

The Efficiency of Multi-unit Electricity Auctions*

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Abstract

Using a complete information, game-theoretic model, we analyze the performance of different electricity auction structures in attaining efficiency (i.e., least-cost dispatch). We find that an auction structure where generators are allowed to bid for load "slices" outperforms an auction structure where generators submit bids for different hours in the day.

1 Introduction

The electric power industry around the world is undergoing a process of privatization deregulation and restructuring. This transition is fueled by technological and social change that led to a fundamental reexamination of conventional wisdom concerning natural monopolies and economies of scale in this industry.

While the restructuring approaches implemented or proposed in various parts of the world and within the US are diverse in many aspects they share several important elements which include competitive generation, a spot energy market and a power auction. The purpose of the auction is to provide a mechanism through which generators can submit bids to supply electricity. The most challenging aspect of designing an electricity auction is that daily demand, which fluctuates from hour to hour, must be satisfied by a set of suppliers, with different cost, in a least-cost manner. Even in a centralized model with known generator costs determining the optimal dispatch is a computationally difficult problem. It is an even greater challenge to design an auction where generators voluntarily chose to be efficiently dispatched.

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In any electricity auction, generators must submit bids which indicate the minimum prices at which they are willing to generate electricity. In some auction designs (e.g. the in the UK system) bids can also include state transition costs such as start up and ramping costs as well as constraints on availability and dispatch¹. The structure of the auction and the determination of prices paid to the winning bidders can vary. It is desirable that the designer of an electricity auction define the structure and prices in such a way as to provide generators with the incentive to bid so as to minimize generation costs. We shall refer to the set of generators which minimizes generation costs as the *efficient dispatch* and to an auction which induces an efficient dispatch as an *efficient auction*.

There are several aspects of an auction for electricity that separate it from the vast body of auction literature and make designing an efficient auction a challenging task. The most obvious is the structure of generation costs. Generators have many cost components (e.g., ramp-up costs, no-load costs, etc.) which must be recovered through their sales revenue. In addition generators are subject to intertemporal dispatch constraints that relate their output in different time periods. These characteristics create cost dependencies in *intertemporal* production so that the average cost of generating Q MW of electricity varies with the number of units generated and the dispatch schedule. Such dependencies complicate both the bidding strategies and the bid evaluation protocols.

The long standing tradition of vertical integration and centralized dispatch in the electric utility industry resulted in advanced computational tools for optimal dispatch of generating resources, which take as inputs all the costs and operational constraints for each available resource as well as demand data and reliability requirements. Such tools are designed based on the premise of perfect information about these inputs and produce as outputs two type of decision variables. The optimal commitment schedules specifying the hourly on/off state of each resource are typically produced for a range of the next 168 hours. These schedules and a rough estimate of the hourly output level of each resource are calculated by a unit commitment algorithm which is a mixed nonlinear and integer optimization program run every hour on a rolling horizon basis for the next 168 hours. The unit commitment schedules are used as inputs to an optimal power flow calculation which uses a nonlinear optimization algorithm run repeatedly at short time intervals to obtain the up-to-the-minute output levels of each generator on line. The optimal power flow employs a more realistic model of the power system that takes into consideration transmission and security constraints as well as various physical aspects associated with alternating current (AC) systems.

Some restructuring designs have attempted to preserve the central unit commitment protocols with competitive generation by employing a multipart auction to elicit the inputs needed for the traditional unit commitment algorithms. In such auctions bidders are required to submit for the day ahead, supply functions for energy as well as all the other cost components and dispatch constraints needed for central unit commitment procedures. Such an auction structure is employed in the UK where unit commitment is performed using the

¹See Patrick and Wolak (1996) for an analysis of the United Kingdom auction design.

GOAL algorithm and is part of the proposed restructuring plan for the New York Power Pool. Unfortunately multi-part auctions are not well understood and have limited theoretical foundation that would enable an incentive compatible design of such auctions. Indeed, the UK experience suggests that such auctions are susceptible to gaming and manipulation. Furthermore, recent work by Johnson, Oren and Svoboda (1997) suggest that even if such an auction was made incentive compatible (i.e. bidder would be induced to reveal true costs and constraints) central unit commitment may still be inappropriate in a competitive generation environment. In particular, the authors have demonstrated that unit commitment algorithms designed for an environment with central generation ownership have multiple equally good solutions with varying resource schedules. When generation ownership is dispersed among many independent parties, such variation have diverse profit implications for the different parties. The resulting ambiguity in the bid selection protocols cannot be resolved by tie breaking procedures since it is not practical to compute all the optimal solutions (even one good solution is computationally challenging). Furthermore, the optimal unit commitment produced by a specific algorithm (out of the many possible) is affected by fine tuning of the program which consequently may be systematically biased in favor of some generators to the detriment of others.

