A Transmission-Constrained Unit Commitment Method

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Abstract

This paper presents a transmission-constrained unit commitment method using a Lagrangian relaxation approach. The transmission constraints are modeled as linear constraints based on a DC power flow model. The transmission constraints, as well as the demand and spinning reserve constraints, are relaxed by attaching Lagrange multipliers. In this paper we take a new approach in the algorithmic scheme. A three-phase algorithm is devised including dual optimization, a feasibility phase and unit decommitment. A test problem involving more than 2500 transmission lines and 2200 buses is tested along with other test problems.

1. Introduction

The unit commitment problem at a power utility like PG&E requires economically scheduling generating units over a planning horizon so as to meet forecast demand and system operating constraints. It has been an active research subject due to potential cost saving and the difficulty of the problem. The unit commitment problem is a mixed integer programming problem, and is proved to be NP-hard in [15]. Many optimization methods have been proposed to solve the unit commitment problem (e.g. see [5] for a survey). These methods include priority list methods [4], dynamic programming methods [11, 12, 17], sequential method [8] and Lagrangian relaxation methods [3, 5, 6, 7], etc. Lagrangian relaxation methods are now among the most widely used approaches to solving unit commitment.

At PG&E, the Hydro-Thermal Optimization (HTO) program was developed almost a decade ago based on the Lagrangian relaxation approach [6]. In recent work, the Lagrangian relaxation-based algorithm has been extended to schedule thermal units under ramp constraints ([14]).

In today's power system, generating units of a utility company are normally located in multiple areas that are interconnected via transmission lines. If the transmission constraints are not considered, the schedule obtained might cause some transmission lines to be overloaded. In this paper, we will discuss the unit commitment problem which takes transmission constraints into account.

The transmission constraints will be modeled as linear constraints based on a DC power flow model. This is also the approach taken in other papers, e.g. [9, 13]. Methods for solving the transmission-constrained unit commitment problem have been developed. Pang *et al* have considered the problem in [12]. The approach taken in [12] is a dynamic programming method. Lee develops a sequential method in [8, 9], which sequentially determines the commitment of the next most-advantageous unit to commit. The decision making involves a price-adjustment procedure which resembles a bidding process. In [13], Shaw has proposed a practical method for solving the security-constrained unit commitment problem using the Lagrangian relaxation approach. This approach relaxes not only the demand constraints and the spinning reserve constraints, but also the transmission constraints using multipliers. Shaw describes two methods in his paper [13], a direct method and an indirect method. The former takes full account of the transmission constraints in the optimization phase, while the latter does so only in locating a feasible solution. The conclusion of [13] favors the direct method.

Algorithmically, the dual optimization approach in this paper corresponds closely to that presented in [13]. However, we take a new approach in the algorithmic scheme. A three-phase algorithm is proposed including unit decommitment [10, 16]. A thorough discussion of the three-phase algorithm scheme for solving the unit commitment problem can be found in [15]. A test problem involving more than 2500 transmission lines and 2200 buses is tested along with other test problems.

This paper is organized as follows: Section 2 gives the model of the unit commitment problem to be discussed. A three-phase Lagrangian relaxation algorithm is presented in Section 3. Section 4 gives some numerical results. We conclude the paper in Section 5.

2. Problem formulation

The notational convention and problem formulations in this paper have been influenced by the work of Shaw [13]. We first define the following notation.

t : index for time ($t = 0, \cdots, T$)

b : index for the number of buses ($b = 1, \cdots, B$)

Ω_b : index set of units at bus b

i : index for the number of units ($i \in \Omega_b, b = 1, \cdots, B$)

ℓ : index for transmission lines ($\ell = 1, \cdots, L$)

$\Gamma_{\ell b}$: line flow distribution factor for transmission line ℓ due to the net injection at bus b

F_ℓ : the transmission capacity on the transmission line ℓ

u_{it} : zero-one decision variable indicating whether unit i is up or down in time period t.

x_{it} : state variable indicating the length of time that unit i has been up or down in time period t

t_i^{on} : the minimum number of periods unit i must remain on after it has been turned on

t_i^{off} : the minimum number of periods unit i must remain off after it has been turned off

p_{it} : variable indicating the amount of power unit i is generating in time period t

$p_i^{\min}$: minimum rated capacity of unit i

$p_i^{\max}$: maximum rated capacity of unit i

$C_i(p_{it})$: fuel cost for operating unit i at output level p_{it} in time period t

$S_i(x_{i,t-1}, u_{it}, u_{i,t-1})$: startup cost associated with turning on unit i at the beginning of time period t

D_{bt} : forecast demand requirement at bus b in time period t

R_{bt} : spinning capacity requirement at bus b in time period t

The transmission-constrained unit commitment problem is formulated as the following mixed-integer programming problem: (note that the underlined variables are vectors in this paper, e.g. the components of $\underline{u}$ are all legitimate u_{it}.)

Minimize the total generating cost:

$$\min_{\underline{u},\underline{x},\underline{p}} \sum_{t=1}^{T} \sum_{b=1}^{B} \sum_{i\in\Omega_b} [C_i(p_{it})u_{it} + S_i(x_{i,t-1}, u_{it}, u_{i,t-1})] \quad (1)$$

subject to the following constraints:
Demand constraints

$$\sum_{b=1}^{B} \sum_{i\in\Omega_b} p_{it}u_{it} = D_t \equiv \sum_{b=1}^{B} D_{bt},\ \forall t, \quad (2)$$

$$-F_\ell \le \sum_{b=1}^{B} \Gamma_{\ell b}\Big(\sum_{i\in\Omega_b} p_{it}u_{it} - D_{bt}\Big) \le F_\ell,\ \forall \ell, t, \quad (3)$$

Spinning capacity constraints

$$\sum_{b=1}^{B} \sum_{i\in\Omega_b} p_i^{\max} u_{it} \ge R_t \equiv \sum_{b=1}^{B} R_{bt},\ \forall t, \quad (4)$$

Local reserve constraint

$$\sum_{i\in\Omega_b} p_{it}u_{it} \ge r_{bt}D_{bt},\ r_{bt} \in [0,1],\ \forall b, \quad (5)$$

$$\sum_{i\in\Omega_b} p_i^{\max}u_{it} \ge s_{bt}R_{bt},\ s_{bt} \in [0,1],\ \forall b, \quad (6)$$

where s_{bt} and r_{bt} are scalars used to define the local minimum generation level and local spinning capacity level within bus b. Although constraints (5) and (6) are normally considered within an area instead of a bus, we intend to present a method suitable for a generalized multi-area model.

Local unit constraints
Unit capacity constraints, for all $i \in \Omega_b$, $\forall b$ and $t = 1, \cdots, T$:

$$p_i^{\min} \le p_{it} \le p_i^{\max}, \quad (7)$$

the state transition equations

$$x_{it} = \begin{cases} \max(x_{i,t-1}, 0) + 1, & \text{if } u_{it} = 1, \\ \min(x_{i,t-1}, 0) - 1, & \text{if } u_{it} = 0, \end{cases} \quad (8)$$

the minimum uptime and downtime constraints,

$$u_{it} = \begin{cases} 1, & \text{if } 1 \leq x_{i,t-1} < t_i^{\text{on}}, \\ 0, & \text{if } -1 \geq x_{i,t-1} > -t_i^{\text{off}}, \\ 0 \text{ or } 1, & \text{otherwise,} \end{cases} \tag{9}$$

and the initial conditions on x_{it} at $t = 0$ for $\forall i$.

Remark

For simplicity, ramp constraints are not covered in this paper. The reader who is interested in ramp-constrained unit commitment is referred to [14]. If incorporated, the ramp constraints should be taken care of as are other local unit constraints like (7) within the corresponding bus subproblem, which will be detailed later.

3. The Lagrangian relaxation algorithm

The Lagrangian Relaxation (LR) approach relaxes the demand constraints, the spinning reserve constraints and the bus-interchange constraints using Lagrange multipliers. Algorithmically, this dual approach closely corresponds to the *direct method* in [13].

3.1. Phase 1: the dual optimization

Letting λ_t, μ_t be the corresponding nonnegative Lagrange multiplier of (2) and (4), respectively. And α_{tl}, β_{tl} are corresponding to (3), ν_{bt} and ρ_{bt} to (5) and (6), respectively. We now have the following dual problem:

$$(D) \quad \max_{(\underline{\lambda},\underline{\mu},\underline{\alpha},\underline{\beta},\underline{\nu},\underline{\rho})\geq 0} d(\underline{\lambda}, \underline{\mu}, \underline{\alpha}, \underline{\beta}, \underline{\nu}, \underline{\rho}) \tag{10}$$

where

$$d(\underline{\lambda}, \underline{\mu}, \underline{\alpha}, \underline{\beta}, \underline{\nu}, \underline{\rho})$$

$$\equiv \min_{u_{it},x_{it},p_{it}} \sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{i\in\Omega_b}[C_i(p_{it})u_{it}+S_i(x_{i,t-1}, u_{it}, u_{i,t-1})]$$

$$-\sum_{t=1}^{T}\sum_{b=1}^{B}[\lambda_t(\sum_{i\in\Omega_b} p_{it}u_{it} - D_{bt}) + \mu_t(\sum_{i\in\Omega_b} p_i^{\max}u_{it} - R_{bt})]$$

$$-\sum_{t=1}^{T}\sum_{\ell=1}^{L}[\alpha_{t\ell}(F_\ell + \sum_{b=1}^{B}, {}_{\ell b}(\sum_{i\in\Omega_b} p_{it}u_{it} - D_{bt}))$$

$$+ \beta_{t\ell}(F_\ell - \sum_{b=1}^{B}, {}_{\ell b}(\sum_{i\in\Omega_b} p_{it}u_{it} - D_{bt}))]$$

$$-\sum_{t=1}^{T}\sum_{b=1}^{B}[\nu_{bt}(\sum_{i\in\Omega_b} p_{it}u_{it} - r_{bt}D_{bt})$$

$$+ \rho_{bt}(\sum_{i\in\Omega_b} p_i^{\max}u_{it} - s_{bt}R_{bt})] \tag{11}$$

subject to initial conditions, and the unit constraints. After rearrangement of the terms in (11), the separability of $d(\underline{\lambda}, \underline{\mu}, \underline{\alpha}, \underline{\beta}, \underline{\nu}, \underline{\rho})$ appears.

$$d(\underline{\lambda}, \underline{\mu}, \underline{\alpha}, \underline{\beta}, \underline{\nu}, \underline{\rho}) = \sum_{b=1}^{B}\sum_{i\in\Omega_b} d_i(b; \underline{\lambda}, \underline{\mu}, \underline{\alpha}, \underline{\beta}, \underline{\nu}, \underline{\rho}) +$$

$$\sigma(\underline{\lambda}, \underline{\mu}, \underline{\alpha}, \underline{\beta}, \underline{\nu}, \underline{\rho}), \tag{12}$$

where

$$\sigma(\underline{\lambda}, \underline{\mu}, \underline{\alpha}, \underline{\beta}, \underline{\nu}, \underline{\rho}) = \sum_{t=1}^{T}(\lambda_t D_t + \mu_t R_t)$$

$$-\sum_{t=1}^{T}\sum_{\ell=1}^{L}[\alpha_{t\ell}(F_\ell - \sum_{b=1}^{B}, {}_{b\ell}D_{bt}) + \beta_{t\ell}(F_\ell + \sum_{b=1}^{B}, {}_{b\ell}D_{bt})]$$

$$+\sum_{t=1}^{T}\sum_{b=1}^{B}(\nu_{bt}r_{bt}D_{bt} + \rho_{bt}s_{bt}R_{bt}) \tag{13}$$

is a constant term given the multipliers, and

$$d_i(b; \underline{\lambda}, \underline{\mu}, \underline{\alpha}, \underline{\beta}, \underline{\nu}, \underline{\rho}) =$$

$$\min_{u_{it},x_{it},p_{it}} \sum_{t=1}^{T}[C_i(p_{it})+S_i(x_{i,t-1}, u_{it}, u_{i,t-1})-\lambda_t p_{it}-\mu_t p_i^{\max}$$

$$-\sum_{\ell=1}^{L}(\alpha_{t\ell} - \beta_{t\ell}), {}_{\ell b}p_{it} - \nu_{bt}p_{it} - \rho_{bt}p_i^{\max}]u_{it}. \tag{14}$$

Once again, the minimization in (14) is subject to initial conditions and the unit constraints.

Note that d_i is a unit subproblem corresponding to unit i. We can further define a *bus* subproblem d_b such that

$$d_b(\underline{\lambda}, \underline{\mu}, \underline{\alpha}, \underline{\beta}, \underline{\nu}, \underline{\rho}) \equiv \sum_{i\in\Omega_b} d_i(b; \underline{\lambda}, \underline{\mu}, \underline{\alpha}, \underline{\beta}, \underline{\nu}, \underline{\rho}), \tag{15}$$

and the dual objective (12) is equivalent to

$$d(\underline{\lambda}, \underline{\mu}, \underline{\alpha}, \underline{\beta}, \underline{\nu}, \underline{\rho})$$

$$= \sum_{b=1}^{B} d_b(\underline{\lambda}, \underline{\mu}, \underline{\alpha}, \underline{\beta}, \underline{\nu}, \underline{\rho}) + \sigma(\underline{\lambda}, \underline{\mu}, \underline{\alpha}, \underline{\beta}, \underline{\nu}, \underline{\rho})$$

Each unit subproblem d_i (14) can be solved using dynamic programming. The dual function

$d(\underline{\lambda}, \underline{\mu}, \underline{\alpha}, \underline{\beta}, \underline{\nu}, \underline{\rho})$ is concave, but not necessarily differentiable at all points (e.g. [2]). A subgradient method is employed to solve the dual maximization problem (D).

The dual optimization algorithm is summarized as follows.

Phase 1: Dual optimization

Step 0: $k \leftarrow 0$; initialize $\underline{\lambda}^0$, $\underline{\mu}^0$, $\underline{\alpha}^0$, $\underline{\beta}^0$, $\underline{\nu}^0$, $\underline{\rho}^0$; $b \leftarrow 1$.

Step 1: If stopping criteria are met, stop. If $b > B$, go to Step 4. Otherwise solve the b-th bus subproblem $d_b(\underline{\lambda}^k, \underline{\mu}^k, \underline{\alpha}^k, \underline{\beta}^k, \underline{\nu}^k, \underline{\rho}^k)$ to obtain $(\underline{u}^k, \underline{p}^k)$.

Step 2: If $(\underline{u}^k, \underline{p}^k)$ satisfies (5) and (6) for bus b, go to Step 3. Otherwise update ν_{bt}^k and ρ_{bt}^k as follows then go to Step 1: for $\forall t$,
$\nu_{bt}^k \leftarrow \max[0, \nu_{bt}^k - s^k(\sum_{i\in\Omega_b} p_{it}^k u_{it}^k - r_{bt} D_{bt})]$
$\rho_{bt}^k \leftarrow \max[0, \rho_{bt}^k - s^k(\sum_{i\in\Omega_b} p_i^{\max} u_{it}^k - s_{bt} R_{bt})]$

Step 3: $b \leftarrow b+1$, go to Step 1.

Step 4: For $\forall t, \ell$
$\nu_{bt}^{k+1} \leftarrow \nu_{bt}^k$; $\rho_{bt}^{k+1} \leftarrow \rho_{bt}^k$;
$\lambda_t^{k+1} \leftarrow \max[0, \lambda_t^k - s^k(\sum_{b=1}^B \sum_{i\in\Omega_b} p_{it}^k u_{it}^k - D_t)]$;
$\mu_t^{k+1} \leftarrow \max[0, \mu_t^k - s^k(\sum_{b=1}^B \sum_{i\in\Omega_b} p_i^{\max} u_{it}^k - R_t)]$;
$\alpha_{t\ell}^{k+1} \leftarrow \max[0, \alpha_{t\ell}^k - s^k(F_\ell + \sum_{b=1}^B \iota_{\ell b}(\sum_{i\in\Omega_b} p_{it}^k u_{it}^k - D_{bt}))]$;
$\beta_{t\ell}^{k+1} \leftarrow \max[0, \beta_{t\ell}^k - s^k(F_\ell - \sum_{b=1}^B \iota_{\ell b}(\sum_{i\in\Omega_b} p_{it}^k u_{it}^k - D_{bt}))]$;

Step 5: $k \leftarrow k+1$, $b \leftarrow 1$, go to Step 1. ■

The step size in the algorithm used in our implementation has the following form.

$$s^k = m_k/\|g^k\|, \; m > 0, \tag{16}$$

where g^k is the subgradient of the dual function at iteration k and m_k is an adaptive constant. Our stopping criteria used in the algorithm combine the maximum number of iteration, the change of norm of subgradients at two consecutive iterations and the number of iterations without improvement in the dual objective value.

3.2. Phase 2: the feasibility phase

It is well known that the solution obtained through the dual optimization is generally not feasible. A feasibility phase is therefore required.

Definition 1 With respect to a given spinning capacity requirement $\{R_t\}$, a *reserve-feasible* commitment is a unit commitment $\{u_{it}\}$ that satisfies the spinning reserve capacity constraints (4), (6), unit constraints (8), (9) and the initial conditions x_{i0}. ■

Definition 2 With respect to a given load requirement $\{D_t\}$, a unit commitment $\{u_{it}\}$ is said to be dispatchable if there exists a set of dispatches $\{p_{it}\}$ such that (2), (3), (5) and (7) can be satisfied. ■

In [18], Zhuang proposed a method for obtaining feasible solutions to the single-area unit commitment problem. The basic idea is to obtain a reserve-feasible commitment (in the single-area case, ignore (6) in the definition). In [18] a reserve-feasible commitment is found by projecting the subgradient onto the hour corresponding to the most unsatisfied capacity constraint and increasing only the multiplier μ_t of this hour. A method to calculate the exact amount of the increase in the value of the corresponding μ_t is also provided in [18]. It can be shown that a (single-area) reserve-feasible commitment is dispatchable (ignoring (3) and (5) in the definition in the single-area case) if and only if the minimum load condition below is satisfied.

$$\sum_i p_i^{\min} u_{it} \le D_t, \; \forall t. \tag{17}$$

However, in the presence of transmission constraints, this is not the case. The feasibility phase presented in this paper contains two parts. Part 1 seeks a reserve-feasible commitment by increasing the multipliers μ_t in hours with insufficient spinning capacities. Based on the obtained reserve-feasible commitment, Part 2 looks for a dispatchable commitment while maintaining reserve-feasibility by solving the following linear program (LP) for all hours, $(LP(t))$ for $t = 1, \cdots, T$. Suppose Part 1 of the feasibility phase terminates at iteration k with an obtained commitment $\{u_{it}^k\}$. Let q_{bt} be the variable representing the total generation to be dispatched to bus b in time t. Given a commitment $\{u_{it}^k\}$ let $q_{bt}^{\min} = \sum_{i\in\Omega_b} p_i^{\min} u_{it}^k$ and $q_{bt}^{\max} = \sum_{i\in\Omega_b} p_i^{\max} u_{it}^k$, which constitute an immediate lower bound and upper bound for q_{bt}.

$(LP(t))$

$$\min_{q_{bt}, y_{bt}} \quad \sum_{b=1}^B (y_{bt} - q_{bt}^{\max}) \tag{18}$$

$$\text{s.t.} \quad q_{bt}^{\min} \le q_{bt} \le y_{bt}, \; \forall b \tag{19a}$$

$$q_{bt}^{\max} \le y_{bt} \le \sum_{i\in\Omega_b} p_i^{\max}, \; \forall b \tag{19b}$$

$$\sum_{b=1}^{B} q_{bt} = \sum_{b=1}^{B} D_{bt} \quad (19c)$$

$$-F_\ell \leq \sum_{b=1}^{B} {}_{\ell b}(q_{bt} - D_{bt}) \leq F_\ell, \ \forall \ell \quad (19d)$$

$$q_{bt} \geq r_{bt} D_{bt}, \ \forall b \quad (19e)$$

In $(LP(t))$, y_{bt} is a variable upper bound of q_{bt}. Let y^*_{bt} be the solution of $(LP(t))$. If $\{u^k_{it}\}$ is dispatchable in time t, $y^*_{bt} = q^{\max}_{bt}$ solves $(LP(t))$ and the optimal objective value of $(LP(t))$ is zero. If $\{u^k_{it}\}$ is not dispatchable in time t, $(LP(t))$ yields a positive solution, which indicates that to be dispatchable, more units should be committed in time t, and the target value of the new (bus) capacity requirement is now increased to y^*_{bt} for each bus b in time t. Part 1 of the feasibility phase is therefore rerun with respect to the updated capacity requirement $\{s_{bt} R_{bt}\} \leftarrow \{y^*_{bt}\}$. The phase 2 algorithm is summarized below.

Phase 2 Algorithm

Step 0: $k \leftarrow 0$; $\underline{\mu}^0$ and $\underline{\rho}^0$ and other multipliers are from Phase 1.

Step 1: Given $\underline{\mu}^k$ and $\underline{\rho}^k$ and other multipliers, solve the dual subproblems (14) to obtain $(\underline{u}^k, \underline{p}^k)$.

Step 2: If (4) and (6) are satisfied, go to Step 5 if Step 5 has not been visited, otherwise stop.

Step 3: For $\forall b, t$,
$\mu^{k+1}_t \leftarrow \mu^k_t + s^k \cdot \min(0, R_t - \sum_{b=1}^{B} \sum_{i \in \Omega_b} p^{\max}_i u^k_{it})$;
$\rho^{k+1}_{bt} \leftarrow \rho^k_{bt} + s^k \cdot \min(0, s_{bt} R_{bt} - \sum_{i \in \Omega_b} p^{\max}_i u^k_{it})$.
(other multipliers are kept unchanged.)

Step 4: $k \leftarrow k+1$, go to Step 1.

Step 5: Solve $(LP(t))$ for all t. If the optimal value of $(LP(t))$ is 0 for all t, stop and $\underline{u}^k$ is reserve-feasible and dispatchable. Otherwise, update the (bus) capacity requirements $\{s_{bt} R_{bt}\} \leftarrow \{y^*_{bt}\}, \forall b$, where y^*_{bt} solves $(LP(t))$. (Note $\{R_t\}$ remains unchanged.) Then go to Step 1. ■

Remarks

1. The loop between Step 1 and Step 4 corresponds to the Part 1 of the feasibility phase, and Step 5 initiates Part 2.

2. Note that our proposed method for searching for a reserve-feasible commitment in Part 1 of the feasibility phase at each iteration updates the μ_t corresponding to the hours that the spinning capacity requirement is violated simultaneously, instead of only updating μ_t in one hour at a time as in [18].

3. One easy way to test the robustness of the feasibility phase of a transmission-constrained unit commitment method is to apply it to an instance in which each single bus has enough capacity to handle its own load over the planning horizon. Then the transmission line capacities are graduately reduced to zero. In such a problem, a feasible solution obviously exists, and it is equivalent to solving many single-area unit commitment problems. It can be easily verified that our feasibility phase reduces to solving many single-area problems in such an instance.

4. Should $(LP(t))$ be transformed to the standard form of LP by adding slack or surplus variables: min cx subject to $\mathcal{A}x = b$, $x \geq 0$, it can be seen that c and $\mathcal{A}$ are fixed independent of t. This property can be taken into account in implementation to improve the algorithm's performance. For example, if the dual simplex method is used to solve $(LP(t))$, the optimal basis of the LP in the previous hour can be stored and used as an initial (dual feasible) basis.

3.3. Economic dispatch

With the DC power flow model, the transmission constraints are formulated as linear constraints. If the fuel cost is represented by a quadratic function of the power generation, the transmission-constrained economic dispatch is a quadratic programming problem. Many methods can be used to solve this type of problem, including primal methods, e.g. the reduced gradient method, and dual method, e.g. LR. The drawback of any dual method is in its being unable to detect the (primal) infeasibility of the problem beforehand, which normally results in unboundedness during the dual optimization. In our implementation, the transmission-constrained economic dispatch is solved by the commercial software MINOS, which does general nonlinear programming. We have also tested another method using LR, which can generally handle both the cases with smooth and piecewise quadratic functions as fuel cost functions. If using LR, although its decomposition feature is very efficient, it sometimes suffers the drawback mentioned above.

3.4. Phase 3: unit decommitment

Given a feasible schedule, a unit decommitment (UD) method [10, 16, 15] is used to improve this solution while maintaining feasibility. In [16, 15], a single-area unit decommitment method was devised as a postprocessing method for the LR approach for solving the unit commitment problem. A transmission-constrained unit decommitment algorithm can be devised by repeatedly solving the single area unit decommitment. Given a feasible solution $(\underline{\tilde{u}}^k, \underline{\tilde{p}}^k)$, at iteration k the schedule of bus b can be improved by the unit decommitment with respect to the *current* load assigned to be the bus demand at this iteration, i.e. $\{D_{bt}^k\} \leftarrow \{\sum_{i\in\Omega_b} \tilde{p}_{it}^k \tilde{u}_{it}^k\}$. After the commitments of all buses are refined, a transmission-constrained economic dispatch is performed to redispatch the units in all buses. The decommitment procedure can thus proceed until no improvement in the total cost is made. Since at each iteration the decommitment procedure improves the commitment schedule without affecting the bus injection, the transmission feasibility is retained at each iteration. The algorithm is as follows.

Transmission-constrained UD algorithm

Step 0: a feasible schedule $(\underline{\tilde{u}}^0, \underline{\tilde{p}}^0)$ is given, $k \leftarrow 0$.

Step 1: Given $(\underline{\tilde{u}}^k, \underline{\tilde{p}}^k)$, each bus performs the unit decommitment presented in [16] independently with the current load assigned to be the demand requirement $\{D_{bt}^k\} \leftarrow \{\sum_{i\in\Omega_b} \tilde{p}_{it}^k \tilde{u}_{it}^k\}$ also subject to the satisfaction of (6). The resultant commitment is denoted by $\underline{\hat{u}}^k$.

Step 2: Apply the transmission-constrained economic dispatch with respect to $\underline{\hat{u}}^k$ to obtain a dispatch $\underline{\hat{p}}^k$. Let $(\underline{\tilde{u}}^{k+1}, \underline{\tilde{p}}^{k+1}) \leftarrow (\underline{\hat{u}}^k, \underline{\hat{p}}^k)$.

Step 3: If the total cost of $(\underline{\tilde{u}}^{k+1}, \underline{\tilde{p}}^{k+1})$ is no better than that of $(\underline{\tilde{u}}^k, \underline{\tilde{p}}^k)$, stop; otherwise $k \leftarrow k+1$, and go to Step 1. ■

The above method works best for the cases with many generators in each bus. For other transmission-constrained unit decommitment approaches, the interested reader is referred to [15].

4. Numerical results

The transmission-constrained unit commitment algorithm presented in this paper has been implemented in FORTRAN on an HP 700 workstation. This section presents numerical results of some test problems.

We first apply the proposed algorithm to solve the test problem in [13]. Not knowing the spinning reserve requirements set in [13], we set the spinning capacity requirements to be $R_t = 1.07D_t$ for all t in (4). The result along with that in [13] is summarized in Table 1. We must emphasize that the results of our proposed method (three-phase) in Table 1 and the results in [13] (Shaw) are obtained under different bases: different spinning reserve requirements and different computer facilities. However, the results of the duality gap regarded as a measure of solution error suggest that both methods obtained very accurate solutions in this test problem.

Table 1: Summary of test results

method	Dual value ($)	Total Cost ($)	CPU time (sec)	Duality Gap (%)
Three-phase	1100030	1100153	7.410	0.011
Shaw	1110319	1111049	4.400	0.065

4.1. The test system

The second test problem is based on an IEEE test problem [1]. This test system contains 24 buses, 34 transmission lines and 32 generating units. The parameters of the generating units are summarized in Table 2, in which the unit fuel costs are assume to be a quadratic function of power generation as: $C_i(p_{it}) = a_{i0} + a_{i1}p_{it} + a_{i2}p_{it}^2$, and the unit startup costs S_i are assumed to be constants. This test system contains steam units and combustion turbines as indicated in Table 2.

The transmission line parameters are given in Table 3. In Table 3 two sets of line capacities are presented, the one with smaller line capacities results in more serious congestion than the other. A 24 hour planning horizon is adopted in the test problem, the system load information can be found in Tables 4. The reader can find that the load taken here corresponding to the day with peak load in the year in the IEEE test problem. In this test, the spinning capacity requirements are set to be $R_t = 1.07D_t$ for all t. The proposed algorithm is applied to solve this test problem. The test result is summarized in Table 5.

In case (i) of the transmission line capacity F_ℓ, the test result shows that line 33 is always congested and line 11 becomes congested during peak hours. In this test case, it is noted that no combustion turbine is turned on during the planning horizon. The duality gap is 0.47% in this case. Compared with the total cost in the unconstrained case, the increased cost due to the transmission network is about 0.17% of the total cost of the unconstrained case. In the second case (ii) of F_ℓ, the congestion of the network involves seven lines. There are 12 hours on the planning horizon in which there are 5 lines congested (during the peak hours), 7 hours with 4 lines, 4 hours with 2 lines and 1 hour with 1 line congested. The increased cost due to the transmission system constraints increases to 1.5% of the total cost of the unconstrained case. The congestion is much more serious than in the case (i). It can be observed that when the transmission network gets more congested, the algorithm takes longer time to converge, especially in the dual optimization and in obtaining a feasible solution in the feasibility phase, with larger duality gap. We also found that the duality gap of 1.43% in this test case is primarily due to the poor dual objective value obtained. In another test run, tripling the CPU time spent on the dual optimization improves the dual value to 902758 with the primal value reduced to 912576 for the duality gap of 1.09%. Overall, our experience show that our feasibility phase is very robust in obtaining feasible solutions.

4.2. Direct and indirect approaches

In [13], two methods, the *direct* and *indirect* methods, are defined. Those methods which take transmission constraints into account in the optimization stage are called direct methods and those which do not are called indirect. Indirect methods ignore the transmission constraints in the dual optimization, and deal with them only in the feasibility phase. In this section, we shall compare two algorithms. Using the same naming convention as in [13], we will call them the direct approach and the indirect approach. The *direct approach* is the three-phase algorithm proposed in this paper. The *indirect approach* is essentially the direct approach except that the multiplier $\alpha_{t\ell} = \beta_{t\ell} = 0$, for $\forall t, \ell$ through out the algorithm.

The indirect approach ignores the transmission constraints by setting the corresponding multipliers to be zeros. During the feasibility phase, transmission constraints are enforced through (19d). The major motivations for applying the indirect approach are to speed up the dual optimization, and to save memory space (the direct method essentially requires an additional $2T \times (1 + L)$ multipliers). The indirect approach is

Table 2: Generating unit parameters

b_i	type†	$p_i^{\min}$	$p_i^{\max}$	x_{i0}	t_i^{up}	t_i^{down}	t_i^{cold}	a_{i0}	a_{i1}	a_{i2}	S_i
1_1	C	4	20	1	1	1	2	63.999	20.000	1.000×10^{-6}	40
1_2	C	4	20	2	1	1	2	63.999	20.000	1.000×10^{-6}	40
1_3	S2	10	76	3	6	6	12	133.919	16.193	1.508×10^{-2}	45
1_4	S2	10	76	-2	6	6	12	133.919	16.193	1.508×10^{-2}	45
2_1	C	4	20	1	1	1	2	63.999	20.000	1.000×10^{-6}	40
2_2	C	4	20	2	1	1	2	63.999	20.000	1.000×10^{-6}	40
2_3	S2	10	76	10	6	6	12	133.919	16.193	1.508×10^{-2}	45
2_4	S2	10	76	-2	6	6	12	133.919	16.193	1.508×10^{-2}	45
7_1	S1	15	100	10	10	10	20	199.124	12.468	1.532×10^{-2}	45
7_2	S1	15	100	10	10	10	20	199.124	12.468	1.532×10^{-2}	110
7_3	S1	15	100	10	10	10	20	199.124	12.468	1.532×10^{-2}	110
13_1	S1	20	197	12	12	12	24	209.546	13.928	2.085×10^{-3}	100
13_2	S1	20	197	12	12	12	24	209.546	13.928	2.085×10^{-3}	100
13_3	S1	20	197	12	12	12	24	209.546	13.928	2.085×10^{-3}	100
15_1	S1	3	12	4	2	2	4	21.145	16.193	9.553×10^{-2}	30
15_2	S2	3	12	-9	2	2	4	21.145	16.193	9.553×10^{-2}	30
15_3	S1	3	12	4	2	2	4	21.145	16.193	9.553×10^{-2}	30
15_4	S1	3	12	3	2	2	4	21.145	16.193	9.553×10^{-2}	30
15_5	S1	3	12	4	2	2	4	21.145	16.193	9.553×10^{-2}	30
15_6	S2	20	155	-20	12	12	24	275.606	12.360	8.898×10^{-3}	100
16_1	S2	20	155	5	12	12	24	275.606	12.360	8.898×10^{-3}	100
18_1	S3	40	400	100	48	48	60	577.537	14.253	7.365×10^{-4}	440
21_1	S3	40	400	100	48	48	60	577.537	14.253	7.365×10^{-4}	440
22_1	S2	10	76	-2	6	6	12	133.919	16.193	1.508×10^{-2}	45
22_2	S2	10	76	-2	6	6	12	133.919	16.193	1.508×10^{-2}	45
22_3	S2	10	76	-2	6	6	12	133.919	16.193	1.508×10^{-2}	45
22_4	S2	10	76	-2	6	6	12	133.919	16.193	1.508×10^{-2}	45
22_5	S2	10	76	-2	6	6	12	133.919	16.193	1.508×10^{-2}	45
22_6	S2	10	76	-2	6	6	12	133.919	16.193	1.508×10^{-2}	45
23_1	S2	20	155	5	12	12	24	275.606	12.360	8.898×10^{-3}	100
23_2	S2	20	155	3	12	12	24	275.606	12.360	8.898×10^{-3}	100
23_3	S2	35	350	11	24	24	36	517.669	11.892	5.220×10^{-3}	250

† S1: Steam: fossil-oil; S2: Steam: fossil-coal;
S3: Steam: nuclear; C: Combustion turbine

applied to solve the IEEE test problem given in the previous section. The result is summarized in Table 6. It can be seen in Table 6 that the direct approach obtains a better solution and uses less CPU time. The indirect approach only uses less CPU time in the dual optimization phase (Ph 1), but spends more time to locate a feasible solution. Also the quality of the solution obtained from the feasibility phase of the indirect approach tends to be worse than that from the direct approach, so that it might take longer for unit decommitment to improve the solution. Our comparison between the direct and indirect approaches agrees with what is observed in [13].

Table 3: Transmission line parameters

ℓ	From Bus	To Bus	X (p.u.)	(i) F_ℓ (MW)	(ii) F_ℓ (MW)
1	1	2	0.0139	175	175
2	1	3	0.2112	175	175
3	1	5	0.0845	175	175
4	2	4	0.1267	175	175
5	2	6	0.1920	175	175
6	3	9	0.1190	175	175
7	3	24	0.0839	400	175
8	4	9	0.1037	175	175
9	5	10	0.0883	175	175
10	6	10	0.0605	175	175
11	7	8	0.0614	175	175
12	8	9	0.1651	175	175
13	8	10	0.1651	175	175
14	9	11	0.0839	400	175
15	9	12	0.0839	400	136
16	10	11	0.0839	400	200
17	10	12	0.0839	400	250
18	11	13	0.0476	400	198
19	11	14	0.0418	400	250
20	12	13	0.0476	400	150
21	12	23	0.0966	400	166
22	13	23	0.0865	400	250
23	14	16	0.0389	400	220
24	15	16	0.0173	400	250
25	15	21	0.0245	400	290
26	15	24	0.0519	400	270
27	16	17	0.0259	400	270
28	16	19	0.0231	400	270
29	17	18	0.0144	400	270
30	17	22	0.1053	400	270
31	18	21	0.0129	400	270
32	19	20	0.0198	400	198
33	20	23	0.0108	300	270
34	21	22	0.0678	400	270

4.3. A large scale test problem

The proposed three-phase algorithm is also applied to a test problem based on PG&E's system. The test problem with more than 2500 lines, 2200 buses and 79 dispatchable generating units is derived with necessary modifications to fit the scope discussed in this paper. The peak load is 3957 MW. The test result is given in Table 7. We emphasize that these test results used a program developed for research purpose with little effort spent in speeding up the algorithm performance. The motivation is to see how the algorithm performs on a large scale problem. We observe: (i) the convergence of the dual optimization is not directly affected by the increased number of multipliers corresponding to the transmission constraints but by the increased number of nonzero multipliers. The more congested the network is, the more iterations are required to reach a near-optimal dual solution. We observe that the dual objective value tends to improve more slowly as the congestion of the network increases. (ii) In such a large scale test problem the most significant CPU time-consuming step occurs in the calculation of the norm of the subgradient g in (16) at each iteration. (iii) The CPU time required in this test problem, although high, scales approximate linearly with problem size. Algorithm performance can be improved by taking advantage of the sparsity of any large matrices including the matrix of the distribution factors , and the mul-

Table 4: Load information

System load			Bus load	
hour	load (MW)	% of peak	bus	% of peak
1	2223.0	78	1	3.8
2	2052.0	72	2	3.4
3	1938.0	68	3	6.3
4	1881.0	66	4	2.6
5	1824.0	64	5	2.5
6	1852.5	65	6	4.8
7	1881.0	66	7	4.4
8	1995.0	70	8	6.0
9	2280.0	80	9	6.1
10	2508.0	88	10	6.8
11	2565.0	90	11	0
12	2593.5	91	12	0
13	2565.0	90	13	9.3
14	2508.0	88	14	6.8
15	2479.5	87	15	11.1
16	2479.5	87	16	3.5
17	2593.5	91	17	0
18	2850.0	100	18	0
19	2821.5	99	19	6.4
20	2764.5	97	20	4.5
21	2679.0	94	21	0
22	2622.0	92	22	0
23	2479.5	87	23	0
24	2308.5	81	24	0

Table 5: Test result of IEEE test problem (direct approach)

Case	Dual Value	Primal Value	Duality Gap
Unconstrained	\$898619	\$898683	0.00%
Constrained F_ℓ (i)	\$895980	\$900206	0.47%
Constrained F_ℓ (ii)	\$899662	\$912650	1.43%

Case	CPU Time (sec)			
	Ph 1	Ph 2	Ph 3	Total
Unconstrained	3.72	0.37	2.32	6.41
Constrained F_ℓ (i)	7.81	5.77	6.28	19.86
Constrained F_ℓ (ii)	7.79	12.14	10.92	30.85

Table 6: Test result of the indirect approach

Case	Dual Value	Primal Value	Duality Gap
Constrained F_ℓ (i)	\$895840	\$901289	0.60%
Constrained F_ℓ (ii)	\$895840	\$913542	1.97%

Case	CPU Time (sec)			
	Ph 1	Ph 2	Ph 3	Total
Constrained F_ℓ (i)	5.69	21.74	10.45	37.88
Constrained F_ℓ (ii)	5.80	18.72	9.52	34.04

Table 7: Test result of a large scale problem

Total cost (\$)	CPU time (sec)
Dual optimal: 199216.56	Ph. 1: 1292.47
Primal optimal: 200214.32	Ph. 2: 116.60
Duality gap: 1.469%	Ph. 3: 108.32
	Total: 1512.39

tipliers associated with the transmission constraints. This can be shown to be equivalent to reducing a large scale 'bus-based' model to a small sized 'area-based' one without affecting problem optimality and greatly speeding up the algorithm performance.

5. Conclusions

In this paper we presented a transmission-constrained unit commitment algorithm using the Lagrangian relaxation approach. The transmission constraints are modeled as linear constraints under the DC power flow model. The Lagrangian relaxation approach relaxes not only the demand constraints and the spinning reserve constraints, but also the transmission constraints using multipliers. This algorithm contains three phases. In Phase 1, the dual optimization is performed to determine a commitment, based on which a feasibility phase follows to locate a feasible commitment. The feasibility phase is essentially an extension of that in the single area unit commitment case. A reserve-feasible commitment is first located, then by solving linear programs a dispatchable commitment is determined. Finally the multi-area unit decommitment method is applied, which serves as a post-processing method of the algorithm.

In limited numerical tests, the proposed algorithm is found to be efficient. A large scale problem is also tested. The result suggests that the proposed algorithm can be used to deal with practical-sized transmission-constrained unit commitment problems.

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