



Optimal Placement of Phasor Measurement Units for State Estimation

Final Project Report

Power Systems Engineering Research Center

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Power Systems Engineering Research Center

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for State Estimation**

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Executive Summary

This report describes a study undertaken to determine the optimal locations of phasor measurement units (PMUs) for a given power system. Power systems are rapidly becoming populated by PMUs. PMUs have multiple uses at substations. They provide valuable phasor information for protection and control of power systems during abnormal operation. Under normal operations, they also help in monitoring the system state.

This project focuses on the use of PMU measurements in state estimators. The principle objective was to investigate methods of determining optimal locations for PMUs so that the system state of an entire power system can be observable. Several factors affect how this can be accomplished, such as the available data from existing conventional measurements, the number and location of zero injection buses, the number and location of installed PMUs, and, of course, the system topology.

Two new procedures were used to solve the problem of optimally locating PMUs. One is a numerical procedure where the problem is formulated as an integer optimization problem. The solution algorithm was implemented in a computer program. Several case studies were conducted to evaluate the algorithm's performance.

The other procedure is a topology-based method which finds the strategic locations for PMUs by merging observable islands. Standard test systems were first used to validate the software program for simple cases using the topology-based method. Subsequently, TVA's internal system data were used to obtain results for a large system. We found that the entire system can be made observable by strategically placing PMUs at one-third of the system buses for a given system. Fewer PMUs may be needed if there are zero injection buses in the system.

The software is developed in Matlab using an external optimization program (TomLab) which is used to implement the integer programming solution. The results of the project can be extended to investigate the benefits of adding a small number of PMUs at strategic locations to improve bad data detection and identification capability in the system. Similarly, it may be worthwhile to investigate the placement of a few PMUs for purposes of eliminating the possibility of unobservable states during expected topology changes or contingencies.

Table of Contents

1. State Estimation and Network Observability.....	1
1.1 Introduction.....	1
1.2 Project Objectives and Description.....	1
2. Developed Methods	3
2.1 Integer Programming Based Procedure	4
2.2 Topology Based Procedure.....	12
2.3 Placement Strategy Against Loss of a Single PMU	13
3. Illustrative Examples	14
3.1 IEEE 14-bus system.....	14
3.2 IEEE 30-bus system.....	15
3.3 IEEE 57-bus system.....	17
3.4 IEEE 118-bus system.....	19
4. TVA System Case Studies	22
4.1 TVA System (version 1).....	22
4.2 TVA System (version 2).....	28
5. Conclusions and Future Work	33
References.....	34
Project Publications	34

Table of Figures

Figure 2.1. PMU placement on IEEE 14-Bus System	3
Figure 2.2. IEEE 14-bus system with conventional measurements	5
Figure 2.3. System diagram after the merger of buses 7 and 8.....	11
Figure 2.4. Observable islands of IEEE 14-bus system.....	11
Figure 3.1. IEEE 14-bus system.....	13
Figure 3.2. IEEE 30-bus system.....	15
Figure 3.3. IEEE 57-bus system.....	16
Figure 3.4. IEEE 118-bus system.....	18

Table of Tables

Table 3.1	System information of IEEE 14-bus system	13
Table 3.2	Simulation results for 14-bus system without considering single PMU loss....	14
Table 3.3	Simulation results for 14-bus system considering single PMU loss	14
Table 3.4	System information of IEEE 30-bus system	14
Table 3.5	Simulation results for 30-bus system without considering single PMU loss....	14
Table 3.6	Simulation results for 30-bus system considering single PMU loss	15
Table 3.7	System information of IEEE 57-bus system	16
Table 3.8	Simulation results for 57-bus system without considering single PMU loss....	17
Table 3.9	Simulation results for 57-bus system considering single PMU loss	17
Table 3.10	System information of IEEE 118-bus system	17
Table 3.11	Simulation results for 118-bus system without considering single PMU loss	18
Table 3.12	Simulation results for 118-bus system considering single PMU loss	19
Table 3.13	Measurements information for IEEE 118-bus system	20
Table 3.14	Simulation results for 118-bus system considering conventional measurements.....	20
Table 4.1	Brief information of TVA system (version 1).....	21
Table 4.2	Bus names and numbers with installed PMUs.....	21
Table 4.3	Bus names and numbers where PMUs are scheduled to be installed	22
Table 4.4	Results of case studies for the TVA data (version 1).....	23
Table 4.5	Detailed results for case 1	24
Table 4.6	Detailed results for case 2	25
Table 4.7	Detailed results for case 3	26
Table 4.8	Detailed results for case 4	27
Table 4.9	System information of TVA system (version 2)	27
Table 4.10	Information of installed PMUs in TVA system (version 2).....	28
Table 4.11	Simulation results of TVA system (version 2) without considering conventional measurement, zero injections or PMUs.....	29
Table 4.12	Simulation results of TVA system (version 2) considering conventional measurement and zero injections	30
Table 4.13	Simulation results of TVA system (version 2) considering conventional measurement and zero injections	31

1. State Estimation and Network Observability

1.1 Introduction

Secure operation of power systems requires close monitoring of the system operating conditions. This is traditionally accomplished by the state estimator which resides in the control center computer and has access to the measurements received from numerous substations in the monitored system. By collecting analog measurements and the status data of the circuit breakers from remotely monitored and controlled substations and feeding them as input into state estimation function, state estimation can provide an estimate for all metered and un-metered electrical quantities and network parameters of the power system, detect and filter out gross errors in the measurement set and detect the topology errors in the network configuration. Until recently, available measurement sets did not contain phase angle measurements due to the technical difficulties associated with the synchronization of measurements at remote locations. Global positioning satellite (GPS) technology alleviated these difficulties and led to the development of phasor measurement units (PMU).

