Short-Term Generation Asset Valuation

Chung-Li Tseng, Graydon Barz
Department of Civil Engineering, University of Maryland, College Park, MD 20742, USA
Department of EES & OR, Stanford University, Stanford, CA 94305, USA
chungli@eng.umd.edu, gbarz@leland.stanford.edu

Abstract

In this paper we present a method for valuing a power plant over a short-term period using Monte Carlo simulation. The power plant valuation problem is formulated as a multi-stage stochastic problem. We assume there are hourly markets for both electricity and the fuel used by the generator, and their prices follow some Ito processes. At each hour, the power plant operator must decide to run or not to run the unit so as to maximize expected profit. A certain lead time for commitment decision is necessary to start up a unit. The commitment decision, once made, is subject to physical constraints such as minimum uptime and downtime constraints. The generator's startup cost is also taken into account in our model. In this paper, the Monte Carlo method is employed not only in forward-moving simulation, but also backward-moving recursion of dynamic programming. We demonstrate through numerical tests how the physical constraints affect a power plant value.

1. Introduction

With deregulation of the electricity industry a global trend, utilities and power generators must adjust the new risks of volatile spot prices, which are accompanying the competitive marketplace. Because they are no longer able to rely on embedded cost recovery regulation, they must fundamentally change the way they view power plant operation. For example, the deterministic and cost-based unit commitment problem (e.g. [10]) that schedules power plants to satisfy demand should be replaced, in our opinion, by an optimization problem which, at least, is price-based and takes account of price stochastics. Solving such an optimization problem not only yields optimal commitment decision, on the other hand, reveals the power plant value over the operating period.

Recently the power plant valuation problem has been tackled using financial option theory [2, 4, 5]. A power plant, associated with its *heat rate*, converts a particular fuel to electricity. This conversion involves two commodities with different market prices. Therefore, owning a power plant can be regarded as holding call options of *spark spreads*, defined as the electricity price less the product of the heat rate associated the generator and the fuel price. When the electricity price is high but the fuel price is low such that their ratio is greater than the unit's heat rate, the power plant should run to capture profit due to the price spread, and vice versa. (Since the power plant profits more, when the spread is greater, it is a call option in this sense.) Similar price spread concepts have also been applied to other industries such as oil refining (e.g. [8]).

While using option theory to value a power plant is a novel approach, such an approach overlooks the operational constraints of a power plant. These constraints include the startup time and the minimum uptime and downtime constraints. Using financial option theory to value a power plant implicitly assumes (i) no startup time, i.e., a unit can be started up immediately when favorable prices are observed; (ii) no minimum up/down time constraints, i.e., an on-line unit can be turned down whenever the prices become unfavorable. These assumptions always result in a nonnegative payoff for operating a power plant, suggesting the operator faces no risk of loss. This, unfortunately, is not the case. We shall show in this paper that big risks may be overlooked if these physical constraints are not considered.

In this paper, we formulate the power plant valuation problem as a multi-stage stochastic problem. Over a short-term period, the operator must decide when to run the generator so as to maximize expected profit. The commitment decision must be made before the uncertain prices are revealed. Once the commitment decision is made, plant operation is subject to physical constraints such as minimum uptime and downtime constraints. That is, an on-line unit cannot be turned back down even if market prices become unfavorable before the minimum uptime constraint is fulfilled. Similarly, an off-line unit cannot be turned back on before the minimum downtime constraint is fulfilled. Moreover, in our model, a startup cost associated with turning on a unit is also taken into account.

The major uncertainties considered in this paper are the prices for electricity and the fuel consumed by the generator. We assume there are hourly markets for both electricity and fuel and their prices follow some Ito processes. Although the valuation method presented in this paper can be applied to general Ito price processes, we focus on the ones characterized by mean reversion and lognormally distributed, seasonal prices. We use Monte Carlo simulation to solve this multi-stage stochastic problem. The Monte Carlo method is employed not only in forward-moving simulation, but also backward-moving recursion of dynamic programming.

This paper is organized as follows. In Section 2, physical constraints of a power plant are described and formulated. In Section 3, we review the approach for valuing a power plant using financial option theory. We formulate the power plant valuation problem as a multi-stage stochastic problem and propose solution procedure using simulation in Section 4. In Section 5, we present numerical results. This paper concludes in Section 6.

2. Physical constraints for a power plant

In this paper we focus mainly on fossil or natural gas-fueled steam units. For steam units the dynamics of unit operation are relatively complex. For these units the time required to start up the generator and the cost associated with startup depend on how long the units have been down. This occurs because the water in its boiler must be heated before generation of power can occur. The longer the generator is down, the more heat is lost from its boiler and the longer the time and the greater the expense to reheat the water. Therefore the operation cost of a power unit calls for two cost terms. The first is related to the production of power and depends directly on the amount of fuel consumed. The second term captures the startup costs which vary with the temperature of the boiler.

A thermal generation unit cannot switch between the on-line mode and the off-line mode at arbitrary frequency, due to both the non-zero response time of the unit and the damaging effects of fatigue. In other words, once a thermal unit is shut down (or started up), it is required to stay off-line (on-line) for a minimum period, known as the minimum down- (up-) time, before it can be started up (shut down) again. On the other hand, committing all units all the time would not be feasible either. Generation units become unavailable from time to time, due to planned outages (e.g. for maintenance purpose) or forced outages (e.g. as a result of component failure.)

