016 (0.041)	0.5371 (0.139)
vic_fall	1.8625 (0.292)	3.3192 (0.163)	0.8081 (0.052)	1.8645 (1.123)	0.1078 (0.015)	0.5635 (0.224)	0.9670 (0.017)

The calculated ergodic probabilities can be easier to interpret than the state transition probabilities presented in Tables 2 – 5. The ergodic probabilities for the two states are given in Table 6.

Table 6. Calculated Ergodic State Probabilities

	<u>One- Variance</u>		<u>Two-Variance</u>	
	<u>Low</u>	<u>High</u>	<u>Low</u>	<u>High</u>
	<u>Mean</u>	<u>Mean</u>	<u>Mean</u>	<u>Mean</u>
SERC_wint	0.97	0.03	0.93	0.07
SERC_spr	0.81	0.19	0.82	0.18
SERC_sm	0.98	0.02	0.59	0.41
SERC_fal	0.92	0.08	0.43	0.57
pjmE_wint	0.98	0.02	0.86	0.14
pjmE_spr	0.83	0.17	0.78	0.22
pjmE_sm	0.90	0.10	0.60	0.40
pjmE_fall	0.93	0.07	0.83	0.17
ECAR_wnt	0.87	0.13	0.84	0.16
ECAR_spr	0.94	0.06	0.85	0.15
ECAR_sm	0.93	0.07	0.61	0.39
ECAR_fall	0.83	0.17	0.31	0.69
vic_wint	0.27	0.73	0.56	0.44
vic_spr	0.23	0.77	0.24	0.76
vic_sm	0.72	0.28	0.82	0.18
vic_fall	0.04	0.96	0.07	0.93

For the US data, the calculated ergodic probability for the high mean state in the one variance model is generally lower than it is for the high mean state in the two variance specification. This is not surprising, as the probability of overlap between states generally increases when moving from the one to two variance specification: the means are closer together, and the larger of the two variances is greater than the single variance. The ergodic probabilities for the one variance model support the mean reverting with jumps model: prices are generally in a low state, but occasionally (between 2% and 19% of the time) jump to a significantly higher level. When the variance also changes between states, the time spent at the higher level increases to between 7% and 69%.<sup>8</sup>

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<sup>8</sup> Note that the two times in which the high state in the US markets becomes more likely in the two variance model, the corresponding mean is the lowest observed during the year. This suggests an interpretation similar to that employed in the analysis of the Victoria data. If these two models are excluded, the high state probabilities range from 7% to 41%.

The Victoria data must be interpreted differently because in the one variance model, the high state is more likely during three of the four seasons. In the two variance model, the winter ergodic probabilities switch, making the low state more likely, while the other three seasons remain the same. Thus, for the two variance model, the high mean state is more likely in the spring and fall, while the low state is more likely in the winter and summer. However, the high state means in the spring and fall are approximately the same as the low state means for the winter and summer. It appears that during the relatively high demand winter and summer, patterns are as observed in the US markets: generally low prices with jumps upward. During the off-peak seasons, prices are generally in the low range for the peak seasons, but then periodically fall to lower levels. This result fits with other analyses of the Victoria market, which suggests that at low demand times the market is hyper-competitive, driving prices below marginal costs of generation because of the presence of start-up costs. This is one reason why approximately 1000 Mw of capacity was withdrawn from the market in the spring of 1998 in a successful attempt to raise average electricity prices.

The estimated means for PJM East by season for both the one and two-variance specification are shown in Figure 2. The corresponding variances are shown in Figure 3. Figure 2 clearly shows how the one-variance means differ more than the means calculated under the two-variance specification, but how they follow the same seasonal pattern. Figure 3 shows that the variances follow a similar seasonal pattern as the means, with the difference in the variances in the two-variance specification showing that the high mean state is also the high variance state. That is, there are not two large variances in the two-variance specification, but a low variance and a much higher variance when mean prices are higher.