Synchronized Phase Measurement Unit (PMU) is a monitoring device, which was first introduced in mid-1980s. Phasor measurement units (PMU) are devices, which use synchronization signals from the global positioning system (GPS) satellites and provide the phasors of voltage and currents measured at a given substation. As the PMUs become more and more affordable, their utilization will increase not only for substation applications but also at the control centers for the EMS applications. One of the applications, which will be significantly affected by the introduction of PMUs, is the state estimator.

TVA is in the process of installing phasor measurement units (PMU) for enhanced monitoring of the TVA system. In order to avoid redundant use of PMUs, the optimal locations for the new PMUs must be determined. The objective this project is to make use of a minimum number of PMUs in order to make the system fully observable. Installation of PMUs will be a gradual process, requiring decisions on the best possible locations for a limited number of PMUs at the beginning. Hence, a systematic method is needed for finding the best locations for new PMUs in the presence of other already placed PMUs and/or conventional measurements. This project investigates this issue and provides a practical solution for the PMU placement problem.

1.2 Project Objectives and Description

State estimators provide optimal estimates of bus voltage phasors based on the available measurements and knowledge about the network topology. These measurements are commonly provided by the remote terminal units (RTU) at the substations and include real/reactive power flows, power injections, and magnitudes of bus voltages and branch currents. More recently, synchronized phasor measurements have started to become available at selected substations in the system. One of the issues faced by the planning engineers is how to select the best locations to install new PMUs.

The idea of using direct phasor measurements for system monitoring applications including the specific case of state estimation is not new. Earlier work done by Phadke and his co-workers [1-2] introduces the use of PMUs for such applications. This work is later extended to the investigation of optimal location of PMUs where each PMU is assumed to provide voltage and current phasors at its associated bus and all incident branches [3]. It is therefore possible to fully monitor the system by using relatively small number of PMUs much less than the number of buses in the system. This problem is formulated and solved by using a graph theoretic observability analysis and an optimization method based on Simulated Annealing in [3]. Possible loss or failure of PMUs is not considered in that study.

This project is concerned about the optimal placement of PMUs so that they can provide the maximum benefit for the state estimation function. It is understood that these devices have multiple uses and therefore their placement may have to be based on considerations related to several other applications, however this study limits its scope to the specific application of state estimation. Hence, the objective of the placement problem is to ensure that the entire system remains a single observable island for the given measurement set. In this report, two different procedures are proposed to solve the PMU placement problem.

First, a numerical method based on integer programming will be presented. The formulation of the problem facilitates analysis of network observability and is general enough to account for the existence of zero and non-zero power injections and power flow measurements. There are two different ways to treat zero and non-zero power injection measurements using this method. One is via the use of non-linear constraints, and the other one is based on a special topology transformation. These two methods will be illustrated in this report using the IEEE 14-bus as an example. The procedure can also be extended to account for loss of single PMUs.

Next, an alternative procedure of PMU placement for systems with a large number of conventional measurements and few observable islands is also developed. The efficiency of this method increases with the number of already existing measurements. By placing PMUs at the strategic boundary buses and merging the few existing observable islands, an observable system will be quickly rendered.

This report is organized in four sections. Section 2 contains the details of the developed methods for the solution of the optimal PMU placement problem. The two alternative methods, which are mentioned above, are described first, followed by the discussion of the case where loss of single PMU is considered as an added reliability criterion. Section 3 presents numerical results obtained by applying the developed methods to standard IEEE test systems with typical measurement configurations. Finally, the optimal PMU placement results obtained for two different sets of network and measurement data received from TVA are presented in section 4. Conclusions and final remarks are added at the end of the report.

2. Developed Methods

PMUs provide two types of measurements: bus voltage phasors and branch current phasors. Depending on the type of PMUs used the number of channels used for measuring voltage and current phasors will vary. In this report, it is assumed that each PMU has enough channels to record the bus voltage phasor at its associated bus and current phasors along all branches that are incident to this bus. The objective of the PMU placement problem is to render an observable system by using a minimum number of PMUs. An example of an optimally placed set of PMUs in a 14-bus system is shown below in Figure 2.1.

