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Sensitivity of the loading margin to voltage collapse with respect to arbitrary parameters

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Abstract: Loading margin is a fundamental measure of proximity to voltage collapse. Linear and quadratic estimates to the variation of the loading margin with respect to any system parameter or control are derived. Tests with a 118 bus system indicate that the estimates accurately predict the quantitative effect on the loading margin of altering the system loading, reactive power support, wheeling, load model parameters, line susceptance, and generator dispatch. The accuracy of the estimates over a useful range and the ease of obtaining the linear estimate suggest that this method will be of practical value in avoiding voltage collapse.

Keywords: voltage collapse, index, bifurcation, loading margin, control, sensitivity

1. Introduction

Voltage collapse is an instability of heavily loaded electric power systems characterized by monotonically decreasing voltages and blackout [1,2]. Secure operation of a power system requires appropriate planning and control actions to avoid voltage collapse. This paper describes and illustrates the use of loading margin sensitivities for the avoidance of voltage collapse.

For a particular operating point, the amount of additional load in a specific pattern of load increase that would cause a voltage collapse is called the loading margin. We are interested in how the loading margin of a power system changes as system parameters or controls are altered. *This paper shows how to compute linear and quadratic estimates to the variation of the loading margin with respect to any power system parameter or control.* The effect on the loading margin of changing the following controls and parameters is estimated:

- Emergency load shedding
- Reactive power support, shunt capacitance
- Variation in the direction of load increase
- Interarea redispatch, wheeling
- Changes to load model and load composition
- Varying line susceptance, FACTS device
- Generator redispatch

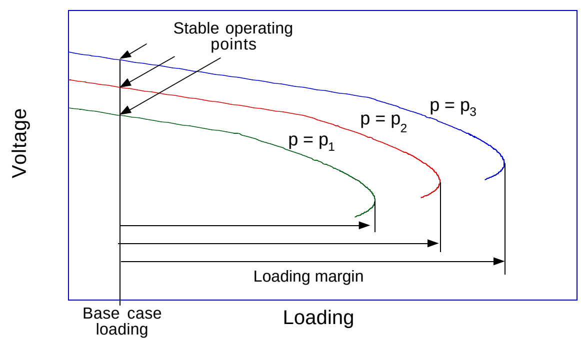

Figure 1A. Nose curves as parameter p varies

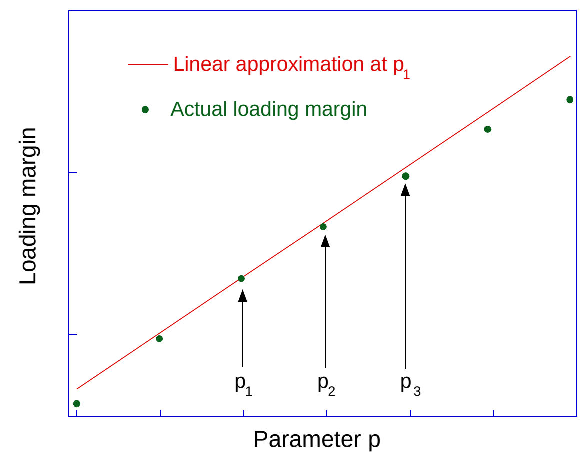

Figure 1B. Loading margin as parameter p varies

Loading margin sensitivities have a simple geometric meaning. Figure 1A shows nose curves of a large power system for three values of a power system parameter. The loading margin is the change in loading between the stable operating point and the nose of the curve corresponding to each parameter setting. (The nose corresponds to a bifurcation point of the power system when it is parameterized by loading.) As the parameter increases, the nose of the curve occurs at a higher loading and the loading margin increases. Figure 1B shows the loading margin as a function of the parameter value. Each nose curve in Figure 1A contributes one point to Figure 1B. The sensitivity of the loading margin with respect to the parameter at the nominal parameter value is given by the tangent linear approximation to the curve in Figure 1B. *The main idea of the paper is that after the loading margin has been computed for nominal parameters, the effect on the loading margin of altering the parameters can be predicted by using linear or quadratic estimates.* Exhaustively recomputing the nose for each parameter change is avoided.