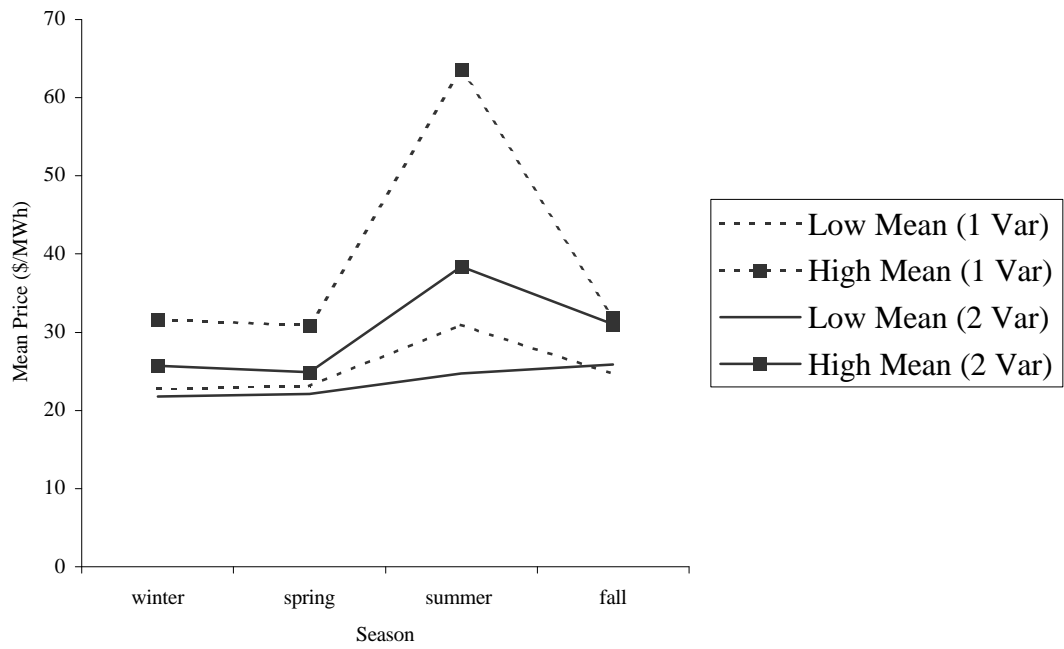


Figure 2. PJM East Mean Prices by Season

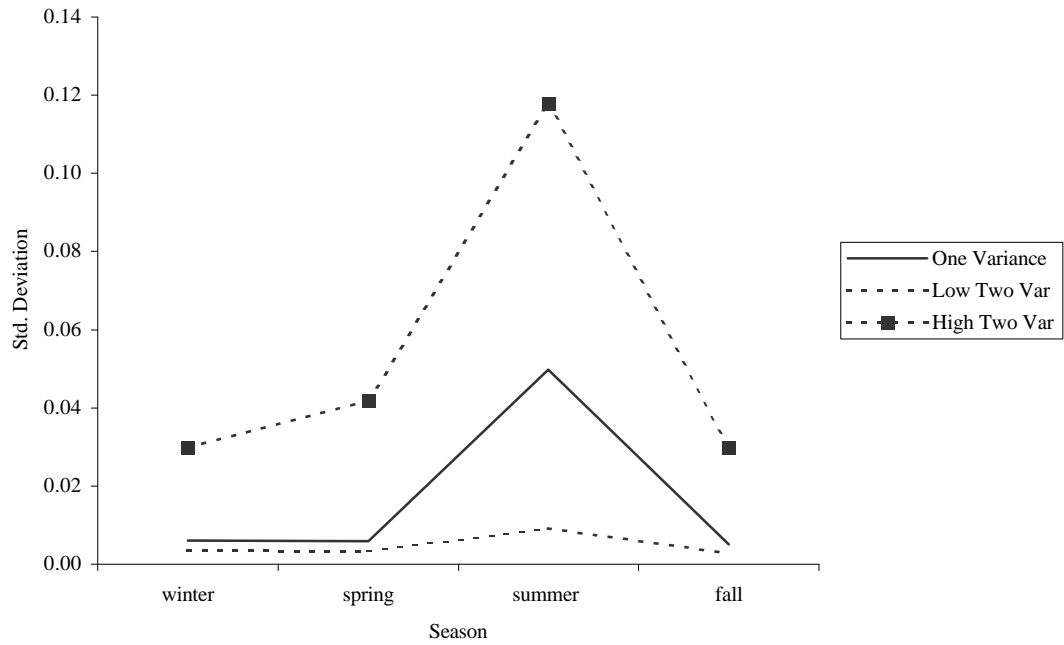


Figure 3. PJM East Variances by Season

## Hypothesis Tests

Three hypotheses are of interest: is the two-state AR(1) model warranted in place of a one state AR(1) model, is a Markov specification warranted rather than a mixtures model, and does a two variance model perform better than a one variance model?

As noted in Engel and Hamilton (1990), standard likelihood ratio (LR) tests are not valid for many hypotheses of interest for these models. For example, testing the null of a one-state AR(1) versus a two-mean Markov model might suggest comparing the two likelihoods. However, since the probability of state two is undefined under the null hypothesis that the means are equal, this is not an appropriate test. While Hansen (1992) has suggested a laborious test for testing this hypothesis, this paper chooses a simpler route.

Engel and Hamilton (1990) note that a Wald test can be used under a slightly modified null hypothesis. To test whether the means differ in a two-state two-variance model,<sup>9</sup> the null would be:

$$\begin{aligned} H_0^A: \quad & \mu_1 = \mu_2 \\ & \sigma_1 \neq \sigma_2 \\ & p_{11} \neq (1 - p_{22}) \end{aligned}$$

To test whether the markov property is significant, the null would be:

$$\begin{aligned} H_0^B: \quad & \mu_1 \neq \mu_2 \\ & \sigma_1 \neq \sigma_2 \end{aligned}$$

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<sup>9</sup> The Wald test is also valid for the one-variance model, with hypotheses modified accordingly.

$$p_{11} = (1 - p_{22})$$

To test whether the variances differ in a two state model, the null would be:

$$H_0^C: \mu_1 \neq \mu_2$$

$$\sigma_1 = \sigma_2$$

$$p_{11} \neq (1 - p_{22})$$

Wald statistics for each of the above three hypotheses are:

$$H_0^A: \frac{[\mathbf{m}_1 - \mathbf{m}_2]^2}{[\text{var}(\mathbf{m}_1) + \text{var}(\mathbf{m}_2) - 2\text{cov}(\mathbf{m}_1, \mathbf{m}_2)]} \approx \mathbf{c}_1^2$$

$$H_0^B: \frac{[p_{11} - (1 - p_{22})]^2}{[\text{var}(p_{11}) + \text{var}(p_{22}) + 2\text{cov}(p_{11}, p_{22})]} \approx \mathbf{c}_1^2$$

$$H_0^C: \frac{[\mathbf{s}_1 - \mathbf{s}_2]^2}{[\text{var}(\mathbf{s}_1) + \text{var}(\mathbf{s}_2) - 2\text{cov}(\mathbf{s}_1, \mathbf{s}_2)]} \approx \mathbf{c}_1^2$$

The 5% critical value for a  $\chi^2_1 = 3.84$ . Results for the above tests for each of the estimated models are shown in the tables below, with test statistics in bold for those cases where the null hypothesis could not be rejected.

Table 7. Statistics Tests For the SERC Region (One and Two Variance)

	$H_0': \mu_1 = \mu_2$	$H_0'': p_{11} = 1-p_{22}$	Wald Test $H_0''': \sigma_1 = \sigma_2$	LR Test $H_0''': \sigma_1 = \sigma_2$
SERC_wint	32.37	<b>1.41</b>		
SERC_spr	90.77	352.74		
SERC_sm	96.37	<b>1.37</b>		
SERC_fal	91.97	5.6		
SERC_wnt	32.38	15.89	<b>2.71</b>	6.34
SERC_spr	12.27	81.47	8.11	102.48
SERC_sm	<b>0.21</b>	10.51	20.48	38.98
SERC_fal	8.51	372.35	24.96	14.04

Table 8. Statistics Tests For the PJM East Region  
(One and Two Variance)

	$H_0': \mu_1 = \mu_2$	$H_0'': p_{11} = 1-p_{22}$	Wald Test $H_0''': \sigma_1 = \sigma_2$	LR Test $H_0''': \sigma_1 = \sigma_2$
pjmE_wint	60.48	<b>0.96</b>		
pjmE_spr	76.16	40.31		
pjmE_sm	24.83	17.59		
pjmE_fall	82.68	<b>3.14</b>		
pjmE_wint	8.72	18.37	5.58	9.96
pjmE_spr	3.95	74.24	6.56	34.44
PJME_sm	61.57	70.00	19.56	42.92
pjmE_fall	36.12	11.14	6.72	12.72

Table 9. Statistics Tests For the ECAR Region (One and Two Variance)