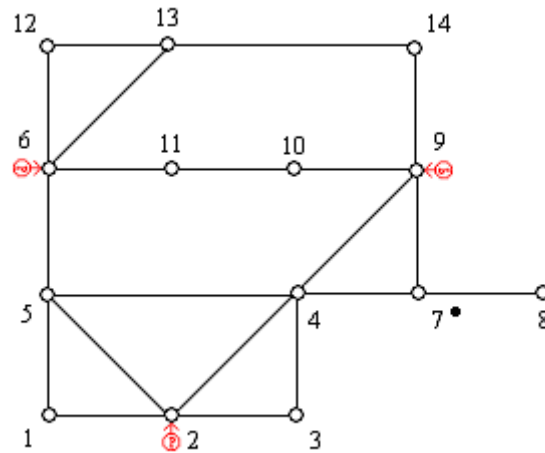


Figure 2.1. Optimal PMU placement for a 14-bus test system

In this system, there are three PMUs placed at buses 2, 6 and 9 respectively. Bus 7 is the only zero injection bus. The PMU at bus 2 can not only measure the voltage phasor of bus 2, but also the current phasors of branches 2-1, 2-3, 2-4 and 2-5. Using Ohm's law, the voltage phasors at buses 1, 3, 4 and 5 can be obtained from the branch currents and the voltage at bus 2. Having determined voltage phasors at buses 1, 2, 3, 4, and 5, the current phasors of branches 1-5, 3-4 and 4-5 can be calculated. Following the same logic, PMU at bus 6 can measure the voltage phasor at bus 6 and the current phasors of branches 6-5, 6-11, 6-12 and 6-13, thus allowing the calculation of the voltage phasors at buses 5, 11, 12, 13 and the current phasor of branch 12-13. PMU at bus 9 can measure the voltage phasor at bus 9 and the current phasors of branches 9-4, 9-7, 9-10, 9-14 and allow the calculation of the voltage phasors at buses 4, 7, 10, 14, and the current phasors of branches 4-7. As voltage phasors of buses 10, 11, 13, 14 are known, current phasors of branches 10-11 and 13-14 can now also be calculated. Using the known current phasors of branches 4-7 and 9-7, and the zero injection at bus 7, the current phasor of branch 7-8 can be derived using the Kirchhoff's Current Law. The only remaining unknown voltage phasor at bus 8 can now be calculated by using the voltage phasor at bus 7 and the current phasor of branch 7-8. Thus the entire system becomes observable by placing only three PMUs at buses 2, 6, 9 and by considering the zero injection at bus 7.

In this report, two different procedures are introduced to solve the PMU placement problem. One is an integer based programming method, and the other one is a topology-based method. Both of these methods will be discussed in detail via the use of the IEEE 14-bus example in the following sections.

2.1 Integer Programming Based Procedure

In this section, a numerical method based on Integer Programming will be presented to solve the optimal PMU placement problem. The formulation of problem is shown as below.

For an n-bus system, the PMU placement problem can be formulated as follows:

$$\begin{aligned} \min \sum_i^n w_i \cdot x_i \\ \text{s.t. } f(X) \geq \hat{1} \end{aligned} \tag{2.1}$$

where

X is a binary decision variable vector, whose entries are defined as:

$$x_i = \begin{cases} 1 & \text{if a PMU is installed at bus } i \\ 0 & \text{otherwise} \end{cases}$$

w_i is the cost of the PMU installed at bus i .

$f(X)$ is a vector function, whose entries are non-zero if the corresponding bus voltage is solvable using the given measurement set and zero otherwise.

$\hat{1}$ is a vector whose entries are all ones.

Inner product of the binary decision variable vector and the cost vector represents the total installation costs of the selected PMUs. Constraint functions ensure full network observability while minimizing the total installation cost of the PMUs.

The procedure for building the constraint equations will be described for three possible cases where there are (1) no conventional measurement or zero injections, (2) flow measurements or (3) flow measurements as well as injection measurements (they may be zero injections or measured injections). Description of the procedure for each case will be given using IEEE 14-bus system example for clarification. However, the entire procedure is actually programmed and successfully tested on different size systems with diverse measurement configurations.

Consider the IEEE 14-bus system and its measurement configuration shown in Figure 2.2. The black dot near bus 7 represents that bus 7 is a zero injection bus (or has an injection measurement installed) while the black box on line 5-6 represents a paired flow measurement on line 5-6.

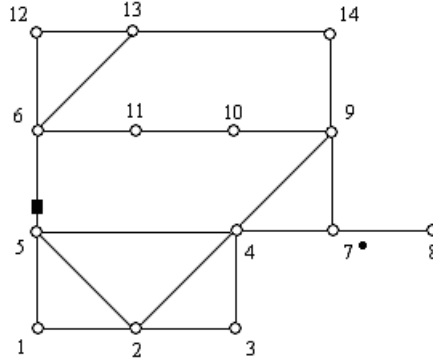


Figure 2.2. IEEE 14-bus system with conventional measurements

Case 1. A system with no conventional measurements and/or zero injections

In this case, the flow measurement and the zero injection are ignored. In order to form the constraint set, the binary connectivity matrix A , whose entries are defined below, will be formed first:

$$A_{k,m} = \begin{cases} 1 & \text{if } k = m \text{ or } k \text{ and } m \text{ are connected} \\ 0 & \text{if otherwise} \end{cases}$$

Matrix A can be directly obtained from the bus admittance matrix by transforming its entries into binary form. Building the A matrix for the 14-bus system yields:

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \quad (2.2)$$

The constraints for this case can be formed as:

$$f(X) = A \cdot X = \begin{cases} f_1 = x_1 + x_2 + x_5 & \geq 1 \\ f_2 = x_1 + x_2 + x_3 + x_4 + x_5 & \geq 1 \\ f_3 = x_2 + x_3 + x_4 & \geq 1 \\ f_4 = x_2 + x_3 + x_4 + x_5 + x_7 + x_9 & \geq 1 \\ f_5 = x_1 + x_2 + x_4 + x_5 + x_6 & \geq 1 \\ f_6 = x_5 + x_6 + x_{11} + x_{12} + x_{13} & \geq 1 \\ f_7 = x_4 + x_7 + x_8 + x_9 & \geq 1 \\ f_8 = x_7 + x_8 & \geq 1 \\ f_9 = x_4 + x_7 + x_9 + x_{10} + x_{14} & \geq 1 \\ f_{10} = x_9 + x_{10} + x_{11} & \geq 1 \\ f_{11} = x_6 + x_{10} + x_{11} & \geq 1 \\ f_{12} = x_6 + x_{12} + x_{13} & \geq 1 \\ f_{13} = x_6 + x_{12} + x_{13} + x_{14} & \geq 1 \\ f_{14} = x_9 + x_{13} + x_{14} & \geq 1 \end{cases} \quad (2.3)$$

The operator “+” serves as the logical “OR” and the use of 1 in the right hand side of the inequality ensures that at least one of the variables appearing in the sum will be non-zero. For example, consider the constraints associated with bus 1 and 2 as given below:

$$\begin{aligned} f_1 &= x_1 + x_2 + x_5 \geq 1 \\ f_2 &= x_1 + x_2 + x_3 + x_4 + x_5 \geq 1 \end{aligned}$$

The first constraint $f_1 \geq 1$ implies that at least one PMU must be placed at either one of buses 1, 2 or 5 in order to make bus 1 observable. Similarly, the second constraint $f_2 \geq 1$ indicates that at least one PMU should be installed at any one of the buses 1, 2, 3, 4, or 5 in order to make bus 2 observable.

Case 2: A system with some flow measurements.

This case considers the situation where some flow measurements may be present. Flow measurement on branch 5-6 in the 14-bus example system will be used to illustrate the approach on how to deal with existing flow measurements. Existence of this flow measurement will lead to the modification of the constraints for buses 5 and 6 accordingly. Modification follows the observation that having a flow measurement along a given branch allows the calculation of one of the terminal bus voltage phasors when the other one is known. Hence, the constraint equations associated with the terminal buses of the measured branch can be merged into a single constraint. In the case of the example system, the constraints for buses 5 and 6 are merged into a joint constraint as follows.

$$\begin{cases} f_5 = x_1 + x_2 + x_4 + x_5 + x_6 \geq 1 \\ f_6 = x_5 + x_6 + x_{11} + x_{12} + x_{13} \geq 1 \end{cases} \Rightarrow \\ f_{5_new} = f_5 + f_6 = x_1 + x_2 + x_4 + x_5 + x_6 + x_{11} + x_{12} + x_{13} \geq 1$$

which implies that if either one of the voltage phasors at bus 5 or 6 is observable, the other one will be observable.

Applying this modification to the constraints for the 14-bus system, the following set of constraints will be obtained:

$$f(X) = \begin{cases} f_1 = x_1 + x_2 + x_5 & \geq 1 \\ f_2 = x_1 + x_2 + x_3 + x_4 + x_5 & \geq 1 \\ f_3 = x_2 + x_3 + x_4 & \geq 1 \\ f_4 = x_2 + x_3 + x_4 + x_5 + x_7 + x_9 & \geq 1 \\ f_{5_new} = x_1 + x_2 + x_4 + x_5 + x_6 + x_{11} + x_{12} + x_{13} & \geq 1 \\ f_7 = x_4 + x_7 + x_8 + x_9 & \geq 1 \\ f_8 = x_7 + x_8 & \geq 1 \\ f_9 = x_4 + x_7 + x_9 + x_{10} + x_{14} & \geq 1 \\ f_{10} = x_9 + x_{10} + x_{11} & \geq 1 \\ f_{11} = x_6 + x_{10} + x_{11} & \geq 1 \\ f_{12} = x_6 + x_{12} + x_{13} & \geq 1 \\ f_{13} = x_6 + x_{12} + x_{13} + x_{14} & \geq 1 \\ f_{14} = x_9 + x_{13} + x_{14} & \geq 1 \end{cases} \quad (2.4)$$

Case 3: A system with both injection measurements (some of which may be zero injection pseudo-measurements) and flow measurements.

This case considers the most general situation where both injection and flow measurements may be present, but not enough to make the entire system observable. Injection measurements whether they are zero injections or not, are treated the same way.

Consider again the same 14-bus system, where bus 7 is a zero injection bus. It is easy to see that if the phasor voltages at any three out of the four buses 4, 7, 8 and 9 are known, then the fourth one can be calculated using the Kirchoff's Current Law applied at bus 7 where the net injected current is known.

There are two different ways to treat the injection measurements and form the constraints. One is to form non-linear constraints for the neighbors of the buses, which have injection measurements installed. The alternative approach involves a topology transformation. These will be discussed separately next.

a) *Forming non-linear constraints*

One way to treat the injection buses is to modify the constraints associated with the neighboring buses of these buses and form a set of non-linear constraints. This is accomplished as shown below.