Loading margin is an accurate measure of proximity to voltage collapse which takes full account of system limits and nonlinearities. (Every paper on other voltage collapse indices implicitly acknowledges the significance of loading margin by using it as the horizontal scale when the performance of the proposed index is graphed.) Moreover, loading margin estimates can be directly associated with costs, allowing for economic comparison of different strategies [4]. Methods to compute the nose and hence the loading margin are well developed [5,6,3,7,8,9]. This paper is different than these references because it assumes a loading margin computation and instead addresses the sensitivity of the loading margin.

Another approach to assessing proximity to voltage collapse uses fast time-domain simulation to predict whether the system will collapse (e.g. [10,11]). This approach has the advantage of better representing the potentially complex series of time dependent events which can influence voltage collapse. For example, the time dependence of generator reactive power limits can be represented. However, sensitivity information is difficult to obtain from time-domain simulations and requires a new simulation for each parameter variation considered. The loading margin and time-domain simulation approaches are complementary. Recent work combines aspects of both approaches [12].

There has been previous work on the sensitivity of various indices for voltage collapse. Tiranuchit and Thomas [13] computed the sensitivity of the minimum singular value of the system Jacobian, and Overbye and DeMarco [14] computed the sensitivity of an energy function index. The first order sensitivity of the loading margin was derived by Dobson and Lu [18]. This paper is an extension and application of [18].

2. Application to test system

The practical use of the sensitivity formulas derived in section 4 and appendix A is illustrated using a particular voltage collapse of the 118 bus IEEE standard test system [23] (see [23] for area and bus numbers and to reproduce the results). The system loading and loading margin is measured by the sum of all real load powers (an L^1 norm). The stable operating point at which we test parameter variation has a total system loading of 5677 MW. Buses critical to the voltage collapse are in area two. The generator dispatch distributes the slack so that generators in each area provide additional real power roughly in proportion to their size. The loads increase proportionally from the base case loading and the voltage collapse occurs at a total load of 7443 MW and a loading margin of 1766 MW. Seven generators reach reactive power limits between the stable operating point and the voltage collapse. (Note that the reactive power limit for generator 4 is increased to avoid complications caused by an immediate instability that would have occurred just prior to the voltage collapse. An immediate instability [17] can be caused by a generator reaching a reactive power limit.)

The sensitivity formulas evaluated at the voltage collapse yield linear and quadratic estimates of the loading margin as a function of any parameter. The performance of these estimates is tested for seven different parameters representative of a range of control actions or system uncertainties. The solid lines and dotted curves in figures 2–8 are the respective linear and quadratic estimates for the loading margin variation as a function of the chosen parameter.

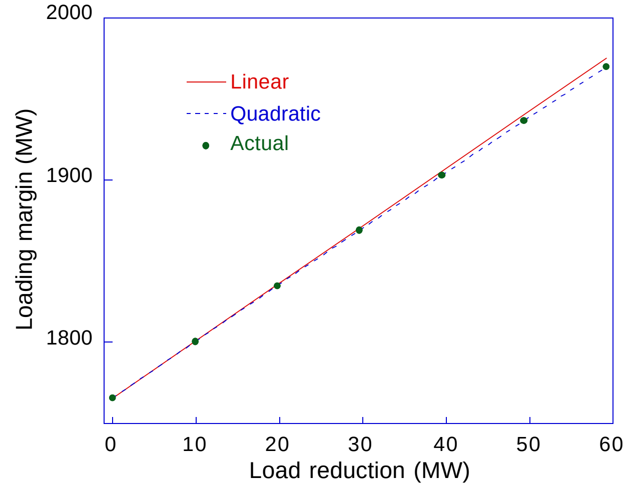

Figure 2. Effect of load shedding at bus 3

The dots in figures 2–8 represent the actual values of the loading margin as computed by combined continuation and direct methods [5]. The large dots represent the loading margin computed assuming that the reactive power limits which apply *at the voltage collapse* remain the same when the parameters are varied. (This assumption was used in deriving our sensitivity formulas.) The small dots represent the actual loading margin allowing different reactive power limits to apply at the voltage collapse. The small dots are computed by enforcing generator reactive power limits as the loading is increased from the stable operating point. In figures 2–6, the assumptions about limits make little difference and the large dots cover the small dots.

Emergency load shedding:

At the stable operating point, bus 3 has a load of 60 MW and 15 MVARs and a voltage of 0.95 p.u. Fig. 2 shows the results for shedding up to 60 MW of base load at constant power factor. Each MW of load reduction increases the loading margin by 3.5 MW, and the relation remains almost linear over the entire range of load shed.

Reactive Power Support:

The largest generator in area two is at bus 10, which is connected by a long transmission line to the high voltage side of the network. At the stable operating point, the generator at bus 10 is near its reactive power limit. Bus 9 represents the midpoint of the transmission line, and is a logical place to consider adding reactive power to alleviate the voltage collapse. Figure 3 shows that the linear estimate is accurate and quantifies the effectiveness of reactive power support at bus 9.

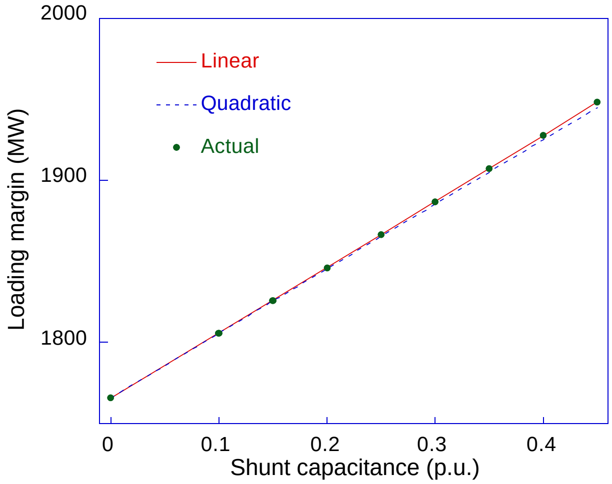

Figure 3. Effect of shunt capacitance at bus 9

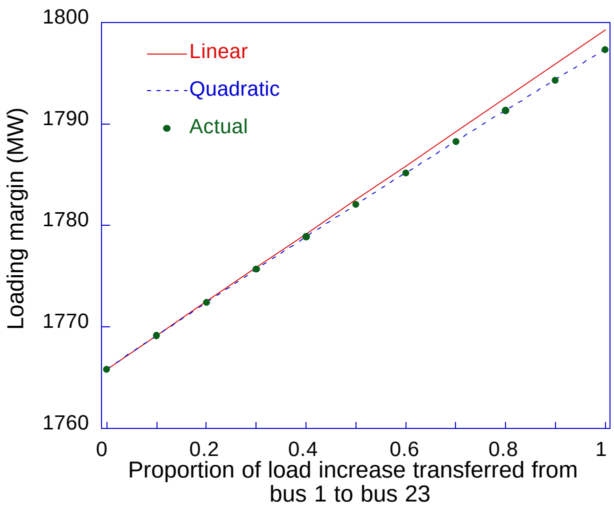

Figure 4. Effect of assumed loading direction

Direction of load increase:

Computing the loading margin requires a direction of load increase to be assumed. Variation in the direction of load increase can result from inaccuracies in forecasting. Thus it is useful to estimate the sensitivity of the loading margin to the direction of load increase. For this example, the direction of load increase is varied by transferring load increase from the critical bus 1 to a less critical bus in the same area, bus 23. For a particular loading factor, the total load remains the same but the proportion of load at bus 23 increases and the proportion at bus 1 decreases. Figure 4 shows that a linear estimate for the change in the loading margin performs well over the full range of variation.

Area interchange:

Recent trends in deregulation are expected to increase wheeling which can affect system security. The nominal interchange between the main area and area 2 is 103 MW.

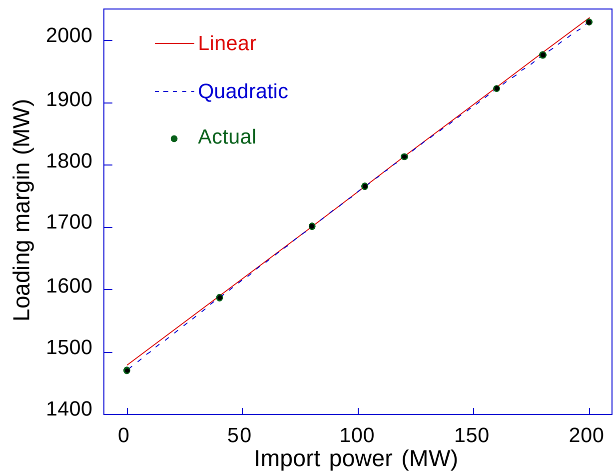

Figure 5. Effect of import into critical area

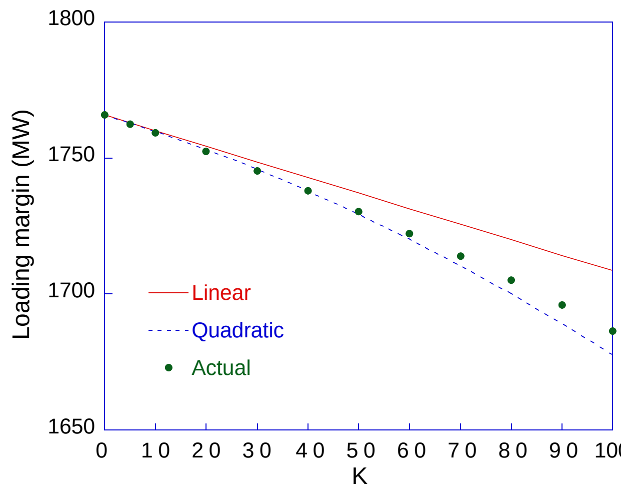

Figure 6. Effect of composition of load model

Figure 5 shows the effects on the loading margin of adjusting the flows between area 2 and the main area. Importing an additional 100 MW results in an increase in loading margin of over 200 MW, which is well predicted by the linear estimate.

Load model:

Load models are important in voltage collapse studies. The sensitivity of the loading margin with respect to parameters of a load model can be used to estimate the effect on the loading margin of using more detailed models. Figure 6 shows the effect on the loading margin of an additional reactive load Q at bus 3 linearly dependent upon the bus voltage V so that $Q = KV$. K can be interpreted as MVARS at a voltage of 1 p.u..

Line susceptance:

Variations in a line susceptance could represent the operation of a FACTS device or could reflect uncertainty in the network data. Figure 7 shows the effect of altering the susceptance of the line connecting bus 9 to bus 10.

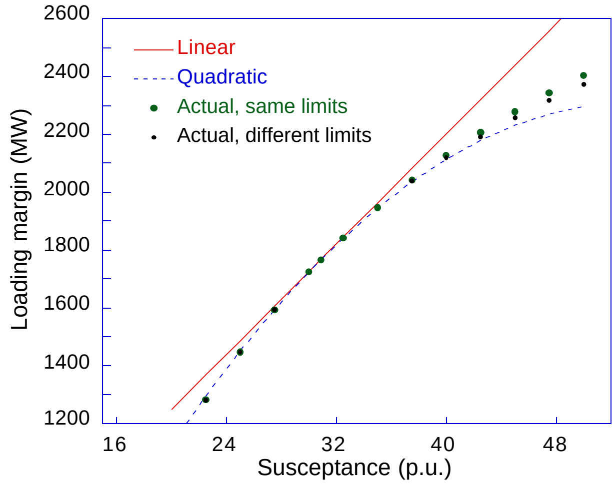

Figure 7. Effect of susceptance of line 9-10

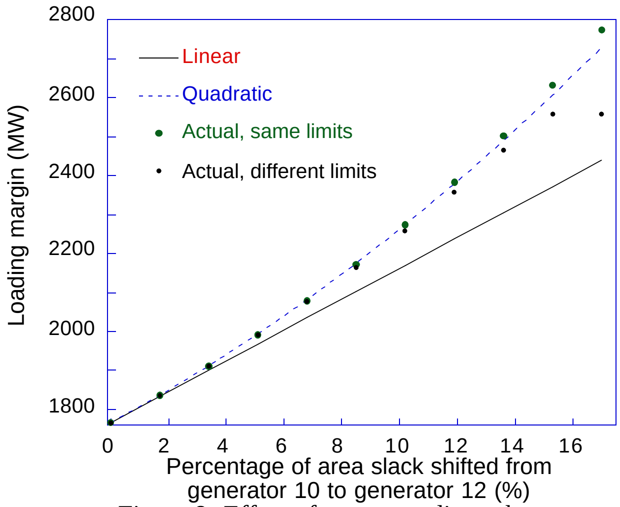

Figure 8. Effect of generator dispatch

For this example, the quadratic estimate is required to obtain accurate results over the full range of variation and the effects of changing limits are noticeable but not significant.

Generator dispatch: Generators 10 and 12 together assume 50% of the slack for area 2 with generator 10 alone picking up 42% for the nominal dispatch. Figure 8 shows the effect of shifting slack from generator 10 to generator 12. The two generators participate equally in the dispatch when 17% of the total area slack is moved from generator 10 to generator 12.

The quadratic estimate is accurate over a much larger range than the linear estimate. Moreover, the effects of limits can be significant. In this case, shifting more than 15% of the total area slack to generator 12 prevents generator 10 from reaching its reactive power limit; additional transfer past this point has little effect on the loading margin.

3. Theoretical background and assumptions

One influential theory of voltage collapse [15] models the power system as differential equations with slowly moving parameters and describes voltage collapse as the dynamic consequence of a saddle node bifurcation. In a saddle node bifurcation, the stable operating equilibrium coalesces with an unstable equilibrium and disappears. The dynamic consequence of a generic saddle node bifurcation is a monotonic decline in system variables.

Although differential equations are the proper setting for understanding voltage collapse and are necessary for explaining why voltages dynamically decrease as a consequence of a saddle node bifurcation, it is possible and very advantageous to compute loading margins to voltage collapse and their sensitivities using static equations. Dobson [19] proves that there is no loss of accuracy in using static models in place of the underlying dynamic models when computing loading margins and their sensitivities.

The derivations and application of the sensitivity formulas require the choice of a nominal stable operating point at which parameters or controls are to be adjusted, and a projected pattern of load increase. The pattern of load increase determines the nominal bifurcation point (nose) and also defines the direction in which the loading margin is measured. *The bifurcation point should be computed by a method that takes into account system limits such as generator reactive power limits as they are encountered.* In general, the limits enforced at the bifurcation are different than those at the stable operating point. The derivation of the sensitivity formulas requires that the system equations remain the same as parameters are varied. In particular, the limits enforced at the bifurcation are assumed to stay the same as parameters are varied.

4. Informal derivation

This section informally derives the first order sensitivity of the loading margin L with respect to any parameter p. See the appendices for a rigorous derivation of this and the quadratic sensitivity formulas.

Suppose that the equilibria of the power system satisfy the equations

$$f(x,\lambda,p)=0 \tag{1}$$

where x is the vector of state variables and λ is the vector of real and reactive load powers. Let λ_0 be the real and reactive powers at the operating equilibrium. We specify a pattern of load increase with a unit vector $\hat{k}$. Then the load powers at the saddle node bifurcation causing voltage collapse are

$$\lambda=\lambda_0+\hat{k}L \tag{2}$$

where L is the loading margin. The choice of norm is arbitrary, $\hat{k}$ is a unit vector in whatever norm is used to measure the loading margin L. Since $\hat{k}$ is a unit vector, it also follows that $L=|\lambda-\lambda_0|$.

At a saddle node bifurcation, the Jacobian matrix f_x is singular. For each (x,λ,p) corresponding to a bifurcation, there is a left eigenvector $w(x,\lambda,p)$ (a row vector) corresponding to the zero eigenvalue of f_x such that

$$w(x,\lambda,p)f_x(x,\lambda,p)=0. \tag{3}$$

The points (x,λ,p) satisfying (1) and (3) correspond to bifurcations and a curve of such points can be obtained by varying p about its nominal value p_*. Linearization of this curve about the bifurcation (x_*,λ_*,p_*) yields

$$f_x|_*\Delta x+f_\lambda|_*\Delta\lambda+f_p|_*\Delta p=0 \tag{4}$$

where f_λ is the derivative of f with respect to the load powers λ and f_p is the derivative of f with respect to the parameter p. '$|_*$' means 'evaluated at (x_*, λ_*, p_*)'. Premultiplication by $w = w(x_*, \lambda_*, p_*)$ yields

$$w f_\lambda|_* \Delta\lambda + w f_p|_* \Delta p = 0 \tag{5}$$

since (3) implies that $w f_x|_* = 0$. Equation (5) can be interpreted as stating that $(w f_\lambda|_*, w f_p|_*)$ is the normal vector at (λ_*, p_*) to the bifurcation set in a load power and parameter space [18].

Using the parameterization of λ by L from (2) yields $\Delta\lambda = \hat{k}\Delta L$ and substitution in (5) gives

$$w f_\lambda \hat{k}\, \Delta L + w f_p\, \Delta p = 0 \tag{6}$$

and hence the sensitivity of the loading margin to the change in parameters is

$$L_p|_* = \frac{-w f_p|_*}{w f_\lambda|_*\, \hat{k}} \tag{7}$$

For the linear estimate we use (7) and

$$\Delta\lambda = L_p|_* \Delta p \tag{8}$$

The same formula holds for multiple parameters p, in which case $w f_p|_*$ is a vector (see appendices). This is useful when approximating the combined effects of changes in several parameters or when comparing the effects of various parameters on the loading margin.

For the quadratic approximation we use (7), (A9) and

$$\Delta\lambda = L_p|_* \Delta p + \frac{1}{2} L_{pp}|_* (\Delta p)^2 \tag{9}$$

5. Discussion

The loading margin sensitivities only depend on quantities evaluated at the nominal bifurcation point. Evaluation of the linear sensitivity is particularly simple. Once the nominal bifurcation point is computed, the linear sensitivity (7) requires computation of the left eigenvector w and the derivative $f_p|_*$ of the power system equations with respect to the parameter. In many cases $f_p|_*$ has only one or two nonzero entries. w can be found by inverse power methods or as a byproduct of a direct method used to refine location of the bifurcation point [18]. Since w is the same regardless of the parameter chosen, it is very quick to compute the sensitivity to any additional parameters.

The quadratic estimate additionally requires solution of a sparse set of linear equations (A6,A8), the right eigenvector v and some second order derivatives. The second order derivatives include the matrix $w f_{xx}|_*$, where $f_{xx}|_*$ is the Hessian tensor. $w f_{xx}|_*$ can be obtained as a byproduct of a direct method that uses a Newton iteration. The other higher order derivatives are more easily obtained and often evaluate to zero. When the quadratic term is small, it increases confidence in the accuracy of the linear estimate. When the quadratic term is not small, it serves as a more accurate estimate.

One source of inaccuracy is the neglect of higher order terms in the estimates. When the computed bifurcation is near a different bifurcation corresponding to voltage collapse of another area of the system, movement of the parameter can cause the voltage collapse to 'shift' from one area to the other. Since the set of critical parameters and loadings could have significant variations in curvature in this case, the linear and quadratic estimates would be useful only over a small parameter range.

Another source of inaccuracy is that the estimates assume a fixed set of equations whereas the form of the equations can change discretely whenever a parameter variation causes power system limits to change. The 118 bus system results are examples in which this does not significantly impair the usefulness of the estimates. However, this source of inaccuracy has the potential to be significant and requires awareness when using the estimates. Future work could address the effect of limits on loading margin sensitivities, perhaps by representing the effect of the limits using homotopy methods [21].

The loading margin and sensitivity computations require only static power system equations but accurately reflect the proximity to voltage collapse of the dynamic power system. In particular, explicit knowledge of load dynamics is not needed.

6. Conclusions

This paper computes linear and quadratic estimates to the variation of the loading margin with respect to any power system parameter or control. These estimates can be used to quickly assess the quantitative effectiveness of various control actions to maintain a sufficient loading margin to voltage collapse. That is, the estimates approximate the change in loading margin for a given change in each control. The estimates are also useful in determining the sensitivity of the loading margin to uncertainties in data. Estimates for any number of parameters or controls require computation of only one nose or bifurcation point.

The sensitivity formulas are rigorously derived in the appendix using bifurcation theory. The quadratic estimate is new and the derivation of the linear estimate improves on previous work in [18]. The derivation is independent of the norm chosen to measure the loading margin.

The practical use of the sensitivity computations is illustrated for a range of system parameters on a voltage collapse of the IEEE 118 bus system. The likely sources of inaccuracy discussed in section 5 include variations in the generator reactive power limits enforced at the nose. The results suggest that the linear estimate is good for many parameters and can sometimes be improved with the quadratic estimate. Direct comparison of different control actions can be made in terms of their effect on the loading margin. The closeness of the estimates over a useful range of parameter variations and the ease of obtaining the linear estimate suggest that the sensitivity computations will be of practical value in avoiding voltage collapse.

7. Acknowledgments

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Appendix A: Derivation of sensitivity formulas

Let $f : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^l \to \mathbb{R}^n$ be a smooth function such that the solutions of

$$0 = f(x, \lambda, p) \tag{A1}$$

are the equilibria of the power system near (x_*, λ_*, p_*). We assume that f has a fold bifurcation at (x_*, λ_*, p_*) satisfying:

F(a) $f(x_*, \lambda_*, p_*) = 0$

F(b) $f_x|_*$ has rank $n - 1$

F(c) $wf_\lambda|_* \hat{k} \neq 0$

F(d) $wf_{xx}|_*(v, v) \neq 0$, where v and w are nonzero vectors satisfying $f_x|_* v = 0$ and $wf_x|_* = 0$.

These are the generic conditions for a fold bifurcation [22]. (They differ slightly from the conditions for a saddle node bifurcation and the distinction between the two bifurcations is discussed in [19]. The fold bifurcation is more appropriate when working with static equations.)

Let λ_0 be the base case loading and let the unit vector $\hat{k} \in \mathbb{R}^m$ be a given direction in loading space. The loading is parameterized by $\ell \in \mathbb{R}$:

$$\lambda(\ell, \hat{k}, \lambda_0) = \lambda_0 + \ell\, \hat{k} \tag{A2}$$

The loading may be measured with any norm, but different norms lead to different unit vectors $\hat{k}$. Let

$$g(x, \ell, p) = f(x, \lambda_0 + \ell\hat{k}, p) \tag{A3}$$

Since $g_x = f_x$, $g_{xx} = f_{xx}$, and $g_\ell = f_\lambda \hat{k}$, g also has a fold bifurcation at (x_*, ℓ_*, p_*) and the corresponding conditions F(a)-(d) are satisfied with g written for f except that F(c) becomes $wg_\ell|_* \neq 0$.

Appendix B proves that near (x_*, ℓ_*, p_*) there is a smooth surface Ψ parameterized by p so that each point on Ψ corresponds to a fold bifurcation. Points on Ψ are of the form $(X(p), L(p), p)$ where $X(p)$ defines the variation of the bifurcating equilibrium with parameter p and $L(p)$ defines the variation of the loading margin with parameter p. In the useful case of one dimensional p, Ψ is a curve. Points of Ψ satisfy

$$g(X(p), L(p), p) = 0 \tag{A4}$$

$$\mu(X(p), L(p), p) = 0 \tag{A5}$$

(A4) states that $(X(p), L(p), p)$ is an equilibrium and (A5) is the condition for bifurcation (μ is defined in Appendix B).

Differentiation of (A4) with respect to p yields

$$g_x X_p + g_\ell L_p + g_p = 0 \tag{A6}$$

Evaluation at (x_*, ℓ_*, p_*) and premultiplication by w leads to $wg_\ell|_* L_p|_* + wg_p|_* = 0$ and the desired first order result

$$L_p|_* = -\frac{wg_p|_*}{wg_\ell|_*} = -\frac{wf_p|_*}{wf_\lambda|_*\,\hat{k}} \tag{A7}$$

The second order term $L_{pp}|_*$ may be found as follows. Differentiation of (A5) (obtained by differentiating (B2)) and evaluation at (x_*, ℓ_*, p_*) yields

$$wg_{xx}|_*(v, X_p|_*) + wg_{x\ell}|_* v L_p|_* + wg_{xp}|_* v = 0 \tag{A8}$$

which, with (A6) evaluated at (x_*, ℓ_*, p_*), is a set of $n+1$ linear equations we may solve for $X_p|_*$. F(b) and F(d) imply that these $n + 1$ equations have rank n and are uniquely solvable for $X_p|_*$.

Differentiation of (A6) gives

$$g_x X_{pp} + 2g_{x\ell} X_p L_p + g_{xx}(X_p, X_p) + 2g_{xp} X_p + g_{\ell\ell} L_p L_p + g_\ell L_{pp} + 2g_{\ell p} L_p + g_{pp} = 0$$

Evaluation at (x_*, ℓ_*, p_*), premultiplication by w, and solving for $L_{pp}|_*$ gives

$$L_{pp}|_* = \frac{-1}{wg_\ell|_*}\Big[2wg_{x\ell} X_p L_p + wg_{xx}(X_p, X_p) + 2wg_{xp} X_p + wg_{\ell\ell} L_p L_p + 2wg_{\ell p} L_p + wg_{pp}\Big]\Big|_* \tag{A9}$$

All terms on the right hand side are known and can easily be expressed in terms of f. If the loading λ appears only linearly in (A1) then (A9) simplifies to

$$L_{pp}|_* = \frac{-1}{wg_\ell|_*}\Big[wg_{xx}(X_p, X_p) + 2wg_{xp} X_p + wg_{pp}\Big]\Big|_* \tag{A10}$$

If, in addition, the parameters p also appear in (A1) as linear terms, then $g_{xp} = g_{pp} = 0$ and the last two terms of the bracket in (A10) vanish.

Appendix B: Construction of Ψ.

The surface Ψ of bifurcation points is constructed as the zero section of a smooth function U. Write B^{ij} for the cofactor of the (i, j) element of a matrix $B \in \mathbb{R}^{n\times n}$. Since condition F(b) states that $g_x|_*$ has rank $n - 1$, we can find i and j such that $(g_x|_*)^{ij} \neq 0$. Since the cofactors of a matrix are smooth functions of the entries of the matrix, there is a neighborhood $S \subset \mathbb{R}^{n\times n}$ of $g_x|_*$ such that $B^{ij} \neq 0$ for $B \in S$. Define the smooth functions $\tilde{w}(B) = (B^{1j}, B^{2j}, ..., B^{nj})$ and $\tilde{v}(B) = (B^{i1}, B^{i2}, ..., B^{in})^T$. Then

$$\tilde{w}(B)B = \det B\, e_j^T \quad \text{and} \quad B\tilde{v}(B) = \det B\, e_i \tag{B1}$$

where e_i is a column vector of all zeros except that the ith position has value one. $\tilde{w}(B)$ and $\tilde{v}(B)$ are non-zero vectors for $B \in S$. Define $w = \tilde{w}(g_x|_*)$ and $v = \tilde{v}(g_x|_*)$. It follows from (B1) and $\det g_x|_* = 0$ that $w\, g_x|_* = 0$ and $g_x|_*\, v = 0$ so that w and v are nonzero vectors satisfying the conditions in the definition of the fold bifurcation.

Define the smooth map $\beta : S \to \mathbb{R}$ by $\beta(B) = \tilde{w}(B)B\,\tilde{v}(B)$ It follows from (B1) that $\beta(B) = B^{ij}\det B$. Since g_x is smooth, there is a neighborhood N about (x_*, ℓ_*, p_*) such that $(x, \ell, p) \in N$ implies $g_x|_{(x,\ell,p)} \in S$. Define $\mu : N \to \mathbb{R}$ by

$$\mu(x, \ell, p) = \beta(g_x(x, \ell, p)) = \tilde{w}\big(g_x(x, \ell, p)\big) g_x(x, \ell, p) \tilde{v}\big(g_x(x, \ell, p)\big) \tag{B2}$$

and define $U : N \to \mathbb{R}^n \times \mathbb{R}$ by $U(x, \ell, p) = \begin{pmatrix} g(x, \ell, p) \\ \mu(x, \ell, p) \end{pmatrix}$. Then Ψ is defined as the zero section of U: $\Psi = U^{-1}(0)$.

The matrix $(U_x, U_\ell)|_* = \begin{pmatrix} g_x|_* & g_\ell|_* \\ wg_{xx}|_* v & wg_{x\ell}|_* v \end{pmatrix}$ is invertible if $a = b = 0$ is the only solution to

$(U_x, U_\ell)|_* \begin{pmatrix} a \\ b \end{pmatrix} = 0$, or, equivalently,

$$g_x|_* a + g_\ell|_* b = 0 \qquad \text{(B3)}$$

$$w g_{xx}|_* v a + w g_{x\ell}|_* v b = 0 \qquad \text{(B4)}$$

Since F(c) implies that $g_\ell|_*$ is not in the range of $g_x|_*$, to satisfy (B3), $b = 0$ and then F(b) implies that $a = \alpha v$ for some scalar α. Then (B4) with $b = 0$ yields $\alpha w g_{xx}|_*(v, v) = 0$ and F(d) implies $\alpha = 0$ and $a = 0$. Thus $(U_x, U_\ell)|_*$ is invertible.

It then follows from the implicit function theorem that there is a neighborhood P of p_* and smooth functions $X : P \to \mathbb{R}^n$ and $L : P \to \mathbb{R}$ such that $\{(X(p), L(p), p) \mid p \in P\} \subset \Psi$ and $U(X(p), L(p), p) = 0$, which can be rewritten as (A4) and (A5).

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