	$H_0^{\prime}: \mu_1 = \mu_2$	$H_0^{\prime\prime}: p_{11} = 1 - p_{22}$	Wald Test $H_0^{\prime\prime\prime}: \sigma_1 = \sigma_2$	LR Test $H_0^{\prime\prime\prime}: \sigma_1 = \sigma_2$
ECAR_wnt	128.15	68.60		
ECAR_spr	238.74	5.45		
ECAR_sm	98.65	39.79		
ECAR_fall	164.39	10.30		
ECAR_wint	32.99	96.18	11.30	39.64
ECAR_spr	58.29	59.52	10.72	91.36
ECAR_sm	103.36	102.67	30.34	97.18
ECAR_fall	46.37	341.50	35.56	40.02

Table 10. Statistics Tests For the Victoria (One and Two Variance)

	$H_0^{\prime}: \mu_1 = \mu_2$	$H_0^{\prime\prime}: p_{11} = 1 - p_{22}$	Wald Test $H_0^{\prime\prime\prime}: \sigma_1 = \sigma_2$	LR Test $H_0^{\prime\prime\prime}: \sigma_1 = \sigma_2$
vic_wint	198.00	103.73		
vic_spr	191.95	216.34		
vic_sm	154.99	128.98		
vic_fall	146.63	<b>0.99</b>		
vic_wint	32.50	235.64	35.35	23.86
vic_spr	170.32	219.18	<b>0.15</b>	<b>0.14</b>
vic_sm	<b>1.90</b>	8.19	7.61	17.16
vic_fall	40.63	5.51	<b>2.45</b>	<b>0.68</b>

Wald tests of  $H_0^A$  ( $\mu_1 = \mu_2$ ) are all greater than the critical value at the 5% level of significance ( $\chi^2_{1, .05} = 3.84$ ) for the single variance model. 14 of 16 tests on the two variance model also reject the null. This provides strong evidence that the two mean model is an appropriate specification.

Note that many of the Wald statistics are quite large (well over 100). This is typical of the Wald statistic.

Wald tests of  $H_0^B$  ( $p_{11} = 1 - p_{22}$ ) are also significant for all of the two-variance models, though 5 of 16 tests fail to reject the null for the one variance models, giving general support to the Markov model. It is not surprising that the two variance model more strongly supports the Markov specification, given that there is generally greater probability of overlap in the two states for the two variance model than for the one variance model.

Wald tests of  $H_0^C$  ( $\sigma_1 = \sigma_2$ ) were also generally significant. The null is rejected for 13 of 16 tests, providing strong support for the model with variances that differ by state. These Wald tests can be compared with the likelihood ratio (LR) tests, and the null is rejected one additional time using the LR test. Interestingly, the LR statistics, which are also distributed as  $\chi^2$ , are generally larger than the associated Wald statistic, in contrast to the findings of Engel and Hamilton (1990). This is reassuring, because Engel and Hamilton had expressed concern about the Wald test having higher values. The LR statistic is more powerful than the Wald statistic in our case.

### **Calculating Option Values**

An important use of the electricity price path characterization presented above is for the determination of option values. Option values are useful not only for electricity traders, but also for electricity generators. For example, a generator may wish to decide if it is likely to be profitable to operate during the following quarter if there are fixed costs which must be incurred, given estimates of the price path parameters in the next quarter. The fixed costs might include contracting with workers, customer charges for gas or coal contracts, and other administrative or overhead costs. This decision is relevant for a peaking unit, which operates primarily in the summer and winter, but has a generally low capacity factor during the shoulder seasons.

Ownership of an electricity generating plant can be viewed as an option to sell electricity at the prevailing market spot price during any hour in the future (see Hsu and Quan 1998) corresponding to a series of options to sell at a specified date. Through operating the generator, the owner is able to produce electricity at the marginal cost of production (equivalent to buying the commodity at fixed price  $K$ ) and to sell electricity at the spot rate. The marginal production costs (the 'strike price') would be incurred only if the option to generate were exercised. Thus the payoff for the generator for each unit of capacity would be  $\max[(P(T)-K, 0)]$ , where  $K$  is the marginal cost of production, and  $T$  is a specified day in the future. This suggests that a generator's availability to operate can be valued using traditional option theory and techniques.

Lo and Wang (1995) show how the traditional Black-Scholes model for valuation of standard European options can be used for valuing options when underlying prices have a drift term which is a function of both price and time.<sup>10</sup> They provide formulae for modifying the volatility parameter when prices follow alternative stochastic prices paths. A special case of this formulation is a mean reverting price path which can be adapted to valuing options when the underlying price path follows a two-regime mean reverting model, as presented here. The value calculated here is for a call option, to sell electricity at price  $P(T)$  for marginal cost  $K$ . This is the fixed cost the owner of a generator would be willing to incur to have the option to generate at time  $T$ .

The Black Scholes valuation for a call option at time  $t$  is:

$$C(P_t, t; K, T, r, \mathbf{s}) = P_t \Phi(d_1) - Ke^{-r(T-t)} \Phi(d_2)$$

where

$$d_1 \equiv \frac{\ln(P_t / K) + (r + 0.5\mathbf{s}^2)(T-t)}{\mathbf{s}\sqrt{T-t}}, \quad d_2 \equiv d_1 - \mathbf{s}\sqrt{T-t}$$

where  $P_t$  is the spot price at time  $t$ ,  $K$  is the strike price,  $T$  is the future date of exercising the option,  $r$  is the risk free interest rate,  $\mathbf{s}$  is the estimated volatility, and  $\Phi$  is the standard normal cumulative distribution function. Lo and Wang (1995) show that the Black Scholes model provides risk neutral valuation of a call option if the estimated volatility is adjusted to reflect the true underlying distribution. In the case of a mean reverting price path, the adjustment is:

$$\mathbf{s}^2 = \frac{\hat{\sigma}^2(2\ln \mathbf{f})}{(\mathbf{f}^2 - 1)}$$

where  $\hat{\sigma}^2$  is the estimated variance from a mean reverting model in logged prices.  $\phi$  is the autoregressive parameter from the AR(1) model.<sup>11</sup>

The modified Black Scholes model was applied using the estimated parameters for PJM East as an example (given in Table 3). The option value is for a typical MWh of electricity to be sold  $T-t$  days in the future. The regime switching mechanism in the model allows an option to be calculated as the outcome of a simple lottery (Varian 1991). The option value in each state can be calculated using the Black-Scholes model with the variance modified using the formula suggested by Lo and Wang. The buyer of an option would thus be buying a weighted average of two options whose values are determined by parameters for state one with probability  $p$  and state two with probability  $(1-p)$ . In other words, the option value for each state was calculated

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<sup>10</sup> Bjerk Sund and Ekern (1995) present an alternative way to derive the value of a European call option if the price  $X(T)$  is mean reverting, in a shipping context.