An alternative approach to the multi-part auction that has been adopted in the California Power Exchange proposed protocol and in the Victoria pool in New Wales, Australia relies on self-commitment. In other words, unit commitment decisions are left to the bidders while the auction structure is simplified to a single price per tender. A tender consists of one or multiple blocks of energy defined in terms of their timing and capacity (e.g. 2 MW supplied for one hour between 1 and 2 PM). All the production costs incurred by a generator (including fixed, intertemporal and dispatch constraints costs) are internalized in such an auction and reflected in the single bid price.

From an auction theory perspective power auctions with self-commitment may be interpreted as multi-unit auctions with dependent valuations, Unfortunately the auction theory literature has little to offer on such auction structures, in general.² A few notable exceptions have addressed the issue of multi-unit auctions. Wilson (1979) began the study of “share” auctions, where bidders with a common valuation, submit demand curves and are awarded a fraction of the shares at a market clearing price. Maskin and Riley (1989) study the design of optimal multi-unit auctions with private valuations. Hausch (1986) studies a two objects auction, where there are two bidders with common valuations who desire both objects. Hausch finds that the seller’s revenue is greater when both objects are sold simultaneously versus sequentially. Rothkopf, Pekec, and Harstad (1995) identify a special class of multi-unit auctions in which bidders can submit a bid for different combinations of objects and the auction is computationally manageable. von der Fehr and Harbord’s (1993) analysis of the UK Electricity Industry is the only other study we know of that identifies an electricity auction as a multi-unit auction with private valuations and attempts to study the strategic bidding behavior of generators. von der Fehr and Harbord assume a complete information

²See McAfee and McMillan (1987) for an excellent survey of the auction literature.

framework where demand is uncertain but its distribution is known, with two generators who have (different) constant marginal costs of generation. They show that the less efficient (higher marginal cost) generator may submit lower bids than the more efficient generator, and hence generation costs may not be minimized in equilibrium.

Our objective in this paper is to address specific types of such auctions that are relevant to the context of electric power. In particular we focus on two alternative ways of structuring a multi-unit power auction and use the framework of games with perfect information to examine the efficiency of their outcome. Specifically, we focus on the question; will the auction structure induce efficient (i.e., least social cost) dispatch. While the analysis employs a very simplistic stylized model of demand and generation cost, we employ the insight of the analysis to outline the practical implementation of a new auction structure that promises efficient self-commitment and dispatch.

2 Electricity Auction Structure

In this paper we will ignore transmission constraints and assume that the power auction treats all the demand and supply as if it was at a single location. This simplification is consistent with the UK system, the proposed California power exchange, the Victoria pool and other systems around the world where transmission constraints and congestion management are handled outside the power auction. We will further assume that there is no demand side bidding which is also consistent with most currently operating and proposed power auction. Under this simplified structure the objective of the power auctioneer is to "fill" a forecasted load curve for a specified time period (say the next 24 hours) with tenders consisting of blocks of energy specified by capacity (MW) duration (hours) and timing. (see Figure 1(a)).

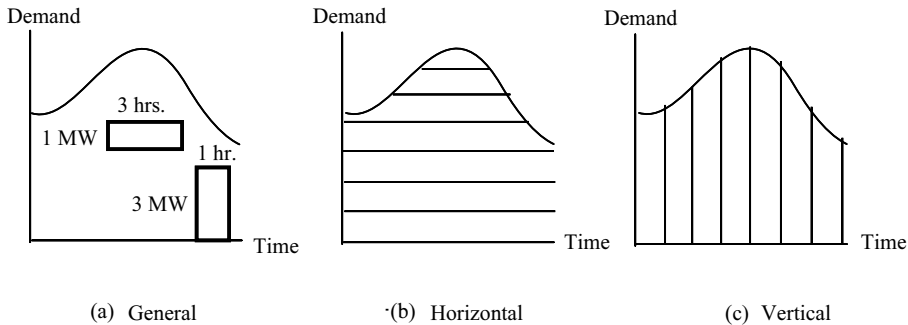


Figure 1: Various auciton structures for electricity demand.

There are many possible ways to structure such an auction in terms of the tenders allowed, the bid evaluation process and the prices paid to winning tenders. In this paper, we will

analyze the performance of two auction structures, a Vertical and Horizontal auction, and examine their ability to guarantee efficient dispatches.

In a Horizontal Auction (HA, see Figure 1(b)), demand is divided into distinct sets that are auctioned sequentially. Demand sets are formed by partitioning daily demand by its duration, i.e., a distinct set for each duration t . Hence, generators submit a supply curve for each set, indicating the price at which they are willing to generate k megawatts for a **duration** of t hours, where $k, t > 0$. Before each auction the results of any previous auctions are made known. The bids for each set are assembled in ascending order and the winning bidders are paid their bid price.

