Solving Unit Commitment by a Unit Decommitment Method[*]

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Abstract

In this paper, we present an efficient and robust method for solving unit commitment problem using a unit decommitment method.

1 Introduction

A problem that must be frequently solved by a power utility is to economically determine a schedule of what units will be used to meet the forecasted demand, and operating constraints such as spinning reserve requirements, over a short time horizon. This problem is commonly referred to as the unit commitment (UC) problem. The UC problem is a mixed integer programming problem, and is in the class of NP-hard problems [11]. Many optimization methods have been proposed to solve the UC problem (e.g. [4]). Among them, the Lagrangian relaxation (LR) methods [1, 4, 5] are the most advanced and widely used approaches. The LR approaches, though popular, are known to require many heuristics which strongly influence their performance [4, 13].

In [7], Li *et al.* proposed a heuristic method for solving UC. Their method mimics the LR approach, and the multipliers are taken from the economic dispatch phase, rather than updated by the subgradient iteration. In [10], a unit decommitment (UD) method was developed as a post-processing tool to improve the solution quality of the existing UC algorithms. In this paper, we consolidate the approaches of both papers and extend them to a more general formulation. The proposed method is a unified unit decommitment method. This method starts with a solution having all available units on-line at all hours in the planning horizon and determines an optimal strategy

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for decommitting units, one at a time. We show that the proposed method may be viewed as an approximate implementation of the LR approach. The multiplier updating rule is similar to that in [7]. Furthermore, we show that the number of iterations required by the method is bounded by the number of units. Empirical tests suggest that the proposed method is more efficient and robust than the LR approach.

This paper is organized as follows. In Section 2, the UC problem is formulated. Section 3 generalizes some important properties of economic dispatch. We generalize the UD method and propose the algorithm for solving UC using the UD method in Section 4. The relation between the proposed method and the LR approach is also discussed. Finally we generate random instances of the UC problems and solve them by the proposed method. The numerical test results and conclusion are given in Section 5.

2 Problem formulation

In this paper the following standard notation will be used. Additional symbols will be introduced when necessary.

i : index for the number of units ($i = 1, \cdots, I$)

t : index for time ($t = 0, \cdots, T$)

u_{it} : zero-one decision variable indicating whether unit i is up or down in time period t

x_{it} : state variable indicating the length of time that unit i has been up or down in time period t

t_i^{on} (t_i^{off}) : the minimum number of periods unit i must remain on (off) after it has been turned on (off)

p_{it} : state variable indicating the amount of power unit i is generating in time period t

$p_i^{\min}$ ($p_i^{\max}$) : minimum (maximum) rated capacity of unit i

$r_i^{\max}$: maximum reserve for unit i

$r_i(p_{it})$: reserve available from unit i in time period t ($\equiv \min(r_i^{\max}, p_i^{\max} - p_{it})$)

$C_i(p_{it})$: fuel cost for operating unit i at output level p_{it} in time period t

$S_i(x_{i,t-1}, u_{it}, u_{i,t-1})$: startup cost associated with turning on unit i at the beginning of time period t

D_t : forecast demand in time period t

R_t : spinning reserve requirement in time period t

The unit commitment problem is formulated as the following mixed-integer programming problem: (the underlined variables denote vectors, e.g. $\underline{u} = (u_{11}, \cdots, u_{IT})$.)

$$\min_{\underline{u},\underline{x},\underline{p}} \sum_{t=1}^{T} \sum_{i=1}^{I} [C_i(p_{it}) u_{it} + S_i(x_{i,t-1}, u_{it}, u_{i,t-1})] \tag{2.1}$$

subject to the demand constraints,

$$\sum_{i=1}^{I} p_{it} u_{it} = D_t, \quad t = 1, \cdots, T, \tag{2.2}$$

and the spinning reserve constraints,

$$\sum_{i=1}^{I} r_i(p_{it}) u_{it} \geq R_t, \quad t = 1, \cdots, T, \tag{2.3}$$

where $r_i(p_{it}) \equiv \min(r_i^{\max}, p_i^{\max} - p_{it})$. There are other unit constraints such as unit capacity constraints,

$$p_i^{\min} \leq p_{it} \leq p_i^{\max}, \; i = 1, \cdots, I; \; t = 1, \cdots, T, \tag{2.4}$$

the state transition equation for $i = 1, \cdots, I$,

$$x_{it} = \begin{cases} \max(x_{i,t-1}, 0) + 1, & \text{if } u_{it} = 1, \\ \min(x_{i,t-1}, 0) - 1, & \text{if } u_{it} = 0, \end{cases} \tag{2.5}$$

the minimum up/down time constraints for $i = 1, \cdots, I$,

$$u_{it} = \begin{cases} 1, & \text{if } 1 \leq x_{i,t-1} < t_i^{\text{on}}, \\ 0, & \text{if } -1 \geq x_{i,t-1} > -t_i^{\text{off}}, \\ 0 \text{ or } 1, & \text{otherwise}, \end{cases} \tag{2.6}$$

and the initial conditions on x_{it} at $t = 0$ for $\forall i$.

In the objective function, we assume that the fuel cost function C_i of a unit, say i, to be a smooth and strictly convex function of the power output(MWh) of the unit. The startup costs $S_i(x_{i,t-1}, u_{it}, u_{i,t-1})$ is an increasing function of the length of time that the unit has been off, i.e. $x_{i,t-1}$. To further simplify the notation, we let $S_i(\underline{u}, t) = S_i(x_{i,t-1}, u_{it}, u_{i,t-1})$.

3 Reserve-constrained economic dispatch

Given a known commitment $\underline{\tilde{u}} = \{\tilde{u}_{it}\}$ satisfying (2.5) and (2.6), economic dispatch (ED) is a problem of allocating system demand among all on-line generating units while satisfying (2.2), (2.3) and (2.4) at any time over the planning horizon, i.e. to determine the corresponding $\underline{\tilde{p}} = \{\tilde{p}_{it}\}$. (In this paper, variables denoted with a *tilde* hat denote fixed realization of the corresponding variables.) If the spinning reserve constraints (2.3) are not considered, the ED problem is a conventional resource allocation (RA) problem (e.g. [6]), which has the form:

$$\min\{\sum_i C_i(p_i) | \sum_i p_i = D;\ p_i^{\min} \leq p_i \leq p_i^{\max}\}.) \tag{3.7}$$

Optimality of such a RA-type ED problem requires that all generators operate at a marginal cost that either equals same fixed value Lambda (Lagrange multiplier) or equals the marginal cost corresponding to the upper or lower bound of a generator's output level, whichever is closer to Lambda. This property is commonly referred to as the 'equal-Lambda' rule (e.g. [12]). We use the term 'an equal-Lambda method' to refer to a method which solves the RA-type ED problem and presents the solution as well as the Lambda. An equal-Lambda method can be implemented very efficiently such that it obtains the optimal solution within strongly polynomial time if $C_i(\cdot)$ are quadratic convex functions [2].

With the presence of the reserve constraints (2.3), the problem becomes a reserve-constrained economic dispatch (RCED). Methods for obtaining approximate solutions for RCED has been proposed, e.g. [8, 9]. In this section, we state some mathematical properties of RCED. Note that the RCED problem is separable in time, it can be solved sequentially by hour t. Define the index set of on-line units at time t with respect to this feasible commitment $J(t; \underline{\tilde{u}}) \equiv \{i | \tilde{u}_{it} = 1\}$. For simplicity, $\tilde{J}_t = J(t; \underline{\tilde{u}})$. The RCED problem in time t is denoted by

$$rced(\tilde{J}_t, t) \equiv \min_{p_i^{\min} \leq p_{it} \leq p_i^{\max}} \{\sum_{i \in \tilde{J}_t} C_i(p_{it}) | \sum_{i \in \tilde{J}_t} p_{it} = D_t; \sum_{i \in \tilde{J}_t} r_{it}(p_{it}) \geq R_t\}, \forall t. \tag{3.8}$$

Also assume $\underline{\tilde{p}} = \{\tilde{p}_{it}\}$ solves $rced(\tilde{J}_t, t)$, if the solution exists.

Proposition 1 The solution of $rced(\tilde{J}_t, t)$ exists, *if and only if*, the following conditions hold.

$$\sum_{i \in \tilde{J}_t} p_i^{\min} \leq D_t \leq \sum_{i \in \tilde{J}_t} p_i^{\max}, \tag{3.9a}$$

$$\sum_{i \in \tilde{J}_t} r_i^{\max} \geq R_t, \tag{3.9b}$$

and

$$\sum_{i \in \tilde{J}_t} p_i^{\max} \geq D_t + R_t. \tag{3.9c}$$

Proof. The *if* part is obvious. To show the *only if* part, note that (3.9b) implies that there exists $\{\tilde{r}_i\}$ such that $\sum_{i \in \tilde{J}_t} \tilde{r}_i = R_t$, $0 \leq \tilde{r}_i \leq r_i^{\max}$, $\forall i \in \tilde{J}_t$. Since $\sum_{i \in \tilde{J}_t} p_i^{\min} \leq D_t \leq \sum_{i \in \tilde{J}_t} p_i^{\max} - R_t = \sum_{i \in \tilde{J}_t}(p_i^{\max} - \tilde{r}_i)$, there exists $\{\tilde{p}_{it}\}$ such that $\sum_{i \in \tilde{J}_t} \tilde{p}_{it} = D_t$, and $p_i^{\min} \leq \tilde{p}_{it} \leq p_i^{\max} - \tilde{r}_i$, $\forall i \in \tilde{J}_t$. (Note: $\sum_{i \in \tilde{J}_t} r_i(\tilde{p}_{it}) \geq R_t$.) ■

Proposition 2 Assume $\{\tilde{p}_{it}; \tilde{u}_{it}\}$ is an optimal solution of RCED. Then there exist two mutually exclusive and exhaustive subsets of $\tilde{J}_t$, $\tilde{\Omega}_t$ and $\tilde{\Lambda}_t$, i.e. $\tilde{\Omega}_t \cup \tilde{\Lambda}_t = \tilde{J}_t$, and $\tilde{\Omega}_t \cap \tilde{\Lambda}_t = \emptyset$, and (Lagrange multipliers) $\tilde{\lambda}_t$, $\tilde{\alpha}_t$ and $\tilde{\mu}_t$, $t = 1, \cdots, T$, such that

$$\left.\begin{array}{ll} C_i'(\tilde{p}_{it}) = \tilde{\alpha}_t, & \text{for } p_i^{\max} - r_i^{\max} < \tilde{p}_{it} < p_i^{\max} \\ C_i'(\tilde{p}_{it}) \le \tilde{\alpha}_t, & \text{for } \tilde{p}_{it} = p_i^{\max}, \end{array}\right\}, \ \forall i \in \tilde{\Omega}_t \tag{3.10}$$

$$\left.\begin{array}{ll} C_i'(\tilde{p}_{it}) = \tilde{\lambda}_t, & \text{for } p_i^{\min} < \tilde{p}_{it} < p_i^{\max} - r_i^{\max} \\ C_i'(\tilde{p}_{it}) \le \tilde{\lambda}_t, & \text{for } \tilde{p}_{it} = p_i^{\max} - r_i^{\max} \\ C_i'(\tilde{p}_{it}) \ge \tilde{\lambda}_t, & \text{for } \tilde{p}_{it} = p_i^{\min} \end{array}\right\}, \ \forall i \in \tilde{\Lambda}_t \tag{3.11}$$

$$\tilde{\mu}_t(\sum_{i \in \tilde{J}_t} \min(p_i^{\max} - \tilde{p}_{it}, r_i^{\max}) - R_t) = 0 \tag{3.12}$$

$$\tilde{\mu}_t = \tilde{\lambda}_t - \tilde{\alpha}_t \tag{3.13}$$

$$\tilde{\lambda}_t \ge 0; \ \tilde{\alpha}_t \ge 0; \ \tilde{\mu}_t \ge 0. \tag{3.14}$$

for $\forall t$. ■

The proof of Proposition 2 is straightforward and is omitted here. An intuitive way to interpret the optimality condition is to divide the units into two categories: $\tilde{\Omega}_t$ is the set of units with 'cheap' reserve but 'expensive generation', and $\tilde{\Lambda}_t$ is the counterpart.

In Appendix, we give an algorithm for obtaining the optimal solution for RCED in time t, and its associated multipliers $\tilde{\lambda}_t$ and $\tilde{\mu}_t$.

3.1 Post optimality analysis

The $\{\tilde{\lambda}_t\}$ and $\{\tilde{\mu}_t\}$ in Proposition 2 are the Lagrange multipliers for constraints (2.2) and (2.3) respectively. Given that $\{\tilde{p}_{it}\}$ solves *rced*$(\tilde{J}_t, t)$, suppose $j \in \tilde{J}_t$, to know whether $\tilde{J}_t \backslash \{j\}$ is a more economic commitment in time t (ignoring other physical constraints, e.g. minimum uptime constraints at this point), one can either directly evaluate *rced*$(\tilde{J}_t \backslash \{j\}, t)$, or estimate the increased dispatch cost due to the decommitment of unit j in time t using the Lagrange multipliers associated with *rced*$(\tilde{J}_t, t)$. We investigate the latter approach next. Assume that decommitting unit j in time t yields generation level $\tilde{p}_{it} + \Delta\tilde{p}_{it}$ for unit $i \in \tilde{J}_t \backslash \{j\}$. The following properties of $\{\Delta\tilde{p}_{it}\}$ can be shown: (i) $\sum_{i \in \tilde{J}_t \backslash \{j\}} \Delta\tilde{p}_{it} = \tilde{p}_{jt}$. (ii) $\Delta\tilde{p}_{it} \le 0$, for $\forall i \in \tilde{\Omega}_t \backslash \{j\}$; $\Delta\tilde{p}_{it} \ge 0$, for $\forall i \in \tilde{\Lambda}_t \backslash \{j\}$. (iii) $\sum_{i \in \tilde{\Omega}_t \backslash \{j\}} \Delta\tilde{p}_{it} + r_j(\tilde{p}_{jt}) = 0$. (The interested reader may think of these properties using the algorithm provided in the Appendix.) Property (i) is due to the load balance equation; (ii) and (iii) show that units in $\tilde{\Omega}_t \backslash \{j\}$, units with 'expensive reserve', are the only units to decrease generation so as to make up the loss of reserve originally provided by unit j; while the units in $\tilde{\Lambda}_t \backslash \{j\}$, units with 'expensive generation', would increase generation to balance the load. Since all the fuel cost functions C_i are assumed smooth and convex, from Proposition 2, we can estimate the increased dispatch cost due to the decommitment:

$$
\begin{aligned}
& \sum_{i \in \tilde{J}_t \backslash \{j\}} (C_i(\tilde{p}_{it} + \Delta\tilde{p}_{it}) - C_i(\tilde{p}_{it})) && (3.15) \\
\approx & \sum_{i \in \tilde{J}_t \backslash \{j\}} C_i'(\tilde{p}_{it}) \Delta\tilde{p}_{it} && (3.16) \\
\approx & \tilde{\lambda}_t \sum_{i \in \tilde{\Lambda}_t \backslash \{j\}} \Delta\tilde{p}_{it} + (\tilde{\lambda}_t - \tilde{\mu}_t) \sum_{i \in \tilde{\Omega}_t \backslash \{j\}} \Delta\tilde{p}_{it} && (3.17) \\
= & \tilde{\lambda}_t \tilde{p}_{jt} - \tilde{\mu}_t \sum_{i \in \tilde{\Omega}_t \backslash \{j\}} \Delta\tilde{p}_{it} \\
= & \tilde{\lambda}_t \tilde{p}_{jt} + \tilde{\mu}_t r_j(\tilde{p}_{jt}). && (3.18)
\end{aligned}
$$

Note that it can be further shown that both approximate relations ($\approx$) in (3.16) and (3.17) can be replaced by inequalities ($\geq$).

4 Solving UC using UD

4.1 A unit decommitment method

In Section 3.1, given a feasible schedule $(\underline{\tilde{u}}, \underline{\tilde{p}})$ (assumed economically dispatched) we discussed the method used to estimate the increased cost due to the decommitment of one unit. Now we incorporate other physical constraints and present the problem of optimally decommitting a unit, say unit j, with other units' commitments fixed (i.e. $\tilde{u}_{it}$ remains unchanged but $\tilde{p}_{it}$ are subject to change for $\forall i \neq j, \forall t$.) The formulation is as follows.

$$
(P_j) \quad \min_{u_{jt} \in \{0,1\}} \sum_{t=1}^{T} [C_j(\tilde{p}_{jt}) u_{jt} + (\tilde{\lambda}_t \tilde{p}_{jt} + \tilde{\mu}_t r_j(\tilde{p}_{jt}))(1 - u_{jt}) + S_j(\underline{u}, t)] \qquad (4.19)
$$

subject to

$$
u_{jt} = \begin{cases} 0 & \text{if } \tilde{u}_{jt} = 0, \\ 1 & \text{if } \tilde{u}_{jt} = 1, \text{ and the removal of } j \text{ from } \tilde{J}_t \text{ would} \\ & \text{result in violation of (3.9a) to (3.9c).} \end{cases} \qquad (4.20)
$$

and the minimum uptime, downtime constraints and the initial conditions for unit j.

Note that in (4.19) u_{jt} and $(1 - u_{jt})$ are two mutually exclusive decisions. If unit j is on-line in time t ($u_{jt} = 1$) the generation cost is $C_j(\tilde{p}_{jt})$; if decommitted, the increased cost of other units is approximated by $\tilde{\lambda}_t \tilde{p}_{jt} + \tilde{\mu}_t r_j(\tilde{p}_{jt})$. The startup cost of unit j is imposed whenever applicable. (P_j) is an integer programming problem and can be solved using dynamic programming. In this paper, the solution of (P_j) will be called the *tentative* commitment of unit j.

In the following algorithm, superscript k denotes the k-th iteration of the algorithm. Let $\tilde{\Theta}_i^k$, $i = 1, \cdots, I$ be the total generating cost (fuel cost and startup cost) of unit i of the feasible schedule $(\underline{\tilde{u}}^k, \underline{\tilde{p}}^k)$; and Θ_i^k, $i = 1, \cdots, I$, the optimal objective value of (P_i^k) solved with respect to feasible solution $(\underline{\tilde{u}}^k, \underline{\tilde{p}}^k)$. We now state the decommitment algorithm.

The UD algorithm

Data: Feasible solution $(\underline{\tilde{u}}^0, \underline{\tilde{p}}^0)$ and the corresponding $\tilde{\Theta}_i^0$, $i = 1, \cdots, I$ are given.

Step 0: $k \leftarrow 0$.

Step 1: Solve (P_i^k) with respect to $(\underline{\tilde{u}}^k, \underline{\tilde{p}}^k)$ and obtain Θ_i^k for all $i = 1, \cdots, I$.

Step 2: Let $m = \arg\max\{\tilde{\Theta}_i^k - \Theta_i^k | i = 1, \cdots, I\}$. If $(\tilde{\Lambda}_m^k - \Lambda_i^k) \leq 0$, stop; otherwise update the commitment of unit m in $\underline{\tilde{u}}^k$ by the tentative commitment obtained in (P_m^k). The resultant unit commitment is assigned to be $\underline{\tilde{u}}^{k+1}$.

Step 3: Perform the RCED on $\underline{\tilde{u}}^{k+1}$ to obtain $\underline{\tilde{p}}^{k+1}$
and evaluate $\tilde{\Theta}_i^{k+1}$, the total generating cost of unit i, $i = 1, \cdots, I$.

Step 4: $k \leftarrow k + 1$, go to Step 1. ■

The algorithm in Step 2 chooses the tentative commitment which can yield most savings to replace the original commitment. That is, the method corrects the units' commitment, one unit at a time.

4.2 Initial feasible solution

Solving UC by means of UD requires finding first an initial feasible solution $(\underline{\tilde{u}}, \underline{\tilde{p}})$. While in theory obtaining a feasible solution of the UC problem is an NP-hard problem [11], it is a relatively easy task in real world instances of that problem. Methods based on priority lists to sequentially commit units (e.g. [3]) can be used to construct an initial feasible solution. An intuitive approach is to initially turn on as many units as possible in all hours without violating the minimum up/down time constraints. However, when initially all units are committed, such a commitment tend to violate (3.9a), i.e. the so called minimum load conditions. In other words, the RCED phase in Step 4 of the UD algorithm may not be feasible. A possible modification is to dispatch the on-line generators so as to equalize the marginal costs to the extent possible, even if the minimum load conditions are not satisfied. That is, when

$$\sum_{i \in \tilde{J}_t} p_i^{\min} > D_t, \tag{4.21}$$

all on-line units are dispatched to their minimum capacities respectively,

$$\tilde{p}_{it} \leftarrow p_i^{\min},\ \forall i, \tag{4.22}$$

and the corresponding Lambda is the minimum of the marginal costs of the corresponding dispatches in (4.22),

$$\tilde{\lambda}_t = \tilde{\mu}_t \leftarrow \min_{i \in \tilde{J}_t} C'(p_i^{\min});\ \tilde{\alpha}_t \leftarrow 0. \tag{4.23}$$

Such a modification of the RCED phase above is based on the expectation that as the decommitment procedure proceeds, the commitment obtained will eventually satisfy the minimum load conditions, thus producing a feasible schedule. From a theoretical perspective, determining a feasible solution of the UC problems is NP-hard as mentioned. However, in extensive numerical tests, we have found that the above approach worked satisfactorily. In all observed cases, the UD method performed well as a UC algorithm and obtained feasible solutions. Furthermore, we have the following theorem.

Theorem 1 With the modification in (4.23) of the RCED phase, the UC algorithm terminates within I iterations, where I is the number of units. ■
The proof of Theorem 1 is somewhat tedious and is omitted here. The key is to show that $\{C_j(\tilde{p}_{jt}^k) - (\tilde{\lambda}_t^k \tilde{p}_{jt}^k + \tilde{\mu}_t^k r_j(\tilde{p}_{jt}^k))\}$ is a nonincreasing sequence in k such that once a unit is selected at some iteration, it will not be selected again in Step 2 of the algorithm. The interested reader is directed to [10], where a similar case was proved.

4.3 UD vs. LR

In this section, we present an intuitive discussion on the relationship between the UD method and the LR method for solving the UC problem. Let λ_t and μ_t $(t = 1, \cdots, T)$ be the corresponding nonnegative Lagrange multipliers to (2.2) and (2.3). Conventional LR approaches solve the following dual problem (D):

$$(D)\ \max_{\underline{\lambda}, \underline{\mu} \geq 0} d(\underline{\lambda}, \underline{\mu}), \tag{4.24}$$

where

$$\begin{aligned} d(\lambda_t, \mu_t) &= \min_{\lambda_t, \mu_t \geq 0} \sum_{i=1}^{I} \sum_{t=1}^{T} [C_i(p_{it}) u_{it} + S_i(\underline{u}, t) \\ &\qquad + \lambda_t (D_t - \sum_{i=1}^{I} p_{it} u_{it}) + \mu_t (R_t - \sum_{i=1}^{I} r_i(p_{it}) u_{it})] \\ &= \sum_{i=1}^{I} d_i(\lambda_t, \mu_t) + \sum_{t=1}^{T} (\lambda_t D_t + \mu_t R_t), \end{aligned} \tag{4.25}$$

and

$$d_i(\lambda_t, \mu_t) = \min_{u_{it}, p_{it}} \sum_{t=1}^{T} [C_i(p_{it})u_{it} + S_i(\underline{u}, t) - \lambda_t p_{it} u_{it} - \mu_t r_i(p_{it}) u_{it}]. \tag{4.26}$$

The minimization problem (4.26) is subject to (2.4) to (2.6) and the initial conditions. The multipliers λ_t and μ_t are then updated by the subgradient rule (i.e., the ascent direction of $d(\cdot, \cdot)$) so as to maximize $d(\cdot, \cdot)$. It is well known that even if (D) is completely solved, it would not yield a feasible unit schedule. Conventional LR approaches thus fall into a category of "two-phase" algorithms, with a feasibility phase following the first phase of dual optimization. In [10, 11], Tseng *et al.* suggested to add a third phase, a UD phase as the post-processing phase, to the conventional two-phase methods. This results in a "three-phase" scheme. Advantages of using such a three-phase scheme other than improving the solution quality were discussed in [10].

With $i = j$ and some arrangement, the dual subproblem $d_j(\lambda_t, \mu_t)$ resembles (P_j). It can be shown that, using duality theory, that given $\tilde{\lambda}_t$ and $\tilde{\mu}_t$ obtained from RCED, if $\{\tilde{p}_{jt}; \tilde{u}_{jt}\}$ solves $d_j(\tilde{\lambda}_t, \tilde{\mu}_t)$, it also solves (P_j). This fact implies that a solution of an LR subproblem is already optimally decommitted (in the sense defined in this paper.) The major difference between LR and UD lies in their updating rules. The LR approaches consider at each step the tentative commitments of *all* units'; while the UD method corrects the commitment of only one unit's in each iteration. The overcorrection of the LR approaches leads to its primal infeasibility at all iterations, while the latter adjusts the commitment, one unit at a time, toward a feasible commitment and then maintains feasibility thereafter. It is fair to say that the UD method is a LR-like method and the differences are that the multipliers λ_t and μ_t are taken from RCED phase rather than updated by the subgradient iteration. Figure 1 gives a illustration of the typical algorithm trajectories of the conventional two-phase methods, the three-phase method [10, 11] and the proposed method.

5 Numerical results and conclusions

We conduct numerical tests to compare the performance of UD and LR. All algorithms are implemented in FORTRAN on a HP 700 workstation. Four cases of systems with combinations of 10, 30 units, and 24, 168 hours of planning horizon are tested. For each case, we randomly generate 100 instances of the UC problem. (Detailed configuration of the random instances are available upon request for the authors.) Each instance is solved by the LR and UD methods. The column under D.G. records the duality gap of the LR approach in terms of the percentage of the dual value. Since the comparisons are normalized to the value of the LR approach, the columns under LR consist of all "ones". Also the two numbers in a parenthesis define a range of the sample points. The mean of the sample points is recorded on the top of the corresponding parentheses. The test results including solution qualities and CPU times required for both methods are summarized in Table 1.

From Table 1, we know that the errors between LR and UD is within 0.2%, and the UD methods takes much less (save about 50% at least) CPU time than the LR approach. To sum up, the numerical testing results show that UD serves as a reliable, efficient and robust alternative to the traditional implementations of the LR approaches for solving UC.

Table 1: Comparison of UD and LR

Case	Solution Quality			CPU Time	
	LR	UD	D.G. (%)	LR	UD
10×24	1	1.0010 (0.9933-1.0105)	0.9 (0.09-2.85)	1	0.2185 (0.1028-0.4773)
10×168	1	1.0008 (0.9976-1.0054)	0.9 (0.38-2.04)	1	0.1500 (0.0978-0.3446)
30×24	1	1.0013 (0.9981-1.0090)	0.28 (0.06-0.81)	1	0.5214 (0.3344-0.8608)
30×168	1	1.0017 (0.9997-1.0058)	0.35 (0.15-1.78)	1	0.2745 (0.1513-0.4152)

Appendix

In the Appendix, we shall present an algorithm to solve RCED and obtain the Lambda and Mu as required in the development of the method presented in this paper.

Algorithm for solving RCED

Step 1: Apply an equal-Lambda method to solve the reserve-unconstrained case:

$$\begin{array}{ll} \min\limits_{p_{it}} & \sum_{i\in\tilde{J}_t} C_i(p_{it}) \\ \text{s.t.} & \sum_{i\in\tilde{J}_t} p_{it} = D_t \\ & p_i^{\min} \le p_{it} \le p_i^{\max},\ \forall i \in \tilde{J}_t. \end{array}$$

Assume $\{\tilde{p}_{it}\}$ is the solution and the Lambda is denoted by $\tilde{\lambda}_t$. If $\sum_{\tilde{J}_t} r_i(\tilde{p}_{it}) \ge R_t$, then $\tilde{\alpha}_t = \tilde{\lambda}_t$, stop and $\{\tilde{p}_{it}\}$ is also an optimal solution of RCED. Otherwise discard $\{\tilde{p}_{it}\}$ and go to Step 2.

Step 2: Apply an equal-Lambda method to solve the following RA problem:

$$\begin{array}{cc} \min\limits_{p_{it}} & \sum_{i\in \tilde{J}_t} C_i(p_{it}) \\ \text{s.t.} & \sum_{i\in \tilde{J}_t}(p_i^{\max} - p_{it}) = R_t \\ & p_i^{\max} - r_i^{\max} \le p_{it} \le p_i^{\max},\ \forall i \in \tilde{J}_t. \end{array}$$

Assume $\{\tilde{p}_{it}\}$ is the solution, and the Lambda is denoted by $\tilde{\alpha}_t$. Let $\tilde{\Omega}_t = \{i | p_i^{\max} - r_i^{\max} < \tilde{p}_{it}\}$ and $\tilde{\Lambda}_t = \tilde{J}_t \backslash \tilde{\Omega}_t$.

Step 3: Apply an equal-Lambda method to solve the following RA problem:

$$\begin{array}{cc} \min\limits_{p_{it}} & \sum_{i\in \tilde{\Lambda}_t} C_i(p_{it}) \\ \text{s.t.} & \sum_{i\in \tilde{\Lambda}_t} p_{it} = D_t - \sum_{i\in \tilde{\Omega}_t} p_{it} \\ & p_i^{\min} \le p_{it} \le p_i^{\max} - r_i^{\max},\ \forall i \in \tilde{\Lambda}_t \end{array}$$

Assume the solution is $\hat{p}_{it}$, $i \in \tilde{\Lambda}_t$, and the Lambda is denoted by $\tilde{\lambda}_t$.

Step 4: Let $\tilde{p}_{it} \leftarrow \hat{p}_{it}$, $i \in \tilde{\Lambda}$. $\{\tilde{p}_{it}\}$ is the solution of RCED. ■

It can be verified that the $\{\tilde{p}_{it}\}$ as well as $\tilde{\lambda}_t$ and $\tilde{\mu}_t$ satisfy the optimality condition stated in Proposition 2.

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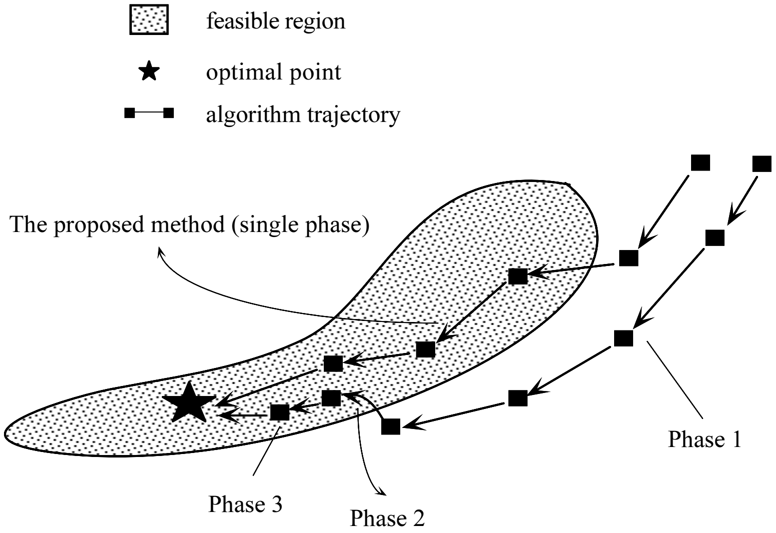

Figure 1: Comparison of algorithm trajectories