In the development, we first introduce the following standard notation. Additional symbols will be introduced when necessary.

t : index for time ($t = 0, \cdots, T$)

u_t : zero-one decision variable indicating whether the unit is up or down in time period t

x_t : state variable indicating the length of time that the unit has been up or down in time period t

t^{on} the minimum number of periods the unit must remain on after it has been turned on

t^{off} : the minimum number of periods the unit must remain off after it has been turned off

t^{cold} : the minimum number of periods required to fully cool down the boiler of a unit after it has been turned off

q_t : decision variable indicating the amount of power the unit is generating in time period t

$q^{\min}$: minimum rated capacity of the unit

$q^{\max}$: maximum rated capacity of the unit

p_t^E : electricity price ($/MWh) in time period t

p_t^F : fuel price ($/MMBtu) in time period t

$C(q_t, p_t^F)$: fuel cost for operating the unit at output level q_t in time period t when the fuel price is p_t^F.

$S(x_{t-1}, u_t, u_{t-1})$: startup cost associated with turning on the unit in time period t

In this paper, the unit of time period is in hours.

2.1. Modeling the cost functions

The generating cost ($) of a thermal generation unit typically includes fuel costs and the startup costs. The *incremental heat rate* (MMBtu/MWh) of a unit is most often modeled as a linear function of the power output (MW) of the unit [11]. Equivalently, the fuel costs are modeled as a quadratic function with the following form:

$$C(q_t, p_t^F) = (a_0 + a_1 q_t + a_2 q_t^2) p_t^F \quad (1)$$
$$\equiv H(q_t) p_t^F, \quad (2)$$

where $H(\cdot)$ captures the input-output characteristic of a generating unit. The input to the unit is in terms of the heat energy requirement (MMBtu/h) and the output is the net electrical output (MW). Each coefficient a_j $(j = 1, 2, 3)$ of $H(\cdot)$ is taken to be nonnegative. In general, $a_2 > 0$, so that the cost function is convex; $a_1 > 0$ since a_1 is the fixed term of the incremental heat rate; and $a_0 > 0$ because $a_0 p_t^F$ captures the no-load cost. In the approach of using financial options to value a power plant, as mentioned in the introduction of this paper, a power plant's cost function is normally simplified by setting $a_2 = a_0 = 0$, and a_1 is addressed as the (constant) heat rate the generator.

As aforementioned, the startup costs vary with the temperature of the boiler. In practice, it is assumed that the boiler cools at an exponential rate inversely proportional to the cooling constant ρ_i. The function is given by

$$S(x_{t-1}, u_t, u_{t-1})$$
$$= [b_1(1 - \exp(-\tfrac{t(x_{t-1})}{\rho})) + b_2] u_t (1 - u_{t-1}) \quad (3)$$
$$\equiv S(x_{t-1}) u_t (1 - u_{t-1}) \quad (4)$$

where $t(x_t) = \max(0, -x_t)$ is the amount of time the unit has been down; b_1 represents the cold start fuel cost for unit i, and b_2 combines the labor costs plus the fixed operating and maintenance expenses of the plant amortized over the unit. To limit the size of the state space, we assume that $\exp(-t(x_{t-1})/\rho)$ in (4) becomes negligible when $t(x_t) \geq t^{\text{cold}}$.

2.2. Deterministic formulation

Assuming that the future prices for electricity and fuel are fully and perfectly known, we present the deterministic formulation of the generation asset valuation problem over a short term period.
(P)

$$J^* = \max_{\mathbf{u},\mathbf{x},\mathbf{q}} \sum_{t=1}^{T} (p_t^E q_t - C(q_t, p_t^F) - S(x_{t-1})(1 - u_{t-1})) u_t \quad (5)$$

subject to the following physical constraints:
State transition constraints

$$x_t = \begin{cases} \min(t^{\text{on}}, \max(x_{t-1}, 0) + 1), & \text{if } u_t = 1, \\ \max(t^{\text{cold}}, \min(x_{t-1}, 0) - 1), & \text{if } u_t = 0, \end{cases} \quad (6)$$

Minimum up/down time constraints

$$u_t = \begin{cases} 1, & \text{if } 1 \leq x_{t-1} < t^{\text{on}}, \\ 0, & \text{if } -1 \geq x_{t-1} > -t^{\text{off}}, \\ 0 \text{ or } 1, & \text{otherwise}, \end{cases} \quad (7)$$

Unit capacity constraints

$$q^{\min} \leq q_t \leq q^{\max}, \; t = 1, \cdots, T, \quad (8)$$

Initial conditions on u_t and x_t at $t = 0$. That is, (u_0, x_0, q_0) equals to some initial schedule, say $(\tilde{u}_0, \tilde{x}_0, \tilde{q}_0)$.

2.3. Solving the deterministic model

The deterministic problem can be solved by dynamic programming. The corresponding equations are

$$F(u_0, x_0) = \begin{cases} 0 & \text{if } (u_0, x_0) = (\tilde{u}_0, \tilde{x}_0) \\ -\infty & \text{otherwise} \end{cases} \quad (9a)$$

$$F(u_t, x_t) = \max_{(u_{t+1}, x_{t+1}) \in W} [(p_t^E g_t - C(g_t, p_t^F) - S(x_t)(1 - u_t)) u_{t+1} + F(u_{t+1}, x_{t+1})] \text{ subject to } (7), \quad (9b)$$

where $t = 0, \cdots, T - 1$, and g_t is the solution of the following problem

$$\max \quad p_t^E q_t - (a_0 + a_1 q_t + a_2 q_t^2) p_t^F$$
$$\text{s.t.} \quad q^{\min} \leq q_t \leq q^{\max}, \quad (10)$$

that is,

$$g_t \equiv \min\left(q^{\max}, \max\left(q^{\min}, \frac{1}{2a_2}(\frac{p_t^E}{p_t^F} - \frac{a_1}{2a_2})\right)\right). \quad (11)$$

Also the decision space W is given by

$$W = \{(1, x_+) | 1 \leq x_+ \leq t^{\text{on}}, x_+ \in Z\} \cup$$
$$\{(0, x_-) | -1 \geq x_- \geq -t^{\text{cold}}, x_- \in Z\}. \quad (12)$$

The optimal value J^* of this deterministic problem is obtained from the last step of the dynamic programming algorithm as

$$J^* = F(u_0, x_0). \quad (13)$$

The above recurrence relation is the *backward* dynamic programming approach. The $F(u_t, x_t)$, known as cost-to-go, defines the optimal cost from time t to the end of the period considered. The state transition diagram is given in Figure 1.