In a *Vertical* auction (VA, see Figure 1(c)), a generator’s bid consists of a single supply curve indicating the price at which they are willing to generate K megawatts for one hour submitted at the start of the auction (in the Victoria pool the supply curve could be different for each hour but all the curves must still be submitted at the beginning of the day for the next 24 hours). From the submitted supply curves, the auctioneer constructs a cumulative supply curve. The auctioneer then uses the same cumulative supply curve to dispatch generators for each hour in the day. All generators chosen for dispatch in an hour are paid the marginal price, i.e., the highest accepted bid price, in that hour.

In the next section, we assume a simple model for demand and generation costs and demonstrate why a Vertical auction does not guarantee an efficient dispatch in equilibrium. We then prove that a Horizontal auction *does* guarantee efficiency in our model. We conclude with the reasons for and the intuition behind these results.

3 Model

An auction for electricity must assure that the demand over an entire day will be satisfied by a set of generators. For simplicity, we assume that the day is divided into two time periods (representing high and low demand), whereas the demand over the length of a day, is represented by a step function (Figure 2(a)). We always assume that daily demand is public information and known to the generators.

Throughout the paper we assume that generators have two costs, which are publicly known, associated with generation; a fixed “start-up” cost, f , to begin to generate and a variable cost per MWh, v , once the plant is up and running. Due to this cost structure, there exist cost dependencies in *intertemporal* production. Finally we assume that all generators own one generating unit each and have the same capacity (normalized to 1 MW)³ : At any point in time a generator is either idle or is generating at an output of 1 MW, i.e., output levels are integer. We assume a framework with three generators, $G1$, $G2$, and $G3$, whose cost of generating 1 and 2 MWh are given in Figure 3 .

³The capacity constraint implies that generators can not supply more than 1 MW at any point in time, but places no restrictions on the duration for which they can generate.

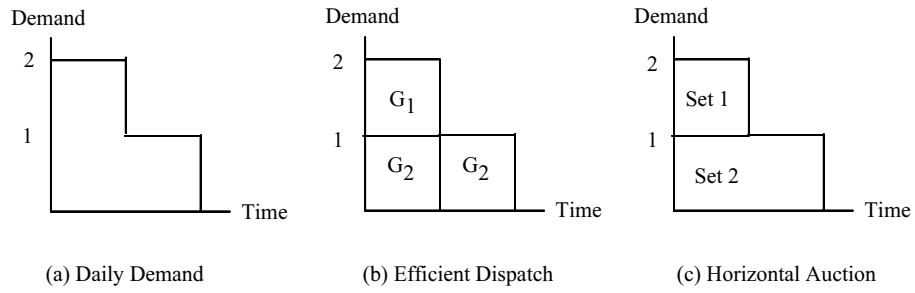


Figure 2: Stylized electricity demand.

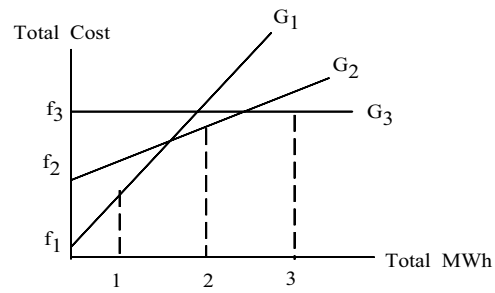


Figure 3: Generation costs.

We rescale all fixed costs by setting $f_1 = 0$. Due to capacity constraints, the auctioneer must dispatch two generators in the $t = 1$ and one generator at $t = 2$ and wishes to minimize generation costs.

3.1 Vertical Auction (VA)

In a Vertical auction, due to the assumption on capacity, each generator submits a supply curve which consists of one point : the minimum price at which the generator is willing to generate 1 MWh. The auctioneer then dispatches generators in increasing order of bids. We have found that even in our simple model of cost and demand, a Vertical auction does not support an efficient dispatch in its set of Nash Equilibria.

Proposition 1 *In a complete information setting, a Vertical Auction cannot guarantee an efficient dispatch in equilibrium.*

Proof: Assume $G1$, $G2$, and $G3$ bid a, b , and c , respectively, where $b < a < c$. The auctioneer collects the bid and finds the least-cost dispatch. Given a daily demand in Figure 2(a), an efficient dispatch requires $b < a < c$ to hold, which implies $G2$ is dispatched both periods and $G1$ is dispatched one period (Figure 2(b)). We argue that such bids can not occur in equilibrium as a result of many conflicting issues: The bidders' desire to increase their bids (and hence their revenue) as much as possible while still ensuring dispatch, the discontinuity of their profits as a function of bids, and the restriction of a singleton bid.