<sup>11</sup> This is a special case of the bivariate trending O-U process presented by Lo and Wang, with  $\sigma_x=0$ .

independently, and the weighted average of the two values is the reported option value at time  $T$ . The strike price can be interpreted as the marginal production cost of a generator.

Option values were calculated for a one-state model, and for the two-state one-variance and two-state two-variance models. The option value in each state for a two-state model was evaluated at the expected underlying electricity price in that state. The expected price in the one state model is the weighted average of the two-state one-variance model mean prices.

The weighting probabilities are calculated in two different ways. One weighting scheme uses the ergodic probabilities  $\pi = P[S=1]$ . The second method assumes that it is known with certainty that at  $t=0$ , the unobserved state is state two (the high mean state). This allows the weighting probabilities to incorporate the information contained in the transition probability matrix. This produces dramatically different option values in the short term, but values are practically indistinguishable after thirty time periods. The probability of being in state one at time  $T$ , conditional on information available at  $t=0$ , is calculated as:

$$P_0[S_t=1] = \pi + \lambda^T(\pi_0 - \pi)$$

where  $\pi_0 = P_0[S_0=1]$ ,  $\pi$  is the state one ergodic probability and  $\lambda \equiv 1+p+q$ .

Some caveats should be made. The proposed approach is clearly a simplified model of a call option when prices follow a two-state AR(1) process. It only evaluates values at the mean price in each state, ignoring short run information available from prices observed at  $t$ . Since a daily load weighted average of electricity prices is being used, the option value is for a typical unit of electricity, that is, one randomly sold during the day. For an individual generator, it might be more appropriate to use an hourly weighted average price. And this neglects short-run generation

issues like start-up costs and ramping rates. A monte carlo model such as that used by Tseng and Barz (1998) would be required to incorporate additional realism.

Table 11. Calculated Option Values (in \$) in PJM East for a Typical MWh T-t Days in the Future<sup>12</sup>

	Strike		Time Until Exercise			
	Price		T-t=1	T-t=7	T-t=30	T-t=91
Ergodic Probability	\$20	1-State	14.213	14.229	14.412	15.514
		1 Variance	14.236	14.253	14.539	15.773
		2 Variance	10.290	10.362	11.206	13.073
	\$30	1-State	4.220	4.716	6.429	9.223
		1 Variance	4.508	5.467	7.214	9.816
		2 Variance	3.440	4.140	5.975	8.600
	\$40	1-State	0.002	0.410	2.224	5.359
		1 Variance	2.360	2.443	3.522	6.149
		2 Variance	0.585	1.978	4.339	7.434
	\$50	1-State	0.000	0.010	0.662	3.117
		1 Variance	1.353	1.377	1.859	3.942
		2 Variance	0.025	0.869	3.199	6.542
Conditional Probability with $P_0[S_0=2]=1$	\$20	1-State	14.213	14.229	14.412	15.514
		1 Variance	32.422	15.314	14.539	15.773
		2 Variance	16.252	11.286	11.207	13.073
	\$30	1-State	4.220	4.716	6.429	9.223
		1 Variance	22.526	6.484	7.214	9.816
		2 Variance	7.145	4.825	5.976	8.600
	\$40	1-State	0.002	0.410	2.224	5.359
		1 Variance	15.488	3.208	3.522	6.149
		2 Variance	1.215	2.305	4.339	7.434
	\$50	1-State	0.000	0.010	0.662	3.117
		1 Variance	8.881	1.824	1.859	3.942
		2 Variance	0.052	1.012	3.200	6.542

<sup>12</sup>  $r = \ln(1.05)/364$ . The strike price  $K$  varies as shown, with model parameters as estimated for PJM East in the earlier table. Option values evaluated at the mean price level in each state.