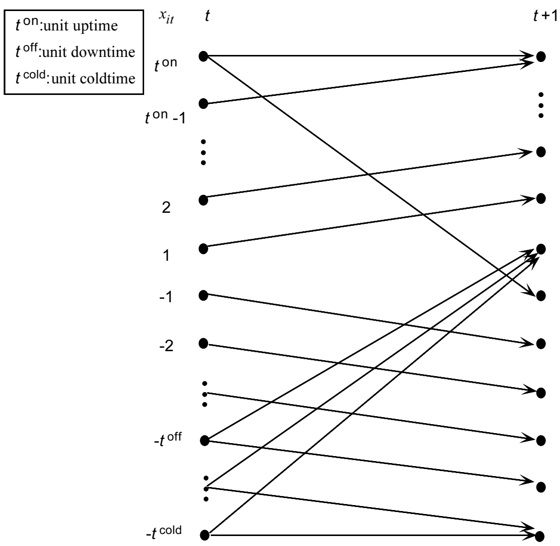

Figure 1: The state transition diagram for u_t

3. The approach using financial option theory

In this section, we briefly review the approach of using financial option theory to value a power plant. This approach (e.g. [4]) assumes that a unit can be started up immediately, and hence commitment decision can be made after the prices of electricity and fuel are observed. Also the minimum uptime and downtime constraints, and the startup cost are left out. The value of a power plant over the period $t = 1, \cdots, T$ can be represented as follows.

$$\sum_{t=1}^{T} E_0[\max\left(p_t^E g_t - (a_0 + a_1 g_t + a_2 g_t^2) p_t^F, 0\right) | p_0^E, p_0^F], \tag{14}$$

where g_t given in (11) is also a function of p_t^E and p_t^F; and E_0 denotes the expectation operator given the price information available at $t = 0$. In their approaches, a_0 and a_2 are set to be 0. The power plant value for per MWh electricity generation is

$$\sum_{t=1}^{T} E_0[\max\left(p_t^E - a_1 p_t^F, 0\right) | p_0^E, p_0^F]. \tag{15}$$

The term in the parentheses in (15) is the so-called *spark spread*, and (15) captures the sum of the payoff of (European) spark spread *call* option expired at each t. The power plant value obtained by either (14) or (15) is always nonnegative.

4. Multi-stage Stochastic model

Obviously price uncertainties plead for a stochastic model. In this paper we formulate the generation asset valuation problem as a multi-stage decision problem. The decision variable u_t is now the commitment decision which has to be made before the uncertainties p_t^E and p_t^F are revealed. On the other hand, the generation level q_t can be adjusted in real time and is obtained after the prices at time t are known. Namely, optimal generation level q_t can be obtained deterministically by solving (10). That is, we shall always set $q_t = g_t$ as defined by (11) in the rest of this paper. In reality, since committing a generation unit involves reheating the water in boiler, commitment decisions indeed have to be made a certain time before operation takes place. In this paper, for simplicity we model the commitment decision lead time (unit startup time) to be one hour. However, the proposed valuation method is flexible enough to be extended to more complicated situations. Thus, this modeling intends to capture the characteristics of real-world power plant operation.

We assume that the operator is risk neutral so that his objective is to maximize the expected profit with respect to the random price vectors $(\mathbf{p}^{\text{E}}, \mathbf{p}^{\text{F}})$, which represent the subjective price probability distributions believed by the operator. At time t, the operator observes the prices p_t^E for electricity and p_t^F for fuel, he then decides the commitment of the unit for the next hour. The operator's decision takes one hour to process to become effective.

Let $J_t(u_t, x_t; p_t^E, p_t^F)$ denote the power plant value for the remaining period starting at time t. At each time t, the operator either has no commitment decision to make because the minimum uptime or downtime constraints are not yet satisfied, or can decide to turn on or turn down the unit at the next hour. However, once committed, a unit needs to fulfill the minimum uptime or downtime requirement regardless the subsequent price changes. The dispatcher's problem is then one of solving the recursive relationship:

$$J_t(u_t, x_t; p_t^E, p_t^F) =$$

$$(p_t^E q_t - (a_0 + a_1 q_t + a_2 q_t^2) p_t^F) u_t - S(x_{t-1})(1 - u_{t-1}) u_t +$$

$$\max_{u_{t+1}, x_{t+1}, q_{t+1}} E_t[J_{t+1}(u_{t+1}, x_{t+1}; p_{t+1}^E, p_{t+1}^F) | p_t^E, p_t^F] \tag{16}$$

subject to (6) to (8). In (16), E_t denotes the expectation operator given the price information available at time t. Note that the maximization on the right-hand side of (16) compares the values of no more than two cases, $u_{t+1} = 1$ or 0.