Suppose we have $b < a < c$ as required in an efficient dispatch. For these bids to constitute an equilibrium, these bids are profitable for $G1$ and $G2$, i.e., $a \geq f_1 + v_1$ and $b \geq v_2$, and each generator must have no incentive to change its bid given its opponents' bids. However, the generators *do* have an incentive to deviate at these values for a and b . Suppose $a > f_1 + v_1$. Since $G2$ submits the lowest bid, she is dispatched in both periods and is the price-setter in the second period. Because she's paid her bid in hour 2, it is in her favor to make b as high as possible and still below a , i.e., $G2$ will make her bid as high as possible while still ensuring dispatch in both periods, and therefore bids $b = a - \varepsilon^4$. If $G2$ bids $a - \varepsilon$, it is not optimal for $G1$ to bid a : he will want to undercut $G2$'s bid by ε , replacing $G1$ as the lowest bidder. By doing so, he foregoes ε in revenue in the first period, but wins dispatch in the second period at his bid price of $a - 2\varepsilon$.

G_1 has an incentive to undercut G_2 's bid even when $a = f_1 + v_1$. Suppose $b < a < c$ and $a = f_1 + v_1$. Since the generators' average costs' are decreasing in quantity, G_1 is better-off undercutting G_2 's bid of $b = f_1 + v_1 - \varepsilon$ and winning dispatch in both hours at prices above average cost. Hence, we have shown that the efficient dispatch can not be supported in equilibrium. ♦

⁴Where ε is finite and is the smallest bid increment.

3.2 Horizontal Auction (HA)

We can learn from the failure of a Vertical Auction to guarantee efficiency in equilibrium that it is necessary to account for cost dependencies in electricity generation when designing an auction. An auction structure which does exactly this is a Horizontal Auction. Below we provide a simple example which illustrates the intuition behind the ability of a HA to guarantee efficiency.

Assume the same framework of demand, costs and generators as in section 3.1. In a HA (Figure 2(c)) the set with the longest duration is auctioned first, i.e., Set 2. After the winner of Set 2 is made public knowledge, Set 1 is auctioned. In order for an efficient dispatch, $G2$ must win Set 2 and $G1$ must win Set 1.

Proposition 2 *The unique SPNE outcome of the horizontal auction presented in Figure 2(c) is efficient.*

Proof: To prove our claim, we will employ backward induction to identify the NE in the auction for Set 1 and then the Subgame Perfect Nash Equilibrium (SPNE) for the entire game⁵.

At the start of auction of Set 2, we can be in any of three possible states (Figure 4),

State 1 $G1$ won Set 2

State 2 $G2$ won Set 2

State 3 $G3$ won Set 2

Assume that we are in the State i) where $G1$ has won dispatch in both hours: Since $G1$ will already be generating 1 MW at $t=1$, it must withdraw from the auction for Set 1. $G2(3)$'s cost of generating 1 MW is $f_2 + v_2$ ($f_3 + v_3$). Given the assumed generation costs, we know $f_2 + v_2 > f_3 + v_3$, which implies that the unique NE outcome in State i) is that $G3$ wins Set 1.

Using the generator's costs and the same logic as above, we can define the unique NE outcome for the auction of Set 1 in States i), ii) and iii) to be:

NE in State 1 *Given $G1$ won Set 2, $G3$ wins Set 1*

NE in State 2 *Given $G2$ won Set 2, $G1$ wins Set 1*

⁵The SPNE is the appropriate equilibrium concept due to the sequential nature of a Horizontal Auction.

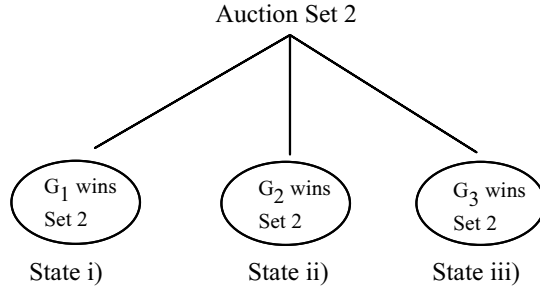


Figure 4: Game tree for sequential auctions.

NE in State 3 *Given G_3 won Set 2, G_1 wins Set 1*

Having identified the NE at each possible subgame, we next step back to the auction of Set 2 and identify the possible SPNE. Define G_1 's bid for Set 2 to be a , G_2 's bid for Set 2 to be b , and G_3 's bid for Set 2 to be c . We will demonstrate that the only bid ordering that can be part of a SPNE is $b < a$, $b < c$, i.e., the only outcome possible in a SPNE is G_2 winning Set 2. In order to complete our proof, we must employ the following two lemmas.

Lemma 1 The bid ordering $a = \min(a, b, c)$ can not be sustained in a SPNE.

Proof: Suppose that the bid ordering $a < b$, $a < c$ can occur in a SPNE. In this equilibrium, G_1 wins Set 2, G_3 wins Set 1, both must incur non-negative profits and G_2 must have no incentive to undercut G_1 . This last condition translates into:

$$a - 1 - f_2 - 2v_2 \leq 0 \quad \rightarrow \quad a \leq f_2 + 2v_2 + 1$$

The maximum G_1 's profit can be, assuming it is unprofitable for G_2 to undercut G_1 , is

$$\pi_1 = f_2 + 2v_2 + 1 - 2v_1$$

But from our cost assumptions this yields, $\pi_1 < 0$, which contradicts the necessary equilibrium condition that G_1 incurs a non-negative profit. \blacklozenge

Lemma 2 The bid ordering $c = \min(a, b, c)$ can not be sustained in a SPNE.

