



# A Zero-Reflection Controller for Electromechanical Disturbances in Power Networks

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## Acknowledgements

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Architectures for Secure and Robust Distributed Infrastructures

## *Approach*

- Use a continuum model to gain insight for the design of generator controls.
- Focus on traveling waves and impedance-matching controllers.

## *Background - general*

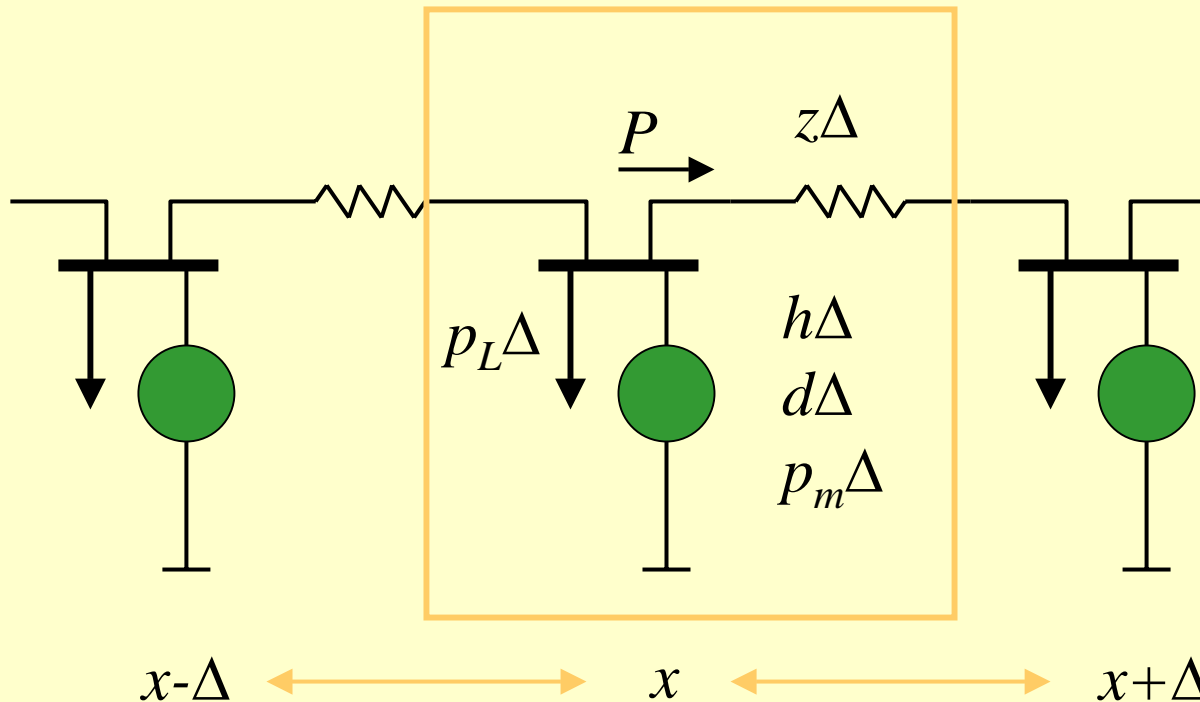
- Power System Models - ODEs, DAEs
- Analysis – simulation, modal analysis, energy methods.
- Control – modal approach.

## *Background – continuum model*

Distributed model, PDE description, electromechanical wave behavior.

- Semlyen (1974) Introduced model, traveling waves, standing waves, parallels to EM.
- Thorp et al. (1998) Develop model to study traveling waves observed in phasor measurements.
- Our work (2002) Controls to eliminate traveling waves.

# A Power System Model



Distributed parameters:

- $z$  impedance per unit length
- $h$  inertia per unit length
- $d$  damping per unit length

Denote  
 $z^{-1} = g - j b$

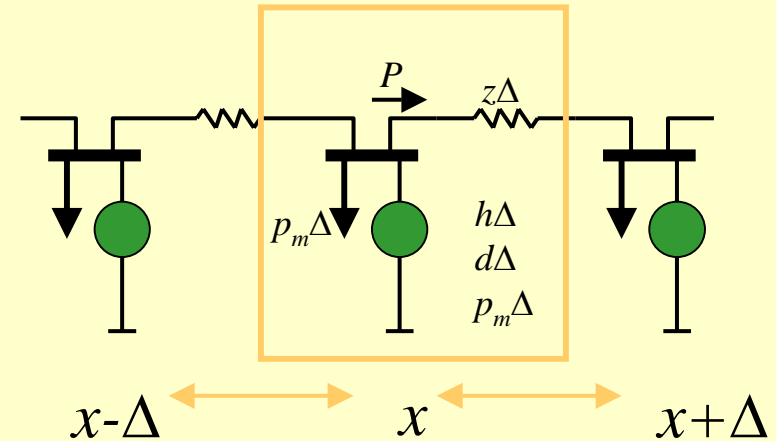
# A Power System Model

$$\frac{2h\Delta}{\omega_s} \frac{d^2\delta}{dt^2} = p_g\Delta - d\Delta \frac{d\delta}{dt}$$

$$-\frac{E^2b}{\Delta} (\sin(\delta(x) - \delta(x - \Delta)) + \sin(\delta(x) - \delta(x + \Delta)))$$

$$-\frac{E^2g}{\Delta} (2 - \cos(\delta(x) - \delta(x - \Delta)) - \cos(\delta(x) - \delta(x + \Delta)))$$

(Swing Equation Model)



Neglect  $g$ ,  $d$ , and examine the limit as  $\Delta \rightarrow 0$

$$\frac{2h}{\omega_s} \frac{\partial \omega}{\partial t} = p_g - \frac{\partial P}{\partial x}$$

## Continuum Model

$$\frac{2h}{\omega_s} \frac{\partial \omega}{\partial t} = p_g - \frac{\partial \mathcal{P}}{\partial x}$$

where

$$P = -E^2 b \frac{\partial \delta}{\partial x}$$

differentiate w.r.t. time

$$\frac{\partial \mathcal{P}}{\partial t} = -E^2 b \frac{\partial \omega}{\partial x}$$

*Telegrapher's Equation  
(we'll use later)*

## Continuum Model

$$\frac{2h}{\omega_s} \frac{\partial \omega}{\partial t} = p_g - \frac{\partial P}{\partial x} \quad \text{where} \quad P = -E^2 b \frac{\partial \delta}{\partial x}$$

combine

$$\frac{2h}{\omega_s} \frac{\partial^2 \delta}{\partial t^2} = p_g - E^2 b \frac{\partial^2 \delta}{\partial x^2}$$

linear undamped wave equation

(a more detailed nonlinear model is found in Thorp et al.)

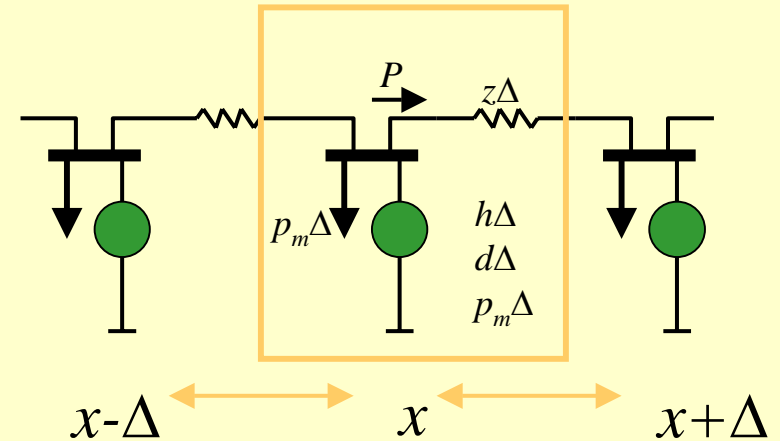
## *Continuum Model - Higher Dimensions*

$$\frac{2h}{\omega_s} \frac{\partial^2 \delta}{a^2} = p_g - E^2 b \nabla^2 \delta$$

Same equation, parameters have different units.

*Does the original discrete model exhibit wave-like behavior?*

# A Power System Model



$$\frac{d\delta_k}{dt} = \omega_k \quad \forall k$$

$$\frac{2H}{\omega_s} \frac{d\omega_k}{dt} = p_k - 0.01\omega_k$$

$$- 6 (\sin(\delta_k - \delta_{k-1}) + \sin(\delta_k - \delta_{k+1}))$$

$$- (2 - \cos(\delta_k - \delta_{k-1}) - \cos(\delta_k - \delta_{k+1})) \quad 2 \leq k \leq 63$$

$$\frac{2H}{\omega_s} \frac{d\omega_1}{dt} = p_1 - 0.01\omega_1$$

$$- 6 \sin(\delta_1 - \delta_2) - (1 - \cos(\delta_1 - \delta_2)) \quad k = 1$$

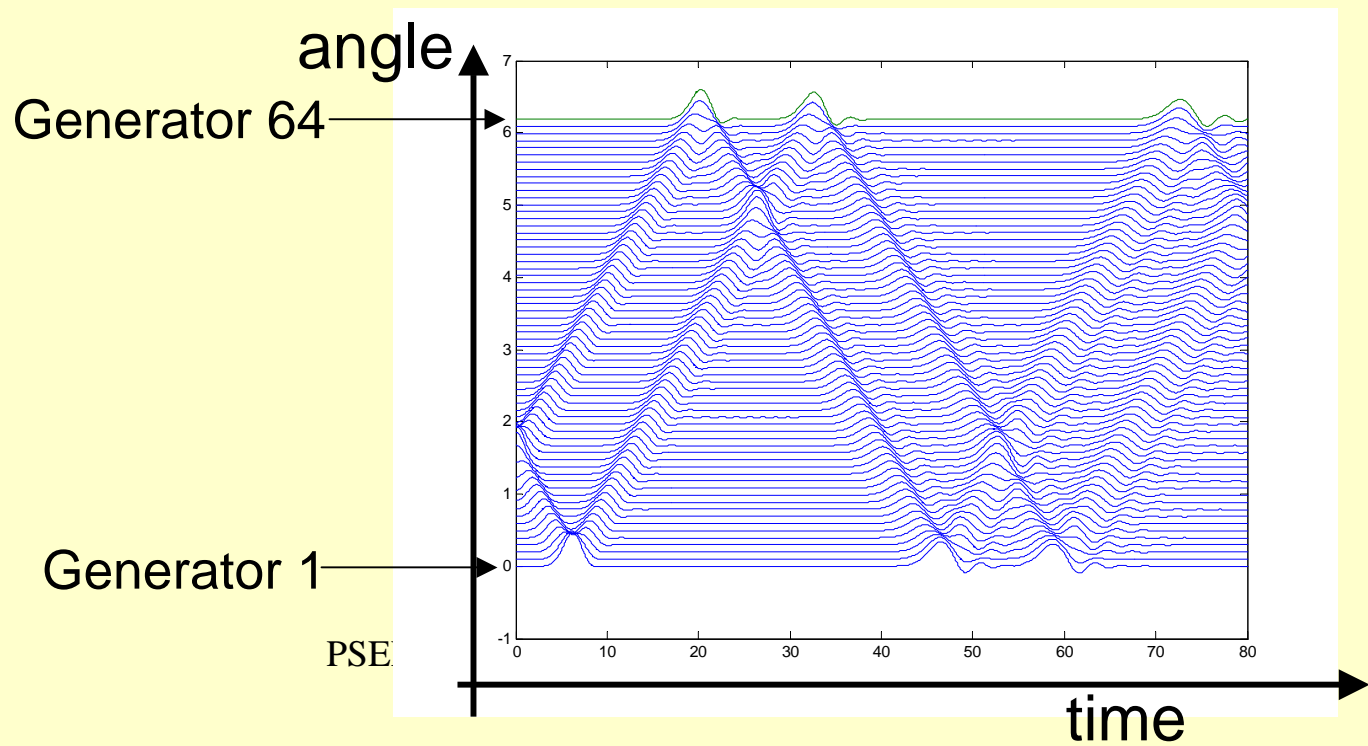
$$\frac{2H}{\omega_s} \frac{d\omega_{64}}{dt} = p_{64} - 0.01\omega_{64}$$

$$- 6 \sin(\delta_{64} - \delta_{63}) - (1 - \cos(\delta_{64} - \delta_{63})) \quad k = 64$$

## Initial Conditions and Disturbance

- Initial conditions: angle varies by  $2\pi$  over the string of generators.

- Gaussian pulse perturbation: 
$$\tilde{\delta}_k = \frac{1}{2} e^{-0.1(k-15.5)^2}$$



# Continuum Model - Characteristic Impedance

Forward traveling wave

$$\frac{\mathcal{P}}{\partial t} = -E^2 b \frac{\partial \omega}{\partial x}$$

$$P^+ = P\left(t - \frac{x}{v}\right) = P(y)$$

$$\omega^+ = \omega\left(t - \frac{x}{v}\right) = \omega(y)$$

$$\frac{\mathcal{P}^+}{\partial y} = \frac{E^2 b}{v} \frac{\partial \omega^+}{\partial y}$$

$$P^+ = \frac{E^2 b}{v} \omega^+$$

*$P^+$  and  $\omega^+$  are in constant proportion in the traveling wave.*

# Continuum Model - Characteristic Impedance

Forward traveling wave

$$P^+ = \frac{E^2 b}{v} \omega^+$$

Characteristic Impedance

$$C_o = \frac{\omega^+}{P^+} = \frac{v}{E^2 b} = \sqrt{\left(\frac{\omega_s}{2h}\right) \frac{1}{E^2 b}}$$

# Reflection Coefficients

Terminating the string of generators such that frequency and power are in constant proportion,  $C = \omega/P$ , leads to the Following Reflection Coefficient:

$$R = \frac{C - C_o}{C + C_o}$$

**$R = 1$**  positive reflection (in frequency)

$C \rightarrow \infty$ , open ended, negative reflection in power

**$R = -1$**  negative reflection (in frequency)

$C = 0$ , infinite bus, angle and frequency are fixed.

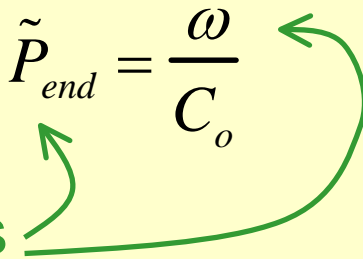
**$R = 0$**  no reflection

# A Zero Reflection Controller

Terminate the string of generators with the characteristic impedance.

Imposed Generator Control Objective:  $\tilde{P}_{end} = \frac{\omega}{C_o}$

deviations from nominal values



Generator Model

$$\frac{d\delta}{dt} = \omega$$
$$\frac{2H}{\omega_s} \frac{d\omega}{dt} = p_g + P_{end} - D\omega$$

Using  $p_g$  as the input, the control objective cannot be achieved... *at least not exactly...*

# A Zero Reflection Controller

Generator Model

$$\frac{d\delta}{dt} = \omega$$
$$\frac{2H}{\omega_s} \frac{d\omega}{dt} = p_g + P_{end} - D\omega$$

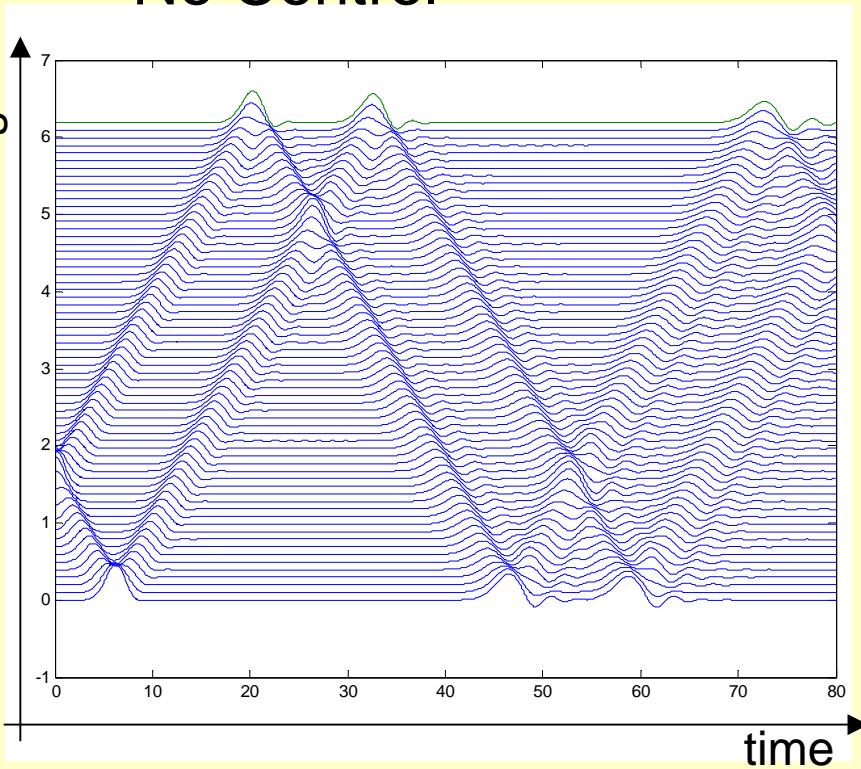
Try the following feedback control

$$\tilde{p}_g = K \left( \tilde{P}_{end} - \frac{\omega}{C_o} \right) - \tilde{P}_{end} + D\omega$$

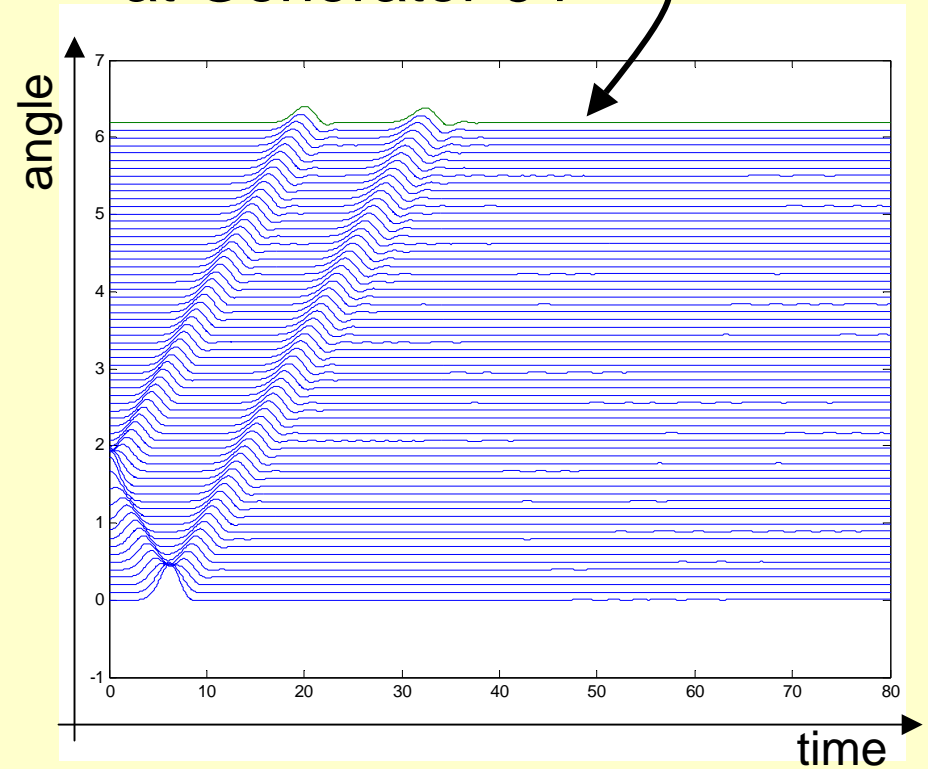
control objective

# Results - 1D Case

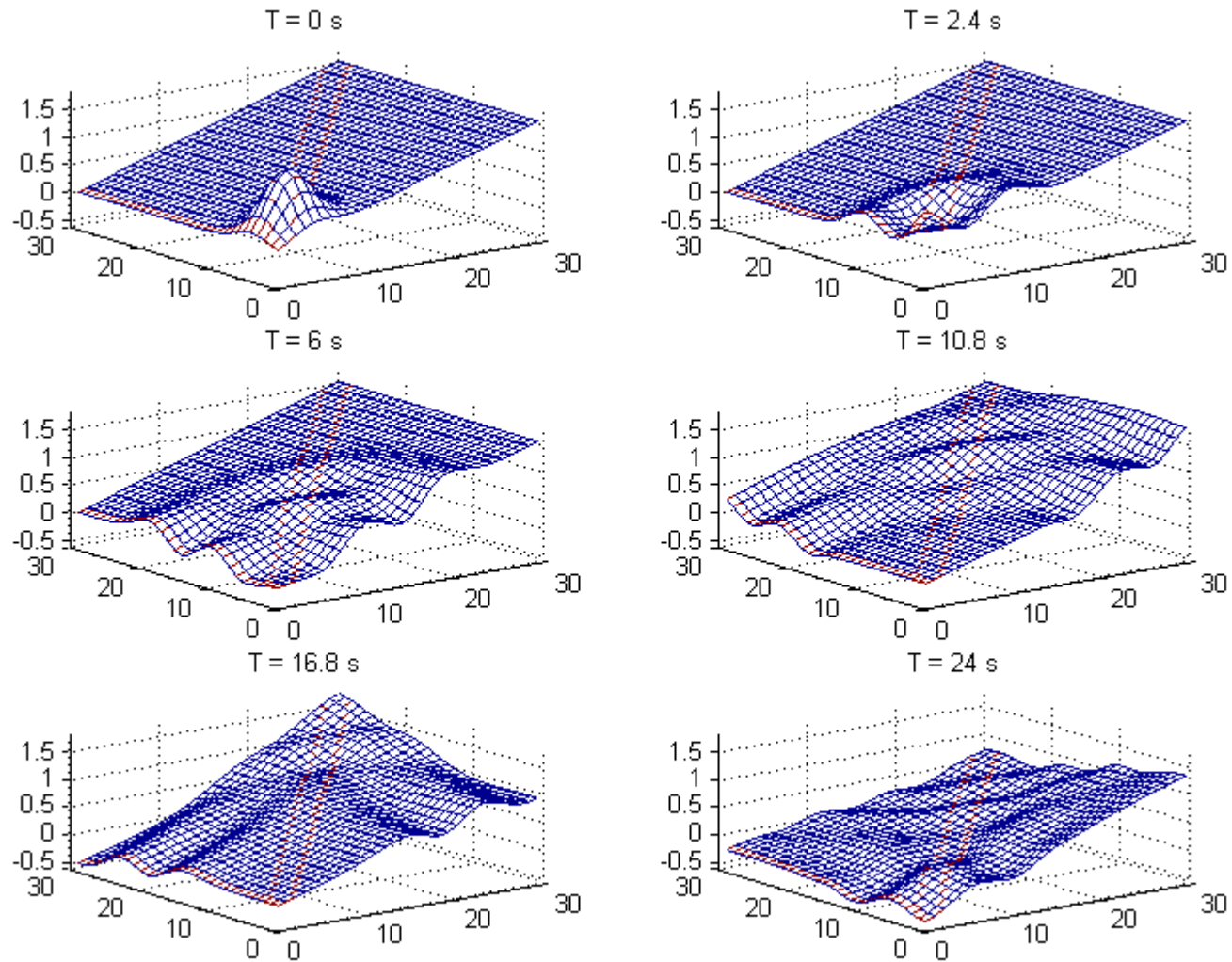
No Control



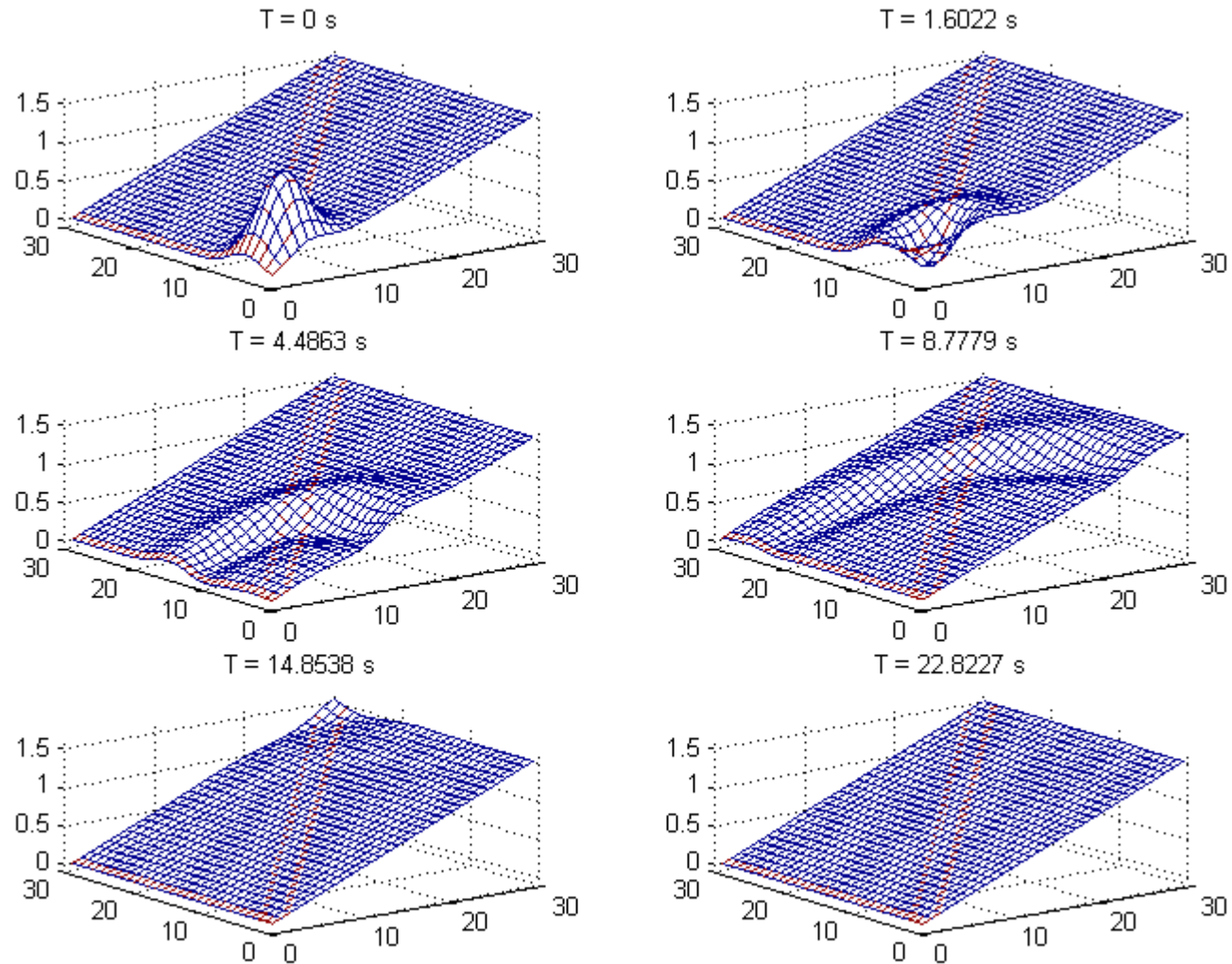
Zero-Reflection Controller  
at Generator 64



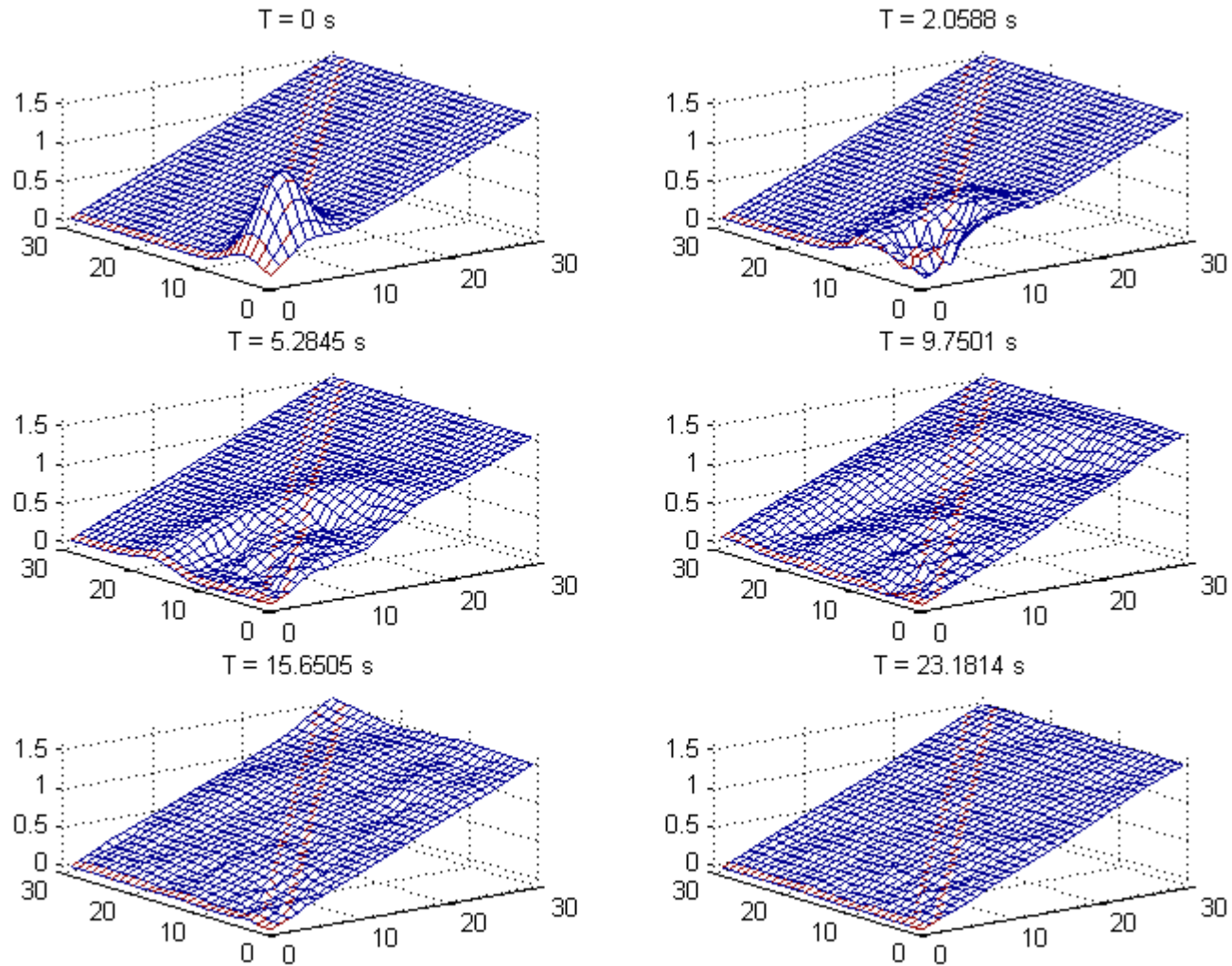
# 2D Grid / no control



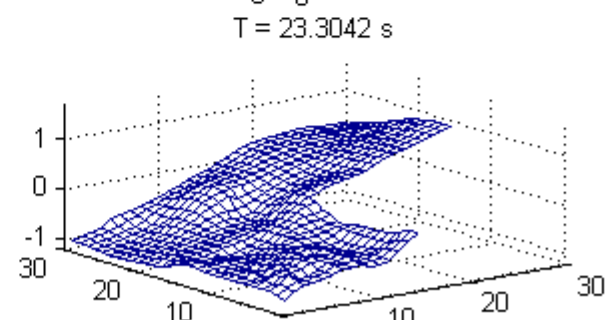
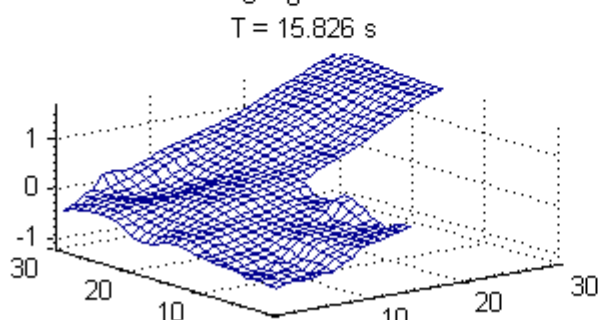
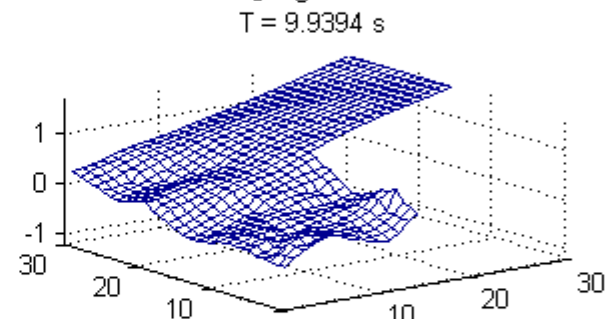
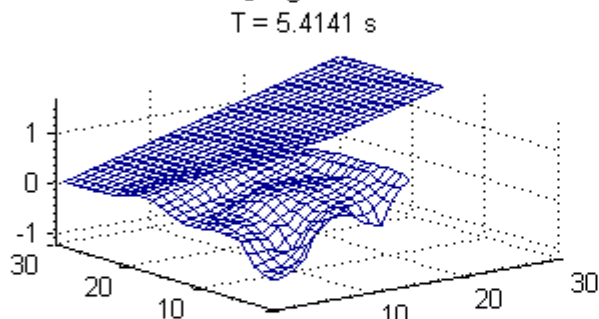
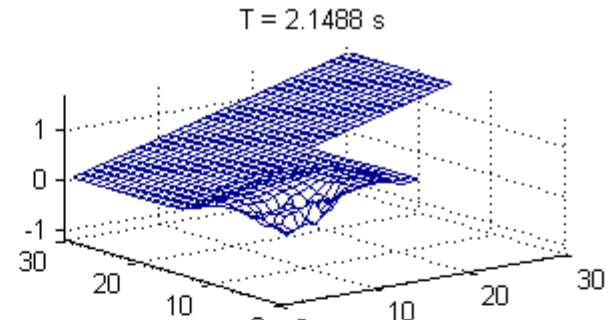
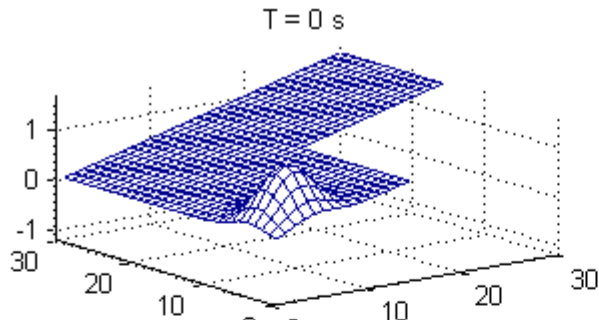
# 2D Grid / Zero Reflection Controller



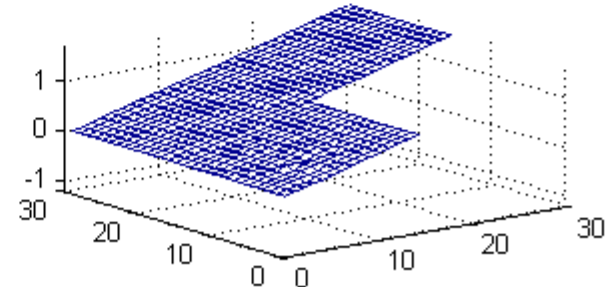
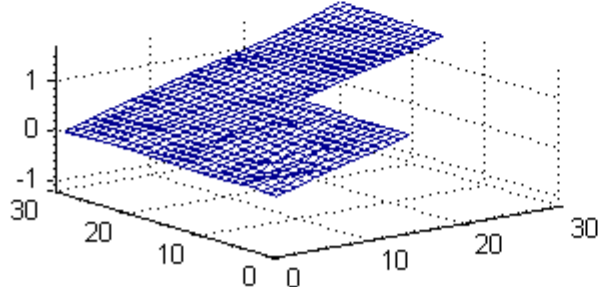
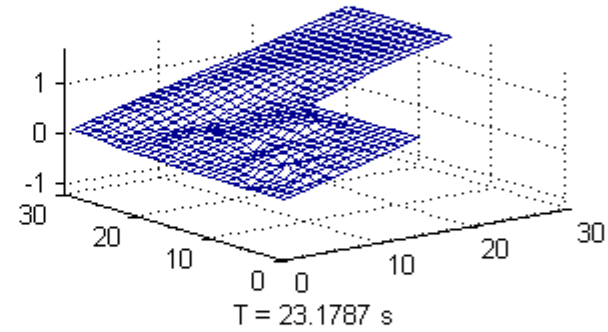
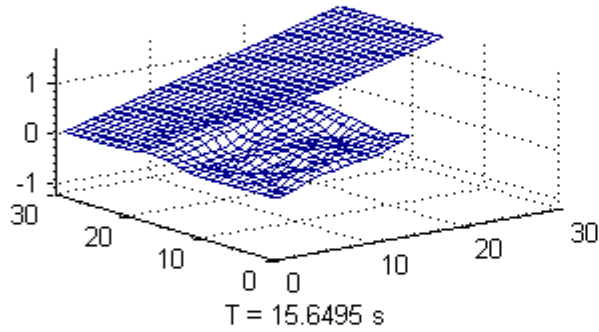
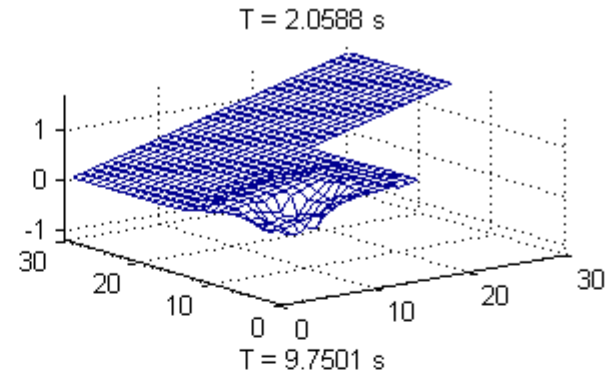
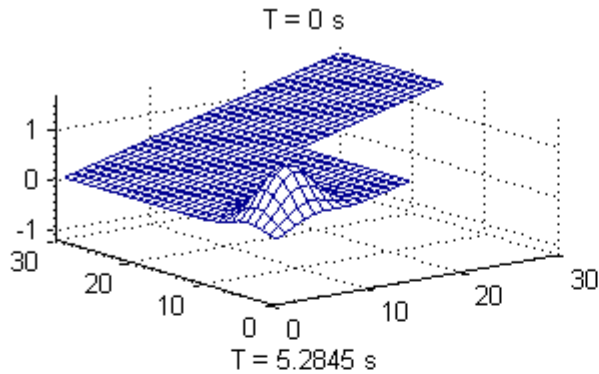
# 2D Grid / Zero Reflection Controller Perturbed Parameters $\pm 90\%$



# *L-Shaped Grid / No Control*



# *L-Shaped Grid / Zero Reflection Controller Perturbed Parameters $\pm 90\%$*



# Conclusions

- Conceptual Contributions:
  - Traveling waves → eliminate reflections.
  - Place controllers where reflections are expected.
  - Controller uses deviations in both power and frequency as inputs.
- Future Work: *lots!*

more realistic models, realistic assessment, irregular topologies, other control opportunities, and more.