The boundary conditions are:

$$J_T(u_T, x_T; p_T^E, p_T^F) =$$

$$(p_T^E q_T - (a_0 + a_1 q_T + a_2 q_T^2) p_T^F) u_T - S(x_{T-1})(1 - u_{T-1}) u_T, \quad (17)$$

for all $(u_T, x_T) \in W$.

4.1. Price Processes

In this paper we assume that price processes for both electricity and fuel are given. Although the valuation method to be presented in this paper can be applied to general Ito price processes, we focus on the following two processes for electricity and fuel respectively:

$$d\ln(p_t^E) = -\mu^E(\ln(p_t^E) - m_t^E)dt + \sigma^E dB_t^E, \quad (18)$$

and

$$d\ln(p_t^F) = -\mu^F(\ln(p_t^F) - m_t^F)dt + \sigma^F dB_t^F, \quad (19)$$

where B_t^E and B_t^F are two Wiener processes with instantaneous correlation ρ. The above commodity price models are characterized by mean reversion and lognormally distributed, seasonal prices. Because, to varying degrees, both electricity and fuel have associated storage costs, their prices are determined to a large degree by the forces of producer supply and consumer demand and less so by investor speculation [1]. This interplay is manifested in the mean-reverting nature of their price processes. Thus, in some sense, the mean reversion parameter μ represents the storability of the commodity. For electricity, which is quite difficult to store, this parameter is large implying little autocorrelation between today's price and tomorrow's price. Furthermore, this parameter in conjunction with σ captures the short and long term price fluctuations and characterizes variance of the lognormal price distribution. This distribution resembles that of other traditional price process models in that price returns are normally distributed and prices are nonnegative. Finally, m_t is a periodic function capturing the cyclical nature of the long-term expected prices. m_t is thus a function of the interplay between the cost of production and consumer demand for the commodity. In a later section, we will describe the procedures for estimating the parameters from historical data using the method of maximum likelihood.

4.2. Valuation using simulation

In this section we demonstrate how Monte Carlo simulation can be used to value a power plant considering the physical constraints described in previous sections. Let's first define

$$\Phi \equiv \{x \in Z | x = t^{\text{on}} \text{ or } -t^{\text{cold}} \le x \le -t^{\text{off}}\} \quad (20)$$

as the set of states at which turning on and off the generation unit are options (see Figure 1). The basic idea is to determine the *indifference locus* $d(p_t^E, p_t^F; t, x_t) = 0$ for each time t and each of the states $x_t \in \Phi$ such that for any pair of (p_t^E, p_t^F) on one side of the locus $(d \ge 0)$, the optimal decision is to turn the unit on for the next hour; and on the other side of the locus $(d < 0)$ the optimal decision is to turn the unit off. Again, the 'optimality' of the decision is in terms of expected value as defined in (16). In terms of function $d(p_t^E, p_t^F; t, x_t)$, at time t the optimal commitment decision for u_{t+1} is as follows.

$$u_{t+1} = \begin{cases} 1 & \text{if } 0 < x_t < t^{\text{on}} \\ 0 & \text{if } 0 > x_t > -t^{\text{off}} \\ 1 & \text{if } x_t = t^{\text{on}}, \text{ and } d(p_t^E, p_t^F; t, x_t) \ge 0 \\ 0 & \text{if } x_t = t^{\text{on}}, \text{ and } d(p_t^E, p_t^F; t, x_t) < 0 \\ 1 & \text{if } -t^{\text{cold}} \le x_t \le -t^{\text{off}}, \text{ and} \\ & \quad d(p_t^E, p_t^F; t, x_t) \ge 0 \\ 0 & \text{if } -t^{\text{cold}} \le x_t \le -t^{\text{off}}, \text{ and} \\ & \quad d(p_t^E, p_t^F; t, x_t) < 0 \end{cases} \quad (21)$$

Our approach corresponds closely to that presented in [3] for valuing path-dependent options. In [3], a critical locus is obtained in each period to determine optimal early exercise policy for American Asian options. In this paper, we extend the approach to a more complicated situation involving multi-stage decision making and intertemporal constraints.

Starting from $t = T-1$, for $x_{T-1} = t^{\text{on}}$, we can initiate a Monte Carlo simulation for *any* p_{T-1}^E to determine a $\hat{p}_{T-1}^F(p_{T-1}^E)$ such that the only two cases $u_T = 1$ or 0 of the maximization on the right-hand side of (16) are equated. That is,

$$E_{T-1}[J_T(u_T = 1, x_T = t^{\text{on}}; p_T^E, p_T^F) | p_{T-1}^E, \hat{p}_{T-1}^F(p_{T-1}^E)]$$

$$= E_{T-1}[J_T(u_T = 0, x_T = -1; p_T^E, p_T^F) | p_{T-1}^E, \hat{p}_{T-1}^F(p_{T-1}^E)]. \quad (22)$$

Similarly, for each x_{T-1} such that $-t^{\text{cold}} \le x_{T-1} \le -t^{\text{off}}$, and any p_{T-1}^E we look for $\hat{p}_{T-1}^F(p_{T-1}^E)$ such that the following equation holds.

$$E_{T-1}[J_T(u_T = 1, x_T = 1; p_T^E, p_T^F) | p_{T-1}^E, \hat{p}_{T-1}^F(p_{T-1}^E)] =$$

$$E_{T-1}[J_T(u_T = 0, \bar{x}_T; p_T^E, p_T^F) | p_{T-1}^E, \hat{p}_{T-1}^F(p_{T-1}^E)], \quad (23)$$

where $\bar{x}_T = \max(x_{T-1} - 1, -t^{\text{cold}})$.