To treat the zero injection bus 7 in the IEEE 14-bus system, constraints associate with its neighboring buses 4, 8 and 9 will be modified as follows,

$$\begin{aligned} f_4 &= x_2 + x_3 + x_4 + x_5 + x_7 + x_9 + f_7 \cdot f_8 \cdot f_9 && \geq 1 \\ f_8 &= x_7 + x_8 + f_4 \cdot f_7 \cdot f_9 && \geq 1 \\ f_9 &= x_4 + x_7 + x_9 + x_{10} + x_{14} + f_4 \cdot f_7 \cdot f_8 && \geq 1 \end{aligned}$$

Note that the operator '.' serves as the logical "AND" in the above equations.

The expressions for f_i can be further simplified by using the following properties of the logical AND (.) and OR (+) operators: Given two sets A and B, where set A is a subset of set B, then $A + B = B$ and $A \cdot B = A$.

For instance, substituting the expression for f_7 in the expression for f_4 , f_4 can be written as:

$$\begin{aligned} f_4 &= x_2 + x_3 + x_4 + x_5 + x_7 + x_9 + f_7 \cdot f_8 \cdot f_9 \\ &= x_2 + x_3 + x_4 + x_5 + x_7 + x_9 + (x_4 + x_7 + x_8 + x_9) \cdot f_8 \cdot f_9 \\ &= x_2 + x_3 + x_4 + x_5 + x_7 + x_9 + x_4 \cdot f_8 \cdot f_9 + x_7 \cdot f_8 \cdot f_9 + x_8 \cdot f_8 \cdot f_9 + x_9 \cdot f_8 \cdot f_9 \\ &= x_2 + x_3 + x_4 + x_5 + x_7 + x_9 + x_8 \cdot f_8 \cdot f_9 \end{aligned}$$

The product $x_4 \cdot f_8 \cdot f_9$ is eliminated because it is the subset of x_4 , which already exists in the expression. Using similar reasoning, $x_7 \cdot f_8 \cdot f_9$ and $x_9 \cdot f_8 \cdot f_9$ are also eliminated.

Note that the expression for f_7 should also include an extra product term given by $f_4 \cdot f_8 \cdot f_9$, however this higher order term will be neglected. In all our simulated cases, this approximation is found to have no effect on the optimization.

Then, substituting the expression of f_8 yields:

$$\begin{aligned} f_4 &= x_2 + x_3 + x_4 + x_5 + x_7 + x_9 + x_8 \cdot f_8 \cdot f_9 \\ &= x_2 + x_3 + x_4 + x_5 + x_7 + x_9 + x_8 \cdot (x_7 + x_8) \cdot f_9 \\ &= x_2 + x_3 + x_4 + x_5 + x_7 + x_9 + x_8 \cdot f_9 \end{aligned}$$

Substituting for f_9 :

$$\begin{aligned} f_4 &= x_2 + x_3 + x_4 + x_5 + x_7 + x_9 + x_8 \cdot f_9 \\ &= x_2 + x_3 + x_4 + x_5 + x_7 + x_9 + x_8 \cdot (x_4 + x_7 + x_9 + x_{10} + x_{14}) \\ &= x_2 + x_3 + x_4 + x_5 + x_7 + x_9 + x_8 \cdot x_{10} + x_8 \cdot x_{14} \end{aligned}$$

Finally, the expression for f_4 simplifies to the following:

$$f_4 = x_2 + x_3 + x_4 + x_5 + x_7 + x_9 + x_8 \cdot x_{10} + x_8 \cdot x_{14}.$$

Applying similar simplification logic to all other expressions, the constraint set can be written as follows:

$$\begin{aligned} f_4 &= x_2 + x_3 + x_4 + x_5 + x_7 + x_9 + x_8 \cdot x_{10} + x_8 \cdot x_{14} && \geq 1 \\ f_8 &= x_4 + x_7 + x_8 + x_9 && \geq 1 \\ f_9 &= x_4 + x_7 + x_9 + x_{10} + x_{14} + x_2 \cdot x_8 + x_3 \cdot x_8 + x_5 \cdot x_8 && \geq 1 \end{aligned}$$

Note that the constraints corresponding to all other buses will remain the same as given in equation (2.4). One exception to this is the constraint for bus 7 where the injection is measured (or known). This constraint will be eliminated from the constraint set. The reason for removing the constraints associated with injection buses is that their effects are indirectly taken into account by the product terms augmented to the constraints associated with the neighboring buses.

The constraints for this case are shown in (2.5).

$$f(X) = \begin{cases} f_1 = x_1 + x_2 + x_5 & \geq 1 \\ f_2 = x_1 + x_2 + x_3 + x_4 + x_5 & \geq 1 \\ f_3 = x_2 + x_3 + x_4 & \geq 1 \\ f_4 = x_2 + x_3 + x_4 + x_5 + x_7 + x_9 + x_8 \cdot x_{10} + x_8 \cdot x_{14} & \geq 1 \\ f_{5_new} = x_1 + x_2 + x_4 + x_5 + x_6 + x_{11} + x_{12} + x_{13} & \geq 1 \\ f_8 = x_4 + x_7 + x_8 + x_9 & \geq 1 \\ f_9 = x_4 + x_7 + x_9 + x_{10} + x_{14} + x_2 \cdot x_8 + x_3 \cdot x_8 + x_5 \cdot x_8 & \geq 1 \\ f_{10} = x_9 + x_{10} + x_{11} & \geq 1 \\ f_{11} = x_6 + x_{10} + x_{11} & \geq 1 \\ f_{12} = x_6 + x_{12} + x_{13} & \geq 1 \\ f_{13} = x_6 + x_{12} + x_{13} + x_{14} & \geq 1 \\ f_{14} = x_9 + x_{13} + x_{14} & \geq 1 \end{cases} \quad (2.5)$$