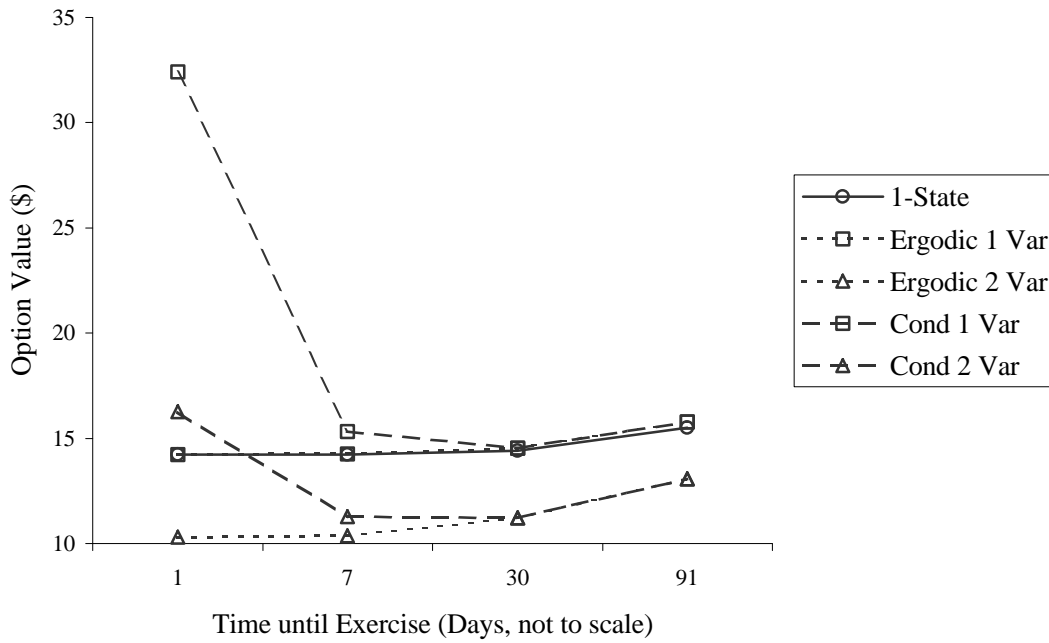


Figure 4. PJM East Option Values for K = \$20

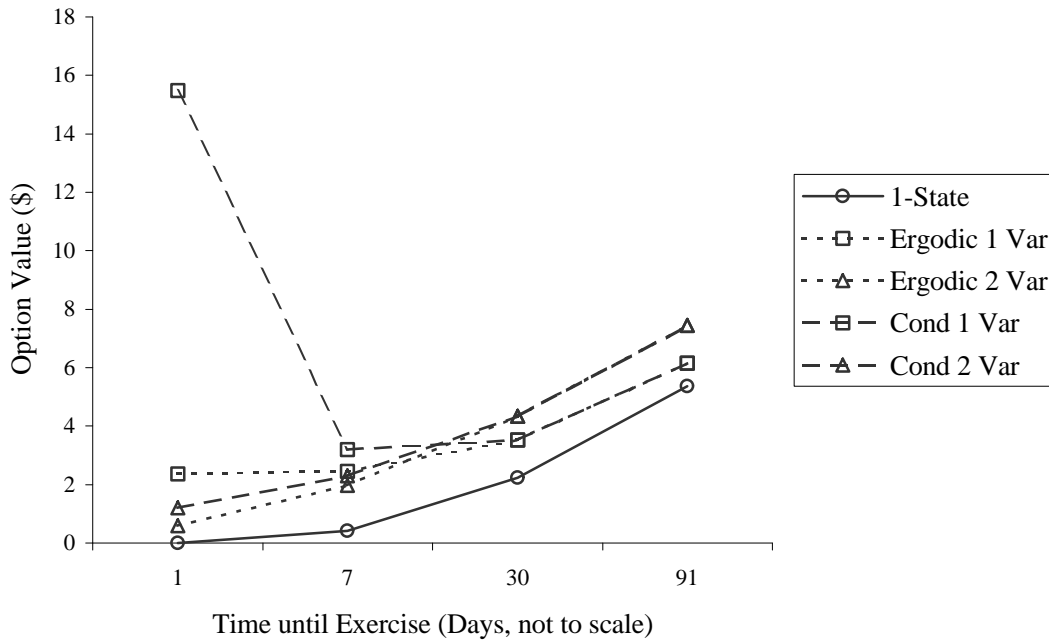


Figure 5. PJM East Option Values for K = \$40

The values in Table 11 show the effect of different model specifications on option values.

Figures 4 and 5 isolate the cases where K=\$20 and K=\$40. Each option grows in value over

time, as prices have the opportunity to move away from their mean value and reflect the long-term price distribution. The one-state model always produces lower values than the two-state, one-variance (from now on the one variance model) model. This difference grows with the strike price. This is to be expected because the high state is still in the money even for high strike prices. Over short time horizons, the two-state two-variance (the two variance model) model produces lower values than the one variance model because of lower expected prices in each state. As the length of time until the option is exercised grows, however, the two-variance model produces higher values for high strike prices. This is due to the greater variance in the high state of the two-variance model. Over time, this means a more diffuse distribution and higher option value.<sup>13</sup> It is important to recognize that while the respective means in the two-variance model are lower than in the one-variance model, the resulting option values do not necessarily follow this pattern. In addition, the high state is more likely in the two-variance specification, thus raising the calculated option value. This is well illustrated in Figures 4 and 5. In Figure 4, with a strike price of  $K=\$20$ , the two variance model with ergodic probabilities always produces the lowest option values. When the strike price rises to  $\$40$  in Figure 5, the two variance model produces the highest values at the longer time horizons.

The conditional probability option values are strictly larger for the two-state models in the first two time periods,  $T-t = 1$  and  $T-t = 7$ , when compared to the ergodic probability values. This is as expected, because the probability of observing the high mean state is increased with the addition of the certain knowledge that the high mean state is observed at time  $t=0$ . The table also shows that the importance of this knowledge decreases rapidly over time. By  $T-t=30$ , the value of this information at  $t=0$  is essentially zero. The two-variance model approaches the ergodic values more slowly than does the one-variance model because state two is expected to last longer under

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<sup>13</sup> This can be seen by analyzing vega, the first derivative of the Black Scholes model with respect to  $\sigma$ . This shows that the Black Scholes option value is strictly increasing with a rise in variance. At long time periods, this can outweigh the value of a high expected price.

the two-variance model, thus the value of the information decreases less rapidly than under the one-variance model.

There are important results from this exercise. The first is that the two-state models show the largest difference in potential value at high strike prices (or production costs) when compared to the one-state model. This reflects the ability to more accurately capture high price realizations. Second, while the one-variance model might seem to predict higher values because of higher mean prices in each state, the two-variance model can produce higher values at high strike prices. In the short term, the one-variance model can produce dramatically higher values. Care must be taken when choosing the time to maturity for option valuation. Third, knowledge about the unobservable state variable can have a great effect on short term values, but the importance of that information decays quickly over time.

## **Conclusions**

Fitting sparsely parameterized models to commodity price data is a useful exercise. Not only does it provide information about price movements, but it is also useful in asset valuation and derivatives pricing. This paper examines a new version of such a model which is specific to electricity prices. The model is more general than the models generally used in energy pricing (Pilopovic 1997), and it accommodates the observed characteristics of electricity price data well. The model relies only on observed electricity spot prices, allowing the use of traditional option models and Monte Carlo techniques for derivative and asset pricing.

The results presented in this paper suggest that electricity dynamics are more complicated than standard spot price models allow. In competitive markets, electricity prices are characterized by stretches of predictable low price behavior interspersed with sharp price spikes. The estimated models strongly support the existence of different means and variances for two mean-reverting

regimes. The switching from one regime to the other is governed by a state specific Markov process, implying that prices can stay in either regime for more than one period. This specification includes the standard model with random jumps (binomial) as a special case that is not supported by the data in our examples.

The two variance model is also preferred to a simple mean switching specification. This supports the analysis of Mount (1999) that electricity supply curves are kinked, with a flat portion which produces stable prices, and a steep portion which results in more volatile prices. The two variance model is also preferred to a one variance model in most cases, using both Wald and LR tests. Note that because prices are estimated in logs, the difference in the estimated means seems smaller than if the difference is in prices. The high option values produced by the two variance specification show the importance of the high variance specification.

The Markov character of the model is preferred over a simple mixture distribution for most data series, both for the one and two variance models. This dynamic of two distinct, and persistent, regimes is quite different than a mixture distribution, where a price spike is not likely to be sustained. This has implications for derivatives pricing, generator entry and exit decisions, asset valuation models, and regulatory decisionmaking. The Markov nature of daily average spot prices suggests that within-day prices might also exhibit such characteristics.

The calculated option values implied by each price process varied in important ways. While in the short term, the one variance model produced higher values at all strike prices, as the time lag increased, the two-variance model increased in value more rapidly, and produced greater values at high strike prices. The importance of knowledge about the unobserved state variable is large in the near term, but declines rapidly with time, even when regimes are relatively persistent. The option values show the importance of using the two state specification if that is indeed representative of the true price path.

Further research includes either identifying seasons or developing a model with time varying coefficients. The results presented here can help to specify the time varying nature of such a model. Developing and parameterizing a model of daily spot prices with time varying parameters and a Markov component is also a potentially useful avenue and the subject of ongoing research. Also, incorporating an ARCH specification which allowed a conditional variance would be a potentially fruitful addition.