In the next step, we try to determine an indifference locus (a function) that fits all the $(p_{T-1}^E, \hat{p}_{T-1}^F(p_{T-1}^E))$ obtained. This procedure theoretically calls for infinite number of price pairs on the locus. In practice, we identify as many indifference price pairs as necessary to obtain an acceptable approximation of the locus. The indifference locus at time $T-1$ and state x_{T-1} can therefore be approximated by

$$d(p_{T-1}^E, p_{T-1}^F; T-1, x_{T-1}) = \hat{p}_{T-1}^F(p_{T-1}^E) = 0. \quad (24)$$

For a given p_{T-1}^E, the process to locate $\hat{p}_{T-1}^F(p_{T-1}^E)$ is similar to finding a root for a univariate function. A simple bisection method can be applied to solve the problem. However, in such a root-finding problem, the function evaluation is very expensive, since each function evaluation includes two independent Monte Carlo simulations (for both u_T=0 and 1). A more sophisticated root-find technique such as curve fitting method (e.g. [7]) can greatly improve the convergence.

With all the indifference loci for time $T-1$ and $x_{T-1} \in \Phi$ obtained, we repeat the same process beginning at time $T-2$. For each (p_{T-2}^E, p_{T-2}^F) pair, first we let $u_{T-1} = 1$. This leads to some state x_{T-1} at $T-1$. Each iteration then produces values for the electricity price and fuel price at time $T-1$. We compare these values with the indifference locus of that corresponding state at $T-1$ to determine the optimal commitment decision, and calculate the corresponding profit. Based on this optimal decision u_{T-1}, we move to a corresponding state at time T and aggregate the corresponding profit at time T. This completes one simulation iteration. By running many simulation iterations, we closely approximate the expected profit for turning on the unit at $T-1$, $E_{T-2}[J_{T-1}(u_{T-1} = 1, x_{T-1}; p_{T-1}^E, p_{T-1}^F) | p_{T-2}^E, p_{T-2}^F]$. We then let $u_{T-1} = 0$, and repeat the same procedures for simulation and obtain $E_{T-2}[J_{T-1}(u_{T-1} = 0, x_{T-1}; p_{T-1}^E, p_{T-1}^F) | p_{T-2}^E, p_{T-2}^F]$, the expected profit for turning off the unit at $T-1$. If these two expected profits for turning on the unit and turning off the unit at $T-1$ are the same, (p_{T-2}^E, p_{T-2}^F) forms a $(p_{T-2}^E, \hat{p}_{T-2}^F(p_{T-2}^E))$ pair. If not, using search technique, update p_{T-2}^F (with p_{T-2}^E fixed), and repeat the same process above (for both $u_{T-1} = 1$ and $u_{T-1} = 0$ again) till a $(p_{T-2}^E, \hat{p}_{T-2}^F(p_{T-2}^E))$ pair is found. After a certain amount of $(p_{T-2}^E, \hat{p}_{T-2}^F(p_{T-2}^E))$ pairs are collected, the indifference locus for this state x_{T-2} is approximated.

By repeating this process working backward in time to time 0, we identify the indifference locus for each t and $x_t \in \Phi$. The last simulation, which begins with the initial conditions at time 0, provides an estimate value of the power plant during the operating period. In summary, the Monte Carlo method in this approach is employed not only in forward-moving simulation, but also backward-moving recursion of dynamic programming.

5. Numerical results

We have implemented the proposed method for valuing a power plant in FORTRAN on a Pentium II personal computer. This section presents numerical test results.

5.1. Test System Parameters

The proposed method has been applied to a natural gas-fueled generating unit with the following input-output characteristics over a 4-day (96 hours) operating period:

$$H(q_t) = 820 + 9.032q_t + 0.00113q_t^2. \quad (25)$$

To obtain the parameters of the price processes of both electricity and fuel, we examine historical price data series of Nymex natural gas prices and electricity prices from both Palo Verde and Norway, taking the logarithm of these prices as our basic data series. Since there is no hourly market for natural gas, we assume that m_t^F is constant within a given day and fit the price process to daily data. Because the model time step is hourly, however, we adjust these parameters accordingly. Specifically, since the model is a continuous-time model, we can deduce the implied hourly fluctuations immediately from the daily parameters.

For electricity, we use historical daily data from the Palo Verde electricity markets and similarly normalize the data to the hourly time step. However, to incorporate a daily price pattern, we then adjust m_t^E by overlaying the daily electricity price pattern (in terms of percentage changes) of the Norwegian market. Although this market is not identical to the newly established hourly market of California, it is the authors' belief that for the purposes of power plant valuation, the cyclical fluctuations incorporated are sufficient. Moreover, while some hourly California electricity prices are available, due to its newness and accompanying scarcity, the hourly California electricity price data was not sufficiently reliable. In the appendix of this paper, we summarize how to use maximum likelihood method to estimate price parameters.

Using the maximum likelihood method, we obtain $\mu^F = 4.17 \times 10^{-6}$ and $\sigma^F = 1.78 \times 10^{-2}$. For electricity we obtain $\mu^E = 0.32 \times 10^{-3}$ and $\sigma^E = 0.20$. As aforementioned, m_t^E captures the cyclical nature of the expected electricity prices. Obtained m_t^E values are summarized in Table 1. Also we assume the instantaneous correlation coefficient between electricity and natural gas is $\rho = 0.4$.

Table 1: Values of hourly m_t^E

t	1	2	3	4	5	6
m_t^E	2.9913	2.9719	2.9600	2.9528	2.9527	2.9699
t	7	8	9	10	11	12
m_t^E	3.0052	3.0341	3.0545	3.0587	3.0597	3.0555
t	13	14	15	16	17	18
m_t^E	3.0491	3.0444	3.0403	3.0362	3.0365	3.0386
t	19	20	21	22	23	24
m_t^E	3.0365	3.0324	3.0272	3.0252	3.0192	3.0005

We shall apply the proposed method to the unit of (25) with various minimum uptime and downtime cases. For simplicity, we assume that $t^{\text{off}} = t^{\text{cold}}$ in all test cases.