This way of forming constraints for zero injection buses or buses which have injection measurements is complicated and time consuming. It is also noticed that nonlinear part will be introduced in the constraints and it will further slow down the integer programming. Hence, the following alternative method is developed for systems with a large number of injections to process.

b) Topology transformation

This alternative method referred here as the topology transformation is developed for handling injection measurements. The main idea is to merge the bus which has the injection measurement, with any one of its neighbors. This is based on the observation that if the voltage phasors of all its neighbors are known, the voltage phasor of this injection bus can be calculated by the Kirchhoff's Current Law.

Figure 2.3 shows the updated system diagram after the merger of buses 7 and 8 into a new bus 8'. The newly created branch 8'-9 reflects the original connection between buses 7 and 9.

Hence, the constraints vector function can be formed as shown in (2.6).

$$f(X) = \begin{cases} f_1 = x_1 + x_2 + x_5 & \geq 1 \\ f_2 = x_1 + x_2 + x_3 + x_4 + x_5 & \geq 1 \\ f_3 = x_2 + x_3 + x_4 & \geq 1 \\ f_4 = x_2 + x_3 + x_4 + x_5 + x_8 + x_9 & \geq 1 \\ f_{5_new} = x_1 + x_2 + x_4 + x_5 + x_6 + x_{11} + x_{12} + x_{13} & \geq 1 \\ f_8 = x_4 + x_8 + x_9 & \geq 1 \\ f_9 = x_4 + x_8 + x_9 + x_{10} + x_{14} & \geq 1 \\ f_{10} = x_9 + x_{10} + x_{11} & \geq 1 \\ f_{11} = x_6 + x_{10} + x_{11} & \geq 1 \\ f_{12} = x_6 + x_{12} + x_{13} & \geq 1 \\ f_{13} = x_6 + x_{12} + x_{13} + x_{14} & \geq 1 \\ f_{14} = x_9 + x_{13} + x_{14} & \geq 1 \end{cases} \quad (2.6)$$

Topology transformation is faster and will not introduce any nonlinear part in constraint set. Yet a word of caution needs to be added here in that, if the optimal solution chooses the newly formed fictitious bus (merger of two actual buses) as a candidate bus, it may place one PMU on one of these two buses or two PMUs on both. In this case, a topology analysis needs to be applied to check the observability of the system. This also assures that the minimum number of PMUs will be placed.

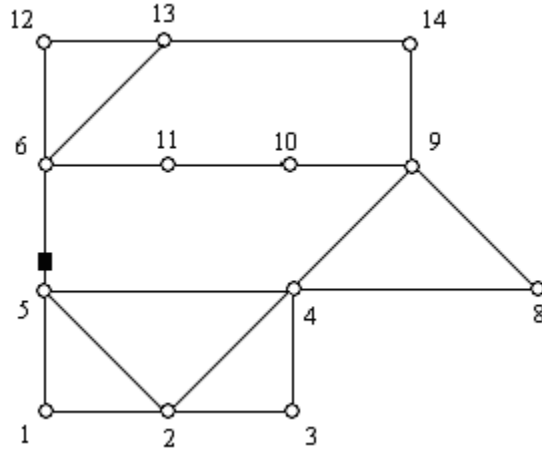


Figure 2.3. System diagram after the merger of buses 7 and 8

2.2 Topology Based Procedure

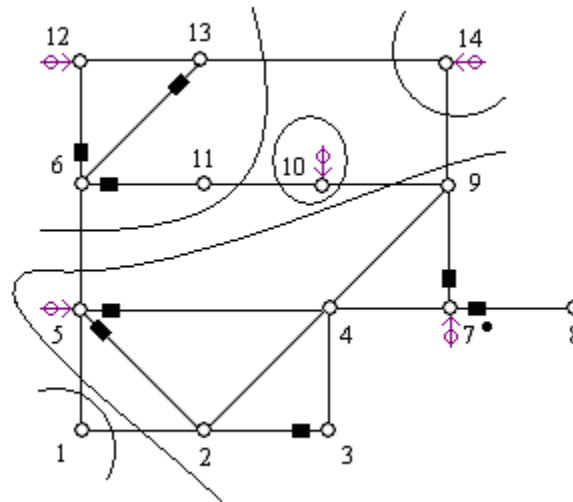


Figure 2.4. Observable islands for the IEEE 14-bus system

The integer programming based procedure is quite effective in systematically placing PMUs in a system where there are very few existing measurements. It can also be used for placing PMUs in a system which is to be exclusively monitored by PMUS only. However, most of today's power systems already have a significant number of conventional measurements and PMUs are planned to be installed to enhance the existing measurement system. For such systems which may be unobservable and have few observable islands, a topology based method can be easier to apply. The method will find the strategic locations of PMUs by merging the observable islands. Since only boundary buses contribute to the process of merging observable islands, these will be the strategic locations for placing PMUs. First a numerical observability analysis is carried out to determine the observable islands. Then the boundary bus, which connects to the maximum number of other islands or the one which has the maximum number of branches connected to other islands will be chosen to place a PMU. In order to take advantage of the injection measurements, after the selection of one PMU location, the numerical observability analysis will be re-applied to update the observable islands.