## APPENDIX A

The daily US spot prices on electricity are from the newsletter "Power Markets Week." There is a sample copy with details on the calculations at <http://www.mhenergy.com/demos/index.html>.

NOTE: Index prices and ranges are for daily prescheduled, on-peak (16 hours) electricity in \$/MWh. Although specific terms vary from market to market, the indexes are based on financially firm or physically firm power where those products are commonly traded, including western trading hubs and the Into market hubs in the East. In ERCOT, the index is based solely on next-day "Unplanned B (UB)." Indexes are based on prices of actual transactions done by both buyers and sellers. In the East, Midwest, and South, the weekly on-peak indexes represent an average daily price for the preceding week, Monday through Friday. Each weekday is given equal weight to determine the weekly index price. The trading week has changed in the West, with power now scheduled on Thursday for both Friday and Saturday delivery. For COB/NOB, Mid-Columbia, Palo Verde, and Four Corners, the weekly index represents the average daily price for six days-Monday through the following Saturday. The index prices are Power Markets Week's assessment of where the bulk of dealmaking occurred and are based on the volume-weighted average.

Volumes represent a daily average for the week for on-peak power on a MW per hour basis. The Into TVA, Into Ameren, and Into Entergy indexes are stand-alone indexes that are not included in the regional indexes. The other market hub indexes, Into Cinergy and Into ComEd, are also included in their respective regions, ECAR and MAIN; those indexes include Into Cinergy and Into ComEd deals as well as deals in those regions. For more information, call Lynn Garner, markets editor, at 202-383-2191. Monthly indexes are volume-weighted averages of on-peak power in \$/MWh for monthly contracts. On-peak at COB and PV is defined as Monday through Saturday.

APPENDIX B

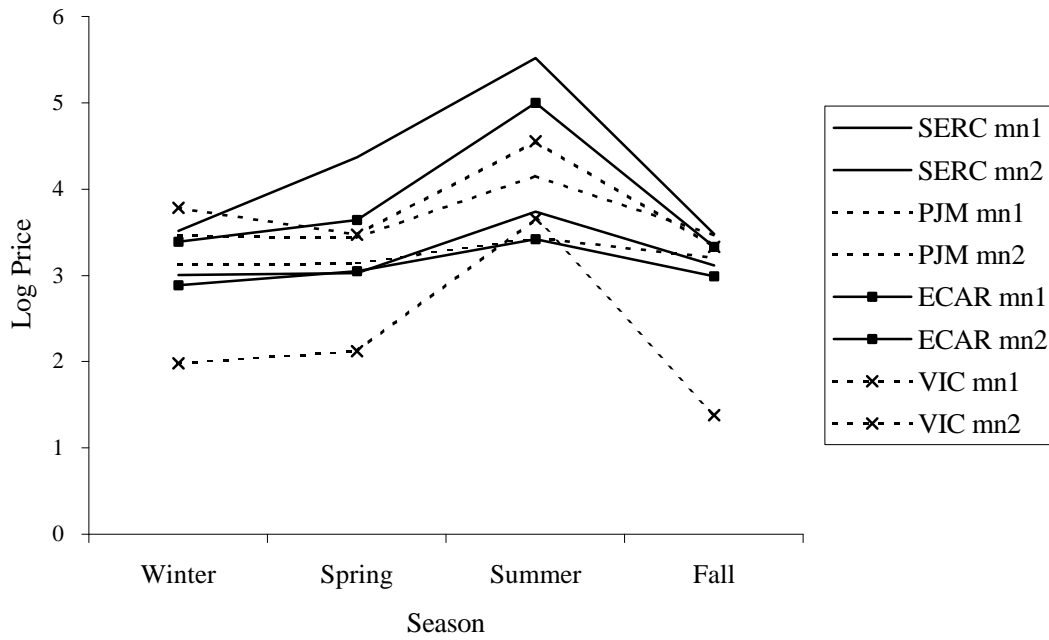


Figure B.1. Means for One Variance Model.

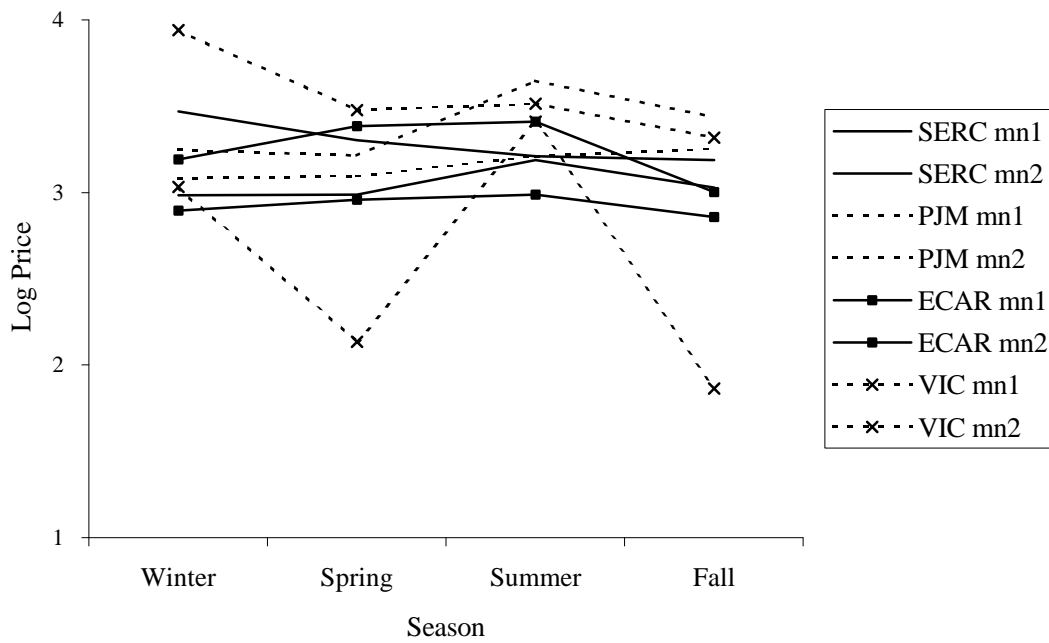


Figure B.2. Means for Two Variance Model.