5.2. Indifference Loci

In Figure 2, we show three indifference loci taken from the test case with $t^{\text{on}} = t^{\text{off}} = 10$. They are the indifference loci at time $t = 95$ with state $x_t = 1$, $t = 89$ with state $x_t = -1$ and $t = 5$ with state $x_t = 1$ respectively. The sample price pairs on each indifference locus obtained in our simulation follows closely to a straight line. These price pairs are then fitted by a linear function with mean square error minimized. Each indifference locus divides the price plane (p_t^E, p_t^F) into two regions: on-line region (the lower right half) and off-line region (the upper left half). At time t, if (p_t^E, p_t^F) falls into the on-line region, the optimal commitment decision for the following hour, $t+1$, is to turn on the unit, and vice versa. The indifference locus at different time t and state $x_t \in \Phi$ appears to have different slopes and interceptions. No specific relation between the locus characteristics and prices has yet been observed.

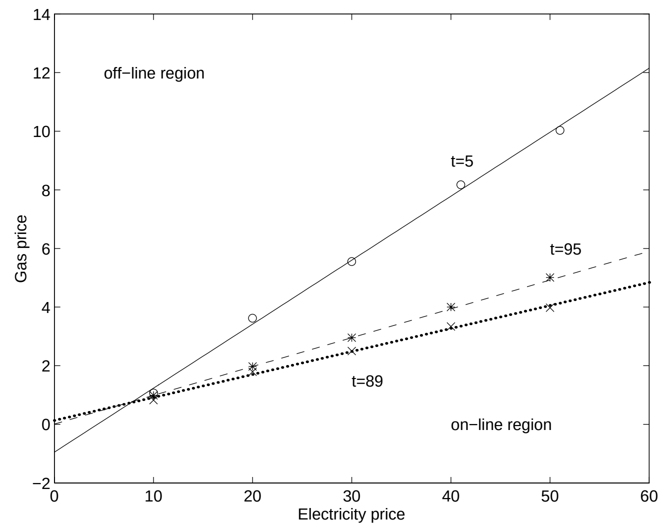

Figure 2: Three indifference loci: $(t, x_t) = (95, 1)$, $(89, -1)$ and $(5, 1)$.

5.3. Power plant value vs. physical constraints

To demonstrate the relation between a power plant value and physical constrains, we applied the proposed method to the generator of (25) under various minimum uptime and downtime constraints. Four cases, $t^{\text{on}} = t^{\text{off}} = 1$, 4, 10 and unconditionally on-line, are tested. The first three cases are all tested twice, one with startup cost considered, the other without. The results are summarized in Table 3.

In Figures 3a to 3c, three frequency charts are depicted. They correspond to three different minimum uptime and downtime constraints, $t^{\text{on}} = t^{\text{off}}$=1, 4, 10, with startup cost considered. In each of the frequency charts, the tallest "spike", representing the most frequent occurrence, corresponds to zero profit, i.e. no commitment at all hours. The outliers on left hand side of the tallest spike correspond to loss. It can be seen that as t^{on} (or t^{off}) increases, the downside risk increases. The case with units on at all hours is considered to be a good measurement of the limiting case for increasing t^{on}, which corresponds to the lowest mean profit, and the highest variance among all test cases. The last column of Table 2 corresponds to value obtained without considering decision lead time. This also represents the value obtained by the approach using financial option theory presented in Section 3. To summarize the test results, based on our specific test system, we observe: (1) with unit startup time (decision lead time) introduced, the (mean) profit drops

Table 2: Power plant value vs. physical constraints

t^{on} (hr)	1	4	10	∞^*
t^{off} (hr)	1	4	10	0
Startup cost S	0	0	0	0
x_0	-1	-4	-10	1
Mean† ($\times 10^6$)	3.06	2.97	2.93	2.87
Variance ($\times 10^{12}$)	62.97	63.97	65.84	69.38
Skewness	14.6	9.26	11	12
Kurtosis	485	154	254	329
Per MWh Profit ($/MWh)	36.23	37.24	36.20	30.37
t^{on} (hr)	1	4	10	$1^\ddagger$
t^{off} (hr)	1	4	10	1
Startup cost S	2050	2050	2050	0
x_0	-1	-4	-10	1
Mean ($\times 10^6$)	3.04	2.97	2.93	3.07
Variance ($\times 10^{12}$)	63.00	63.99	65.85	74.58
Skewness	14.64	9.26	11	16.07
Kurtosis	485	154	254	610
Per MWh profit ($/MWh)	35.94	35.98	35.38	39.32

†: All simulations terminate when the statistical errors are within 0.5% of the mean values.
∗: The unit is on-line unconditionally at all hours.
‡: This corresponds to the case with no decision lead time.

0.33%; (2) With respect to the case with lead time and $t^{on} = t^{off} = 1$, when $t^{on} = t^{off}$ is increased to 4, the profit is decreased by around 2.4%, and is 3.78% when $t^{on} = t^{off}$ is 10. Overall, in this 4-day period, ignoring both startup cost and physical constraints may result in up to 6.5% (=(3.07-2.87)/3.07) difference in power plant value. We expect this deviation to go up when the length of operating period increases.