Proof: The proof to this mimics the proof to Lemma 1.

Now we continue the proof of the original proposition. We have shown that the bid ordering $a < b$, $a < c$ and $c < a$, $c < b$ can not be sustained in SPNE. Therefore it is left

only to show that if there does exist a pure strategy SPNE, the bid ordering in the auction for Set 2 has to be $b < a, < b < c$.

Suppose this is the case: $G2$ wins Set 2 and hence $G1$ wins Set 1. In order for this to constitute a SPNE, no generator will want to deviate from its bids, i.e.,

- $G3$ does not wish to undercut bid $b \implies b - 1 - f_3 - 2v_3 < 0$ or $b < f_3 + 2v_3 + 1$
- $G1$ does not wish to undercut bid $b \implies b - 1 - 2v_2 < f_3 + v_3 - 1$ or $b < f_3 + v_3 + v_1 + 1$
- $G2$ incurs a non-negative profit $\implies b \geq f_2 + 2v_2$

These necessary equilibrium conditions are not in conflict and at the resulting dispatch, both $G1$ and $G2$ incur non-negative profits. Hence the bid ordering, $b < a, b < c$, which results in an efficient dispatch, can be sustained in SPNE as long as $G2$'s bid is within the following range:

$$f_2 + 2v_2 < b < f_3 + 2v_3 + 1$$

◆

The above analysis can be extended to t sets and $n > t$ generators each of whom is the least cost producer of exactly one integer duration (see Appendix A, proposition 3). Under these assumptions we prove that all Subgame Perfect Nash Equilibria of a sequential horizontal auction results in the unique efficient dispatch.

4 Implementing Horizontal Auctions

The above analysis, although based on highly stylized models of the demand, generating cost and auction structure exposes a basic shortcoming of vertical auctions and suggest that horizontal auctions may be more compatible with the notion of self-commitment. In this section we will outline how a horizontal or a "Load Slice" auction may work in a real world environment. We will first assume as in our stylized model a central market which ignores the locational characteristics of supply and demand and a fixed load curve which excludes demand side bidding. Later we will discuss how these assumptions could be relaxed. Under these restrictions the process may proceed as follows:

The auctioneer or exchange operator posts a load forecast for the bid period (say 24 hours) before the bidding process begins (few hours prior to the onset of the bid period). The auction is then done in several round filling up the load curve from the bottom up as illustrated in Figure: 5. In each round the auctioneers solicits bids for a load slice of a specified number of MW with a fixed schedule prescribed by the demand curve (the initial

solicitation is for base load dispatched for the entire bid period then for shoulder load and finally for peak load.) In each round bidders bid tenders consisting of capacity increments in MW that they are willing to commit to the specified schedule and a total price (or an average price per MWh). Since the schedule is known, bidders can easily calculate their total cost for serving a slice including all start-up costs and the costs associated with intertemporal constraints which they can factor into their bid price.

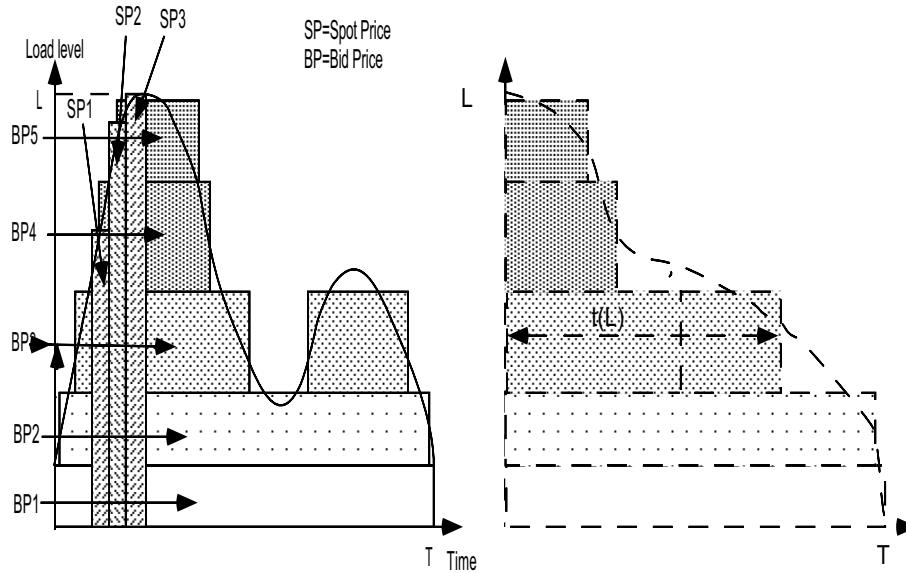


Figure 5: Load slice bidding with spot market selling.

Winning bids in each round are selected based on lowest average price per MWh. Payments to the winning bids can be either discriminatory (i.e., paying each bidder their bid price) or uniform (i.e., paying all winning bids in a round the highest winning price or the lowest rejected price in that round). In either case it should be emphasized that the price per MWh will vary (most likely increase) as we move from lower base load slices to the upper peakload slices. The number of slices (and rounds) is a design parameter of the auction. The slices should be "thin" enough so that the dispatch schedule in each slice is approximately uniform.⁶

The above design may be refined by allowing bidders to also specify a marginal energy price for minor adjustments to their energy output within a specified range (that would not affect the intertemporal costs). Such information might be useful in defining spot energy

⁶An alternative implementation could take the form of a Dutch Auction where the auctioneer posts a price per MWh for a load slice with a specified schedule and raises the price until a dispatch commitment is made. The auctioneer then moves onto the next slice, using the last accepted price as his starting bid, and continues the process until the entire load curve is filled.

prices. As much as it is natural for the suppliers to define their tenders as horizontal slices of loads with prescribed schedules, it is natural for consumers to think of electricity as a time differentiated commodity offered each hour at a uniform spot price. Since energy consumed in a particular hour cannot be traced to a specific supply source it should be sold at a uniform price. Economic efficiency dictates that the spot price at any hour should be set at the marginal cost of the highest load slice active at that hour.

Hence the market can be organized so that on the supply side power is acquired through a load slice sequential auction while on the demand side the energy is sold in a spot market with vertical slices priced uniformly in each time period (see Figure 5). It is possible to show that under some restrictive assumptions about the cost structure on the supply side and about the load pattern such a scheme will break even in the sense that the spot market sales will generate just enough revenue to recover the payments to suppliers (see Appendix B). In general, however, the spot market will run a deficit (due to the intertemporal costs rolled into the supply bids) which must be recovered through a fixed charge or an "uplift charge."

The above scheme can be extended to include demand side bidding and locational factors. Both of these aspect will play a role during the peak load hours and need to be accounted for in the auction rounds for the upper slices. With regard to demand side bidding such bids can be entered as a reservation price in the upper auction rounds. In other words, if bid prices exceed the demand side bids all bids are rejected in favor of demand curtailments. The locational factors, i.e., congestion affects, can be accounted for by running a power flow analysis as the load curve is being filled up. When congestion occurs a locational penalty can be imposed as an adder to the bid price of upper slice bids originating at the constrained locations. This approach discriminates against peaking units in pricing congestion. Since the locational penalties are not imposed while the base load slices are being auctioned off, this amounts to giving priority to such units in congestion management. Intuitively it seems the right thing to do. However we have not yet analyzed the efficiency implications of such an approach.

5 Conclusions

Simulation studies and empirical evidence suggest that central unit commitment is inappropriate in a competitive electricity environment. Yet the traditional approach of auctioning power in an energy only hourly auction is incompatible with self-commitment due to intertemporal costs and nonconvexities in the production function for energy. One approach that has been adopted by the California Power Exchange proposal is a multi-round auction with activity rules which allow bidders to revise their bids so as to account for intertemporal costs resulting from their dispatch schedule. We propose a different approach which structures the auction so that the bid tenders allow bidders to readily account for these costs. Our proposed approach is supported by a game theoretic analysis of a stylized model which

shows that unlike the vertical slice auctions, a sequential horizontal slice auctions will induce efficient dispatch.

Appendix A

Assume there exists a daily demand given in Figure 6, i.e., there exist $t > 0$ demand sets, Set 1, Set 2, ...Set t , where Set t consists of 1 MW demand over t time periods, $t \geq 0$. In a HA, base load demand is auctioned first, i.e., begin by auctioning the demand set with longest duration, Set t . After the winner of Set t is made public knowledge, Set $t - 1$ is auctioned. This process continues until the peak load slice, Set 1, is auctioned. Note that due to capacity constraints, once a generator has won a set, it withdraws from participating in future auctions.

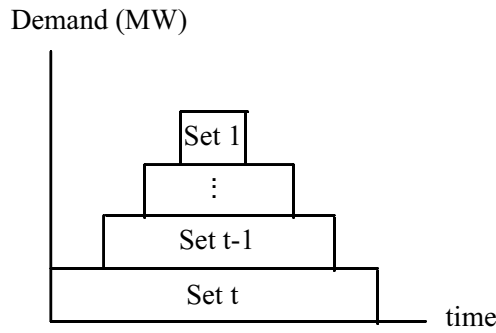


Figure 6: Demand load partitioned for a Horizontal Auction.

We assume there to be $n > t$ generators where generator $G(i)$ is the least-cost producer of i MWh, for $i = 1 \dots n$.⁷ Define $C_{G_x}(q) = f_x + qv_x$ to be generator x 's cost of producing q MW. Figure 7 plots the cost function for such a set of generators.

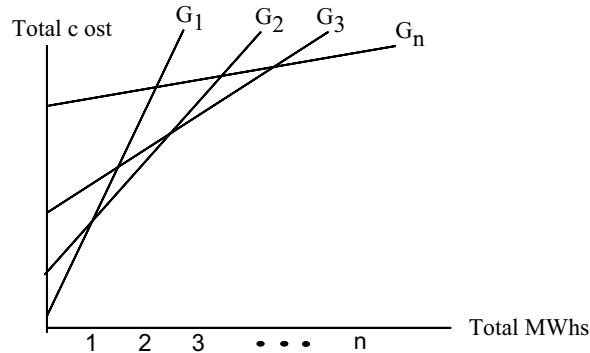


Figure 7: Generation costs for different technology types

⁷Again, due to the capacity constraint, this is interpreted as the most efficient producer of 1 MW over q hours.

Proposition 3 *All Subgame Perfect Nash Equilibria (SPNE) of a sequential horizontal auction result in the unique efficient dispatch.*

Before we proceed to prove the proposition, we shall state two lemmas that are used in the proof.

Lemma 4 *There does not exist a SPNE in which $G(t+1), G(t+2), \dots$, or $G(n)$ are dispatched or, simply put, In a SPNE, generators $G(1) \dots G(t)$ must each win in one auction.*

Lemma 5 *The highest winning bid in the auction of Set j is bounded above by $C_{G(t+1)}(j)$, for $j = 1, \dots, t$.*

Proof (Lemmas 2&3) Without loss of generality, suppose $G(t+1)$ won the auction for Set $x, x < t$. There exists at least one generator $G(y) \in [G(1), \dots, G(t)]$ which is not dispatched and would be made strictly better off by bidding ε above its cost and winning in the auction for Set y . The same reasoning explains why, in equilibrium, generators $G(1), \dots, G(t)$ must each win exactly one auction. The second lemma follows from the fact that generator $G(t+1)$ provides an upper bound on winning bids in any auction round via the threat of undercutting any bid above its own cost.

Equipped with the two lemmas, we now proceed to the proof of the proposition.

Proof of Proposition: Given Lemma 2 and 3, we know that $G(1) \dots G(t)$ will be dispatched in equilibrium and that there exists an upper bound on the winning bids in each auction round. From the viewpoint of $G(1)$, it earns its greatest profit when it waits until the last round to bid aggressively (i.e., it places a bid greater than $C_{G(t+1)}(j)$ for slices $2, \dots, t$). Given $G(1)$ has bid above $C_{G(t+1)}(j)$ for $j = 2 \dots t$, only $G(1)$ and $G(t+1) \dots G(n)$ remain in the last auction, i.e., the auction for Set 1. $G(1)$ can win Set 1 at a profit of $C_{G(t+1)}(1) - C_{G(1)}(1)$. This is greater than any profit $G(1)$ could earn in any of the previous auctions. Therefore $G(1)$'s optimal strategy is to bid above $C_{G(t+1)}(j)$ for Set $j = 2 \dots t$ and bid $C_{G(t+1)}(1) - \varepsilon$ in the final round.

Figure 8 provides the geometric explanation for $G(1)$'s optimal strategy. Recalling Lemma 3, the maximum profit $G(1)$ can hope to make in the auction for Set j is $C_{G(t+1)}(j) - C_{G(1)}(j)$. This difference is decreasing in the duration of a set, i.e., the number of hours the generator is dispatched. Hence it is optimal for $G(1)$ to win in the last auction for Set 1.

Given $G(1)$'s optimal strategy, it is optimal for $G(2)$ to bid above $C_{G(t+1)}(j)$ for slices $j = 3 \dots t$ and then bid $C_{G(t+1)}(2) - \varepsilon$ in the auction for Set 2. The reasoning exactly parallels that for $G(1)$.

Using an iterative reasoning, we can deduce that given $G(1), \dots, G(x)$'s optimal strategy, it is optimal for $G(x+1)$ to bid above $C_{G(t+1)}(j)$ for $j = x+2 \dots t$, and to bid $C_{G(t+1)}(x+1) - \varepsilon$ for Set $x+1$ (for $x+1 < t$). Finally, $G(t)$'s optimal strategy is to bid $C_{G(t+1)}(t)$ in the auction for Set t . ♦

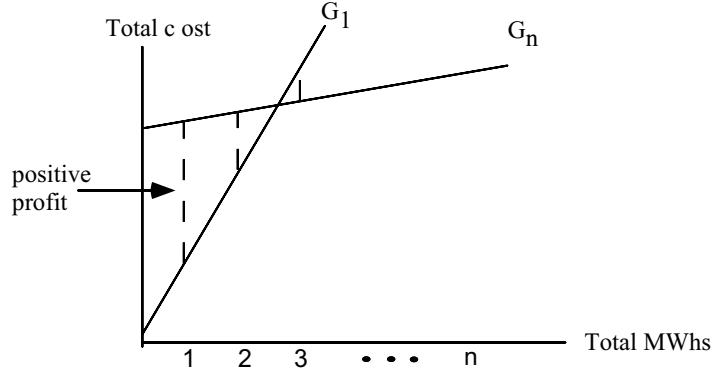


Figure 8: $G(1)$'s profits in successive auctions determined by $G(t + 1)$.

Appendix B

The scheme mentioned in section 4 can be visualized as purchasing power to fill the load curve in uniformly priced “horizontal slices” where the price increases as the slice is higher in the stack. The power is then resold as uniformly priced vertical slices corresponding to different time periods, where the price for each slice is the spot price or the highest marginal price of the operating generation units.

We further assume that the load pattern is unimodal which implies at most one start-up in each load slice, and further assume that there is sufficient competition in each generation technology to make the optimal generation mix feasible. We ignore on/off switching aspects such as ramping-up and assume that a generator's total generation costs at a given load level depend strictly on the dispatch duration (this includes as a special case a two-part cost structure consisting of a start-up and marginal operating cost). Define the cost to generator i of generating at capacity for t hours to be $C_i(t)$. The marginal cost of a generator may depend on the dispatch duration.

In an idealized competitive load slice bidding environment, a generator will commit its capacity at a price per MWh equal to its average cost for the posted dispatch duration. Thus, each load slice will be served by the generating unit with the lowest average cost. Hence, if $t(L)$ denotes the duration corresponding to load level L and \mathbf{L} is the maximum load level, the total cost of serving the load curve is given by

$$\begin{aligned}
 \text{Total Cost} &= \int_0^{\mathbf{L}} \left(\min_i \frac{C_i(t(L))}{t(L)} \right) t(L) dL = \int_0^{\mathbf{L}} \min_i C_i(t(L)) dL \\
 &= \int_0^{\mathbf{L}} \tilde{C}(t(L)) dL = \tilde{C}(T)L(T) + \int_{L(T)}^{\mathbf{L}} \tilde{C}(t(L)) dL
 \end{aligned}$$

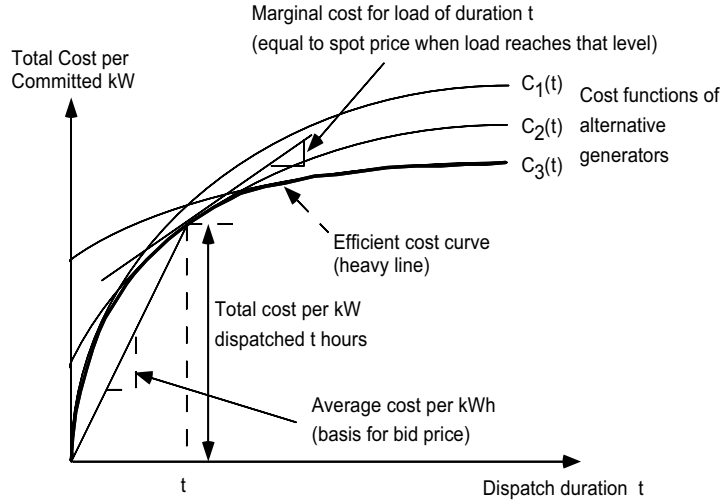


Figure 9: Optimal technology mix (given by the lower envelope).

where $\tilde{C}(t) = \min_i C_i(t)$ and $L(T)$ is the base load level.

Integrating by parts gives us

$$\text{Total Cost} = \tilde{C}(T)L(T) + \tilde{C}(t)L(t)|_T^0 + \int_0^T \tilde{C}'(t)L(t)dt = \tilde{C}(0)L + \int_0^T \tilde{C}'(t)L(t)dt$$

If we include curtailment as one of the supply technologies then the function $\tilde{C}(0) = 0$, so the first term in the above expression vanishes. Also $\tilde{C}'(t)$ is the marginal cost of serving a load level whose duration is t . Since marginal cost is decreasing with duration, this is the highest marginal cost of generating units operating when the load level has duration t . Hence $\tilde{C}'(t)$ is the spot price when the load level corresponds to duration t , and the integral of the spot price times the load over the entire time period equal the total cost. It thus follows that charging the spot price for the entire load in each time slice recovers total acquisition costs.

It should be noted that ramp-up costs and other costs associated with on/off switching cannot be recovered by spot prices that equal marginal costs. This is true in any system whether the acquisition is done through time slice bidding or load slice bidding, as described above. Recovery of switching costs requires some sort of “uplift” of the spot prices. In the above system generators are aware of these costs at the time bids are made and are able to internalize these costs. However, the spot prices will not recover the entire acquisition costs and some sort of fixed charge or uplift of the spot prices will be needed to achieve revenue neutrality.

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