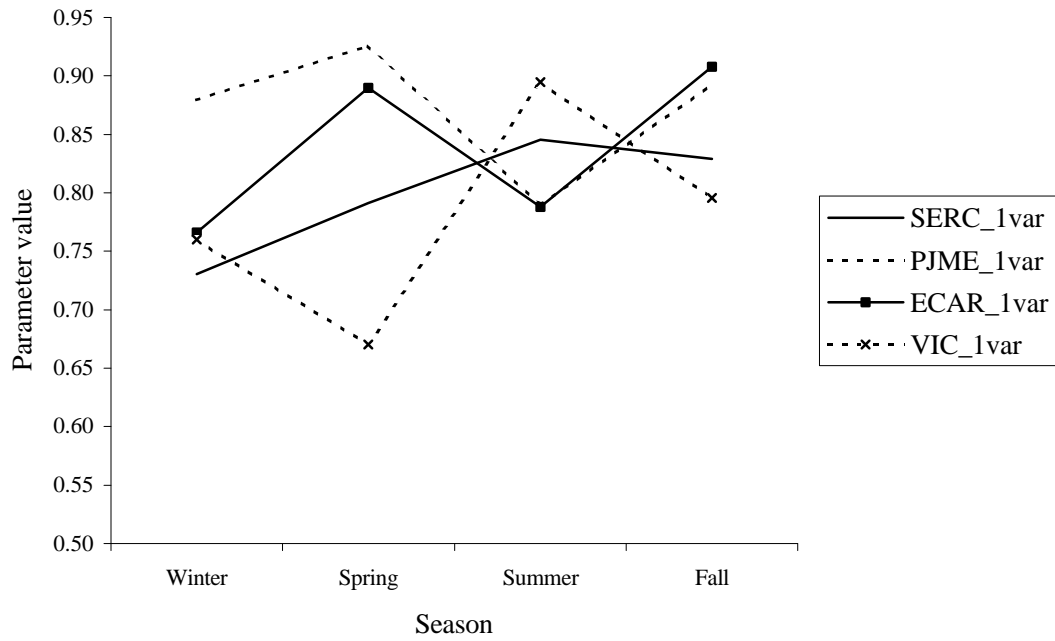


Figure B.3. Autoregressive Parameters for One Variance Model.

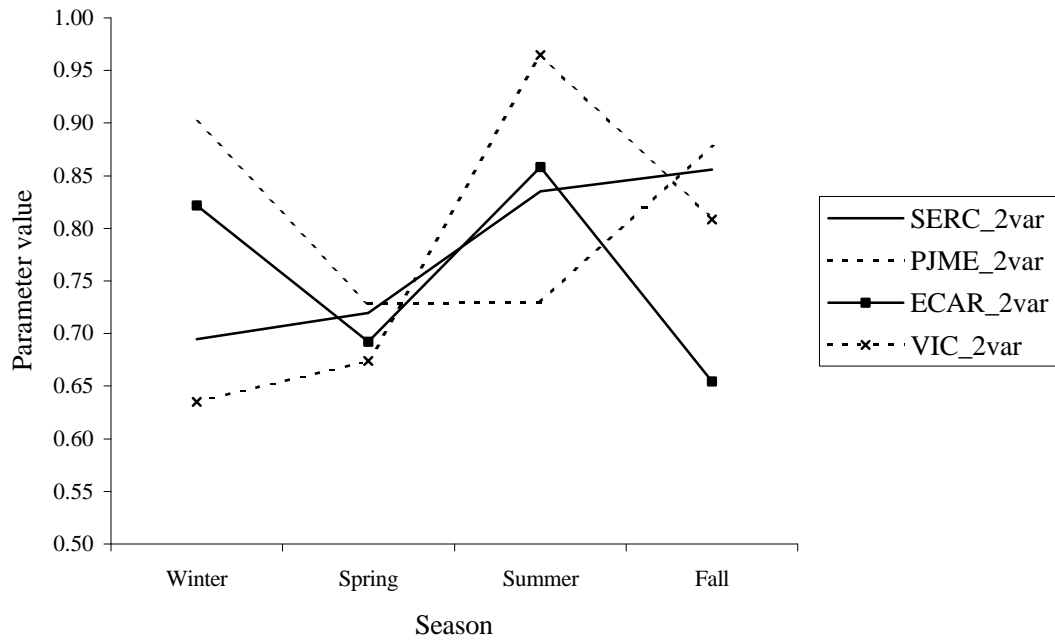


Figure B.4. Autoregressive Parameters for Two Variance Model.

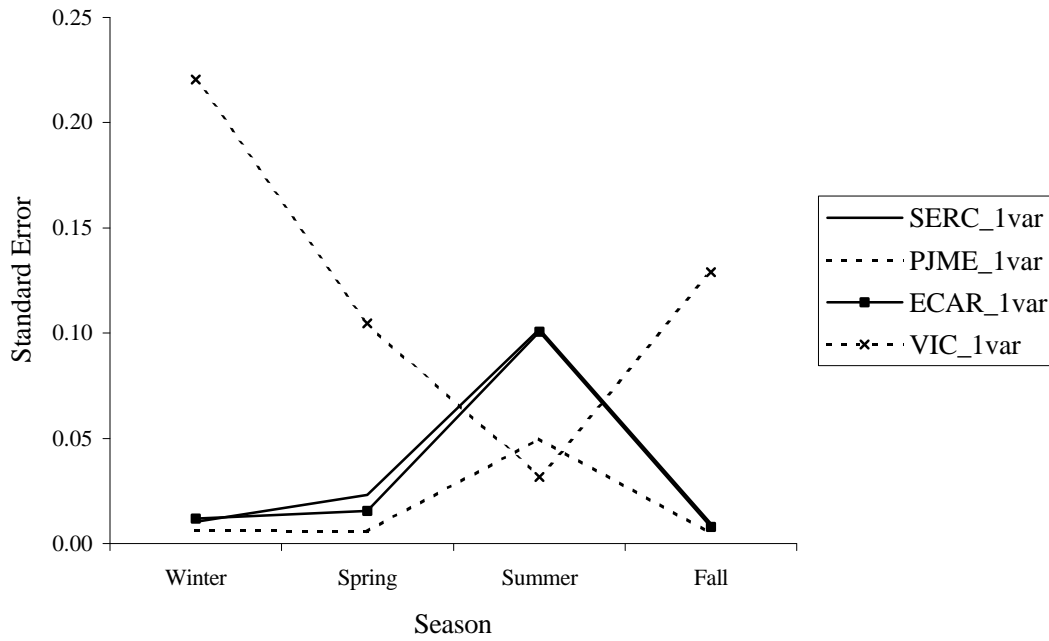


Figure B.5. Standard Error for One Variance Model.