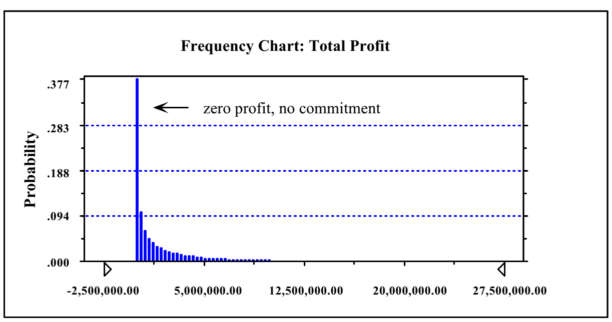

Figure 3a: Frequency chart: $t^{on} = t^{off} = 1$, $S = 2050$

5.4. Value at risk

One advantage of using simulation to value a power plant is that it provides information on the Value at

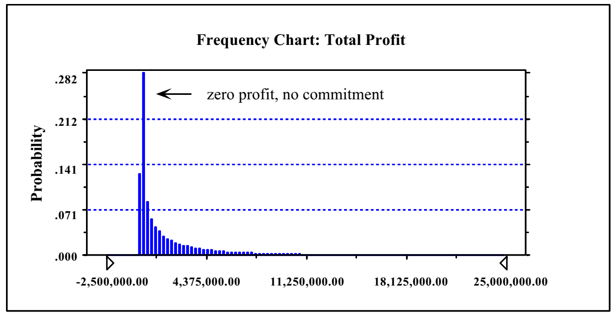

Figure 3b: Frequency chart: $t^{on} = t^{off} = 4$, $S = 2050$

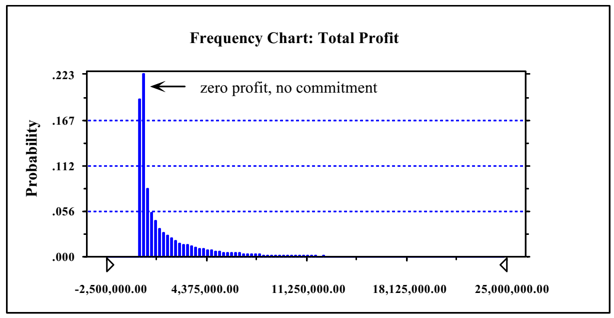

Figure 3c: Frequency chart: $t^{on} = t^{off} = 10$, $S = 2050$

Risk (VaR) [6] of the power plant's operation. VaR is a measure of the amount of money that an institution could lose due to price changes in the underlying markets. The VaR measure depends on the chosen time period and desired confidence level. For example, let the frequency function of the power plant value obtained by simulation be represented by $\Pr(V = v_i)$. The 10% percentile of the cumulative distribution function of the power plant value corresponds to the amount of money that the power plant of (25) may lose during the 4-day operating period, given a 90% confidence level ($\equiv v_{10\%}$), i.e.

$$10\% = \Pr(V \leq v_{10\%}). \qquad (26)$$

This single number $v_{10\%}$ encapsulates the power plant's total operation risk over the 4 days of operation. The VaRs (90% confidence) of the power plant of (25), varied with different physical constraints, are summarized in Table 3. It can be seen that VaR increases as the the length of minimum uptime or downtime increase. Also the incorporation of startup cost increases the VaR of a power plant.

Table 3: Value at risk (VaR)

t^{on} (hr)	t^{off} (hr)	Startup cost S	VaR ($)
1	1	0	-14,277
1	1	2050	-25,437
4	4	0	-26,962
4	4	2050	-33,555
10	10	0	-52.808
10	10	2050	-57,799
∞	0	0	-438,736

6. Future directions and conclusion

In this paper we present a method for valuing a power plant using Monte Carlo simulation. As opposed to the popular approach using financial options, we emphasize the need for incorporating physical constraints into the problem modeling. Without considering physical constraints, the payoff is nonnegative, and the potential risk due to operational limits is greatly overlooked.

While we have tried to model real-world physical constraints into the power plant valuation problem, the unit ramp constraints [9] are still left out. Ramp constraints, which limit the capability of a generator to move between operating levels over short periods of time, can have significant impacts on the power plant value. When subject to ramp constraints, the generation levels of a unit are interdependent in all hours. Incorporating ramp constraints into the valuation problem requires to increase the dimension of the state space of the dynamic programming, and hence increases the complexity of computation. Research using heuristics to take account ramp constraints has been initiated. We will report further results in a future paper.

Appendix

A Estimating parameters using maximum likelihood method

Given a sequence of observations $\{X_0, \ldots, X_n\}$, the joint density function of the sample

$$f(X_0, \ldots, X_n; \theta) = \prod_{i=1}^{n} f(X_i \mid X_{i-1}; \theta) \tag{27}$$

where $f(X_i \mid X_{i-1}; \theta)$ is the conditional density function of X_i given X_{i-1}. (Note because the "influence" of the marginal distribution of the initial point is negligible for large n, we exclude its effect for convenience.) Let $\mathcal{L}(\theta)$ denote the log-likelihood function, the natural logarithm of $f(X_1, \ldots, X_n; \theta)$. Then,

$$\mathcal{L}(\theta) = \sum_{i=1}^{n} \ln[f(X_i \mid X_{i-1}; \theta)] \tag{28}$$

and so the maximum likelihood estimates solve

$$\arg\max_{\theta} \mathcal{L}(\theta) \tag{29}$$

Recall the price process model letting $X_t \equiv \ln(P_t)$:

$$dX_t = -\mu(X_t - m_t)dt + \sigma dB_t \tag{30}$$

This equation has solution

$$X_t = e^{-\kappa t} X_0 + \alpha \left(1 - e^{-\kappa t}\right) + \sigma \int_0^t e^{-\kappa(t-s)} dB_s \tag{31}$$