Consider the IEEE 14-bus system example shown in Figure 2.4, where there are 5 initially observable islands. Boundary buses are identified as buses 1, 2, 5, 6, 9, 10, 11, 13 and 14. Among them bus 9, 10 and 14 are connected two 2 different observable islands respectively, and have the maximum number of neighbor islands. Bus 9 is chosen to install a PMU. Numerical observability analysis is executed and the system is found to become observable. This implies that installing one PMU at bus 9 merges all of the five observable islands into one observable system. The logic is simple and easy to implement, provided that there are few observable islands and consequently few boundary buses. Effective utilization of this method will be illustrated in section 4 on the TVA system.

2.3 Placement Strategy Against Loss of a Single PMU

So far it is assumed that those PMUs which are placed by the proposed method, will function perfectly. While PMUs are highly reliable, they are prone to failure just like any other measuring device. In order to guard against such unexpected failures of PMUs, the above placement strategy is extended to account for single PMU loss. In this study, this objective is achieved by choosing two independent PMU sets, a primary set and a backup set, each of which can make the system observable on its own. If any PMU is lost, the other set of PMUs will guarantee the observability of the system.

The primary set of PMUs is chosen by building the constraint functions according to the procedures described in subsections above and solving the integer-programming problem. The backup set is chosen by removing all the x_i terms in the constraint functions, where bus i is in the primary set, in order to avoid picking up the same bus which appears in primary set. Then the integer-programming problem is solved to obtain the backup set.

3. Illustrative Examples

This section contains various simulation examples, which are carried out using the IEEE 14-bus, 30-bus, 57-bus and 118-bus systems. TOMLAB/MINLP and MILP [4] software package is used to solve the Integer Linear/Nonlinear Programming problem. Detailed system information and simulation results are given in the following sub-sections.

3.1 IEEE 14-bus system

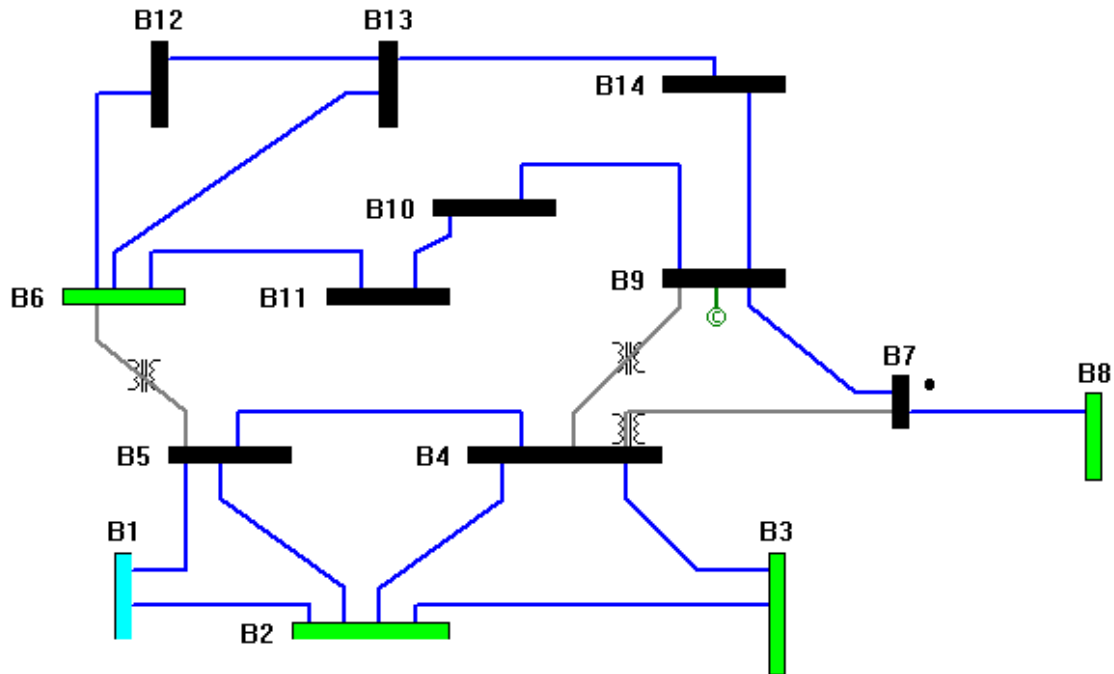


Figure 3.1. IEEE 14-bus system

IEEE 14-bus system is shown in Figure 3.1. The Information of the system and zero injections are given in the Table 3.1.

Table 3.1 System information of IEEE 14-bus system

System	# of branches	# of zero injections	Zero injection buses
IEEE 14-bus	20	1	7

Case 1. Effect of considering zero injections

In this case, Integer Programming method is used to solve the optimal PMU placement problem without considering the loss of single PMU. Simulations are carried out with and without considering zero injections. Results are given in Table 3.2.

Table 3.2 Simulation results for the 14-bus system without considering single PMU loss

Ignore zero injections		Consider zero injections			
		Non-linear constraints		Topology transformation	
# of PMUs	Loc. (bus #)	# of PMUs	Loc. (bus #)	# of PMUs	Loc. (bus #)
4	2, 6, 7, 9	3	2, 6, 9	3	2, 6, 9

Case 2. Considering single PMU loss

This case determines the optimal PMU locations that will maintain observability under the loss of any single PMU. Integer Programming is used, simulations are carried out with and without considering zero injections. Results are given in Table 3.3.

Table 3.3 Simulation results for 14-bus system considering single PMU loss

Ignore zero injections		Consider zero injections			
		Non-linear constraints		Topology transformation	
# of PMUs	Loc. (bus #)	# of PMUs	Loc. (bus #)	# of PMUs	Loc. (bus #)
9	1, 2, 3, 6, 7, 8, 9, 10, 13	7	1, 2, 4, 6, 9, 10, 13	7	1, 2, 4, 6, 9, 10, 13

3.2 IEEE 30-bus system

IEEE 30-bus system is shown in Figure 3.2. The Information of the system and zero injections are given in the Table 3.4.

Table 3.4 System information of IEEE 30-bus system

System	# of branches	# of zero injections	Zero injection buses
IEEE 30-bus	41	5	6, 9, 11, 25, 28

Case 1. Effect of considering zero injection

In this case, Integer Programming method is used to solve the optimal PMU placement problem without considering the loss of single PMU. Simulations are carried out with and without considering zero injections. Results are given in Table 3.5.

Table 3.5 Simulation results for 30-bus system without considering single PMU loss

Ignore zero injections		Consider zero injections			
		Non-linear constraints		Topology transformation	
# of PMUs	Loc. (bus #)	# of PMUs	Loc. (bus #)	# of PMUs	Loc. (bus #)
10	2, 4, 6, 9, 10, 12, 15, 18, 25, 27	7	3, 5, 10, 12, 18, 23, 27	8	2, 3, 6, 10, 12, 18, 23, 27

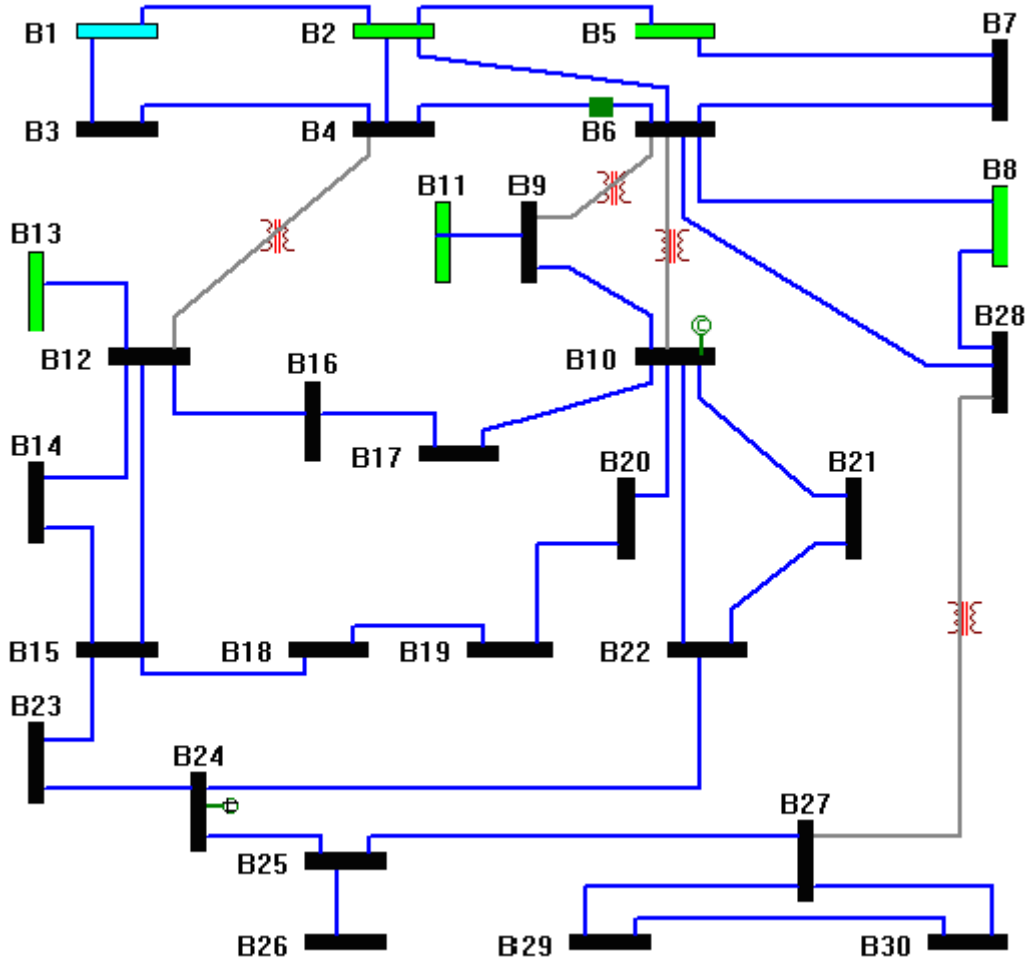


Figure 3.2. IEEE 30-bus system

Case 2. Considering single PMU loss

In this case, single PMU loss is considered