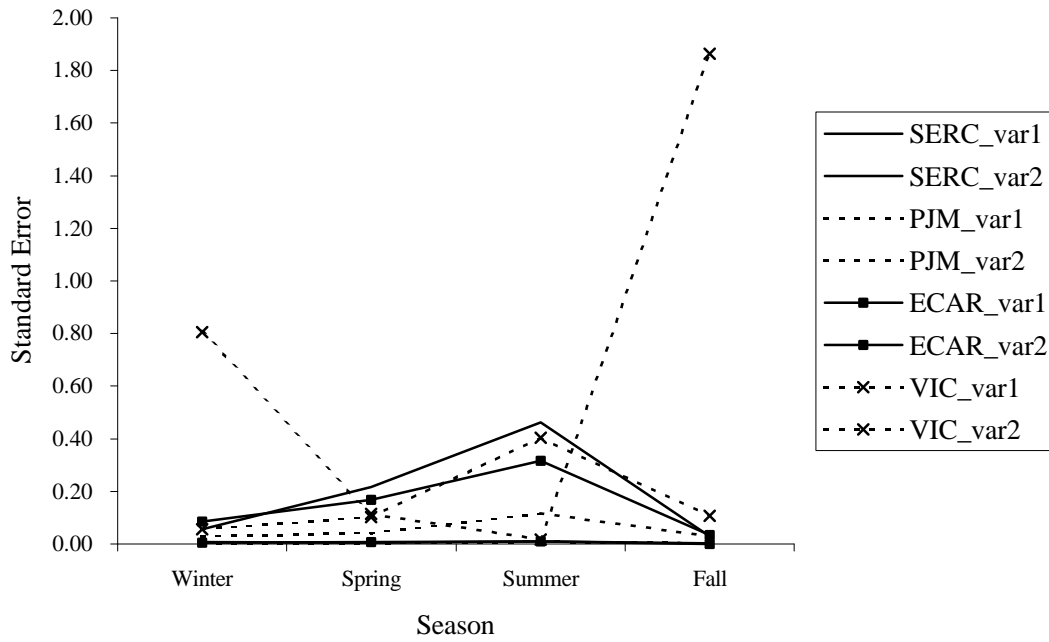


Figure B.6. Standard Error for Two Variance Model.

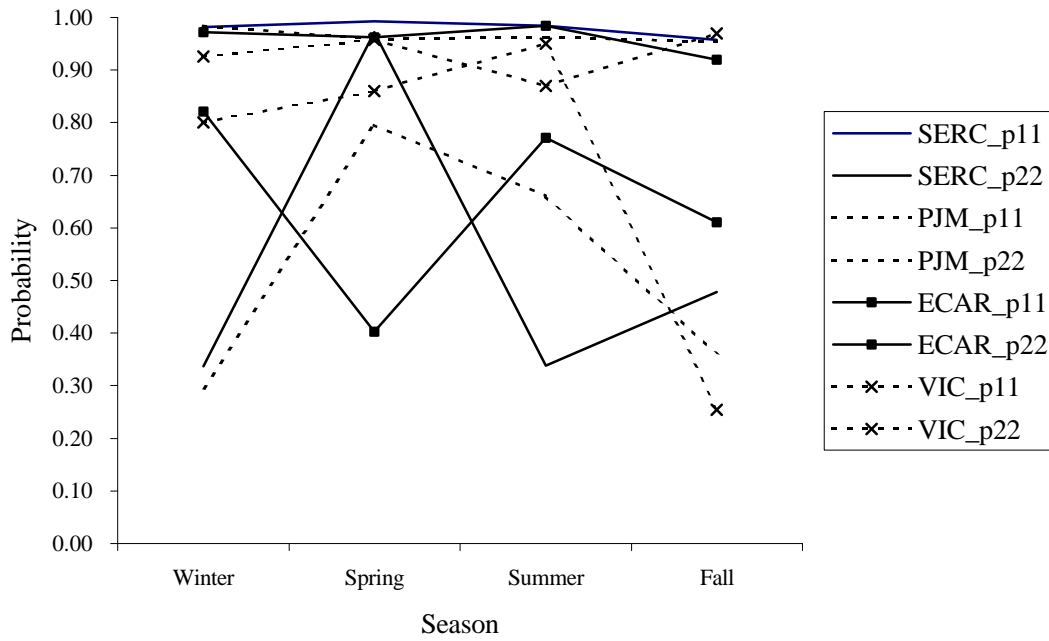


Figure B.7.  $p(p_{11})$  and  $q(p_{22})$  for One Variance Model.

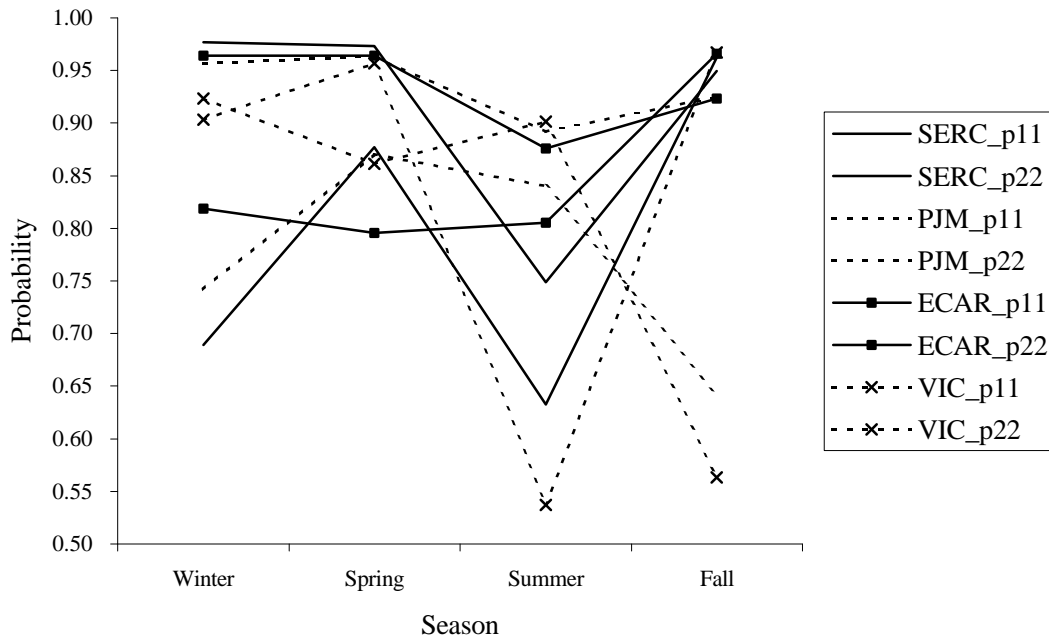


Figure B.8.  $p(p_{11})$  and  $q(p_{22})$  for Two Variance Model.

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