Solving for the conditional density, we find

$$f(X_i \mid X_{i-1}; \alpha, \kappa, \sigma) =$$

$$\frac{1}{\sqrt{2\pi\hat{\sigma}^2}} \exp\left[-\frac{(X_i - (e^{-\kappa} X_t + \alpha\left(1 - e^{-\kappa}\right)))^2}{2\hat{\sigma}^2}\right], \tag{32}$$

where

$$\hat{\sigma}^2 \equiv \frac{\sigma^2}{2\kappa}\left(1 - e^{-2\kappa}\right) \tag{33}$$

Thus, given the conditional density of X_i of our model, we find the log-likelihood function is

$$\mathcal{L}(\alpha, \kappa, \hat{\sigma}) = \textstyle\sum_{i=1}^{n} \ln[f(X_i \mid X_{i-1}; \alpha, \kappa, \hat{\sigma})]$$

$$= -\tfrac{n}{2}\left(\ln[2\pi] + \ln[\hat{\sigma}^2]\right) +$$

$$-\frac{1}{2\hat{\sigma}^2} \sum_{i=1}^{n} \left(X_i - (e^{-\kappa} X_{i-1} + \alpha(1 - e^{-\kappa}))\right)^2 \tag{34}$$

To find the maximum likelihood parameters, first order conditions imply

$$\begin{aligned} 0 &= \frac{\partial \mathcal{L}(\alpha, \kappa, \hat{\sigma})}{\partial \alpha} \\ &= \frac{1}{\hat{\sigma}^2} \sum_{i=1}^{n} \left((X_i - e^{-\kappa} X_{i-1}) - \alpha(1 - e^{-\kappa})\right) \quad (35) \\ 0 &= \frac{\partial \mathcal{L}(\alpha, \kappa, \hat{\sigma})}{\partial \kappa} \end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{\hat{\sigma}^2}\sum_{i=1}^{n}((X_i-\alpha)(X_{i-1}-\alpha)- \\
&\quad e^{-\kappa}(X_i-\alpha)^2) \qquad (36) \\
0 &= \frac{\partial\mathcal{L}(\alpha,\kappa,\hat{\sigma})}{\partial\hat{\sigma}} \\
&= \frac{n}{\hat{\sigma}} - \frac{1}{\hat{\sigma}^3}\sum_{i=1}^{n}\left(X_i-(e^{-\kappa}(X_{i-1}-\alpha)+\alpha)\right)^2 \qquad (37)
\end{aligned}
$$

Thus the maximum likelihood estimators are

$$
\begin{aligned}
\alpha &= \frac{1}{(n+1)}\sum_{i=0}^{n}X_i - \frac{1}{(n+1)}\left(\frac{X_0-e^{-\kappa}X_n}{1-e^{-\kappa}}\right) \\
&\approx \frac{1}{n+1}\sum_{i=0}^{n}X_i \ \text{ for } n \text{ large} \qquad (38) \\
\kappa &= -\ln\left[\frac{\sum_{i=1}^{n}(X_i-\alpha)(X_{i-1}-\alpha)}{\sum_{i=0}^{n-1}(X_i-\alpha)^2}\right] \qquad (39) \\
\hat{\sigma}^2 &= \frac{1}{n}\sum_{i=1}^{n}\left(X_i-(e^{-\kappa}(X_{i-1}-\alpha)+\alpha)\right)^2 \qquad (40) \\
\sigma^2 &= \frac{2\kappa\hat{\sigma}^2}{(1-e^{-2\kappa})} \qquad (41)
\end{aligned}
$$

Therefore, by replacing the prices in our original data series with the natural logarithm of prices, we may apply these formulas to determine the parameters of the model for electricity and fuel.

References

[1] G. Barz and B. Johnson, "Modeling the price of commodities that are costly to store: the case of electricity", presented at the Chicago Risk Management Conference, Chicago, May 1998.

[2] S. Deng, B. Johnson and A. Sogomonian, "Exotic electricity options and the valuation of electricity generation and transmission", presented at the Chicago Risk Management Conference, Chicago, May 1998.

[3] D. Grant, G. Vora and D. Weeks, "Path-dependent options: extending the Monte Carlo simulation approach", *Management Science*, vol. 43, no. 11, 1997, pp. 1589-1602.

[4] M. Hsu, "Spark spread options are hot!", *Electricity Journal*, vol. 11, no. 2, 1998, pp. 28-39.

[5] M. Hsu, "Using Financial Options theory to value and hedge natural gas power assets", presented at the EPRI's Pricing Energy in a Competitive Market Conference, Washington, D.C., June 1998.

[6] J. P. Morgan and Company, *RiskMetrics*™, Technical document, J. P. Morgan and Company, New York.

[7] D. G. Luenberg, *Linear and Nonlinear Programming*, Addison-Wesley, 1984.

[8] D. Shimko, "Options on futures spreads: hedging, speculation and valuation", *Journal of Futures Markets*, vol. 14, no. 2, 1994, pp. 183-213.

[9] A.J. Svoboda, C.L. Tseng, C.A. Li, and R.B. Johnson, Short Term Resource Scheduling with Ramp Constraints, *IEEE Transactions on Power Systems*, Vol. 12, No.1, 1997, pp. 77-83.

[10] C. L. Tseng, "On Power System Generation Unit Commitment Problems", Ph.D. Dissertation, Department of Industry Engineering and Operations Research, University of California at Berkeley, 1996.

[11] A.J. Wood and B.F. Wollenberg, Power Generation, Operation and Control, John Wiley & Sons, New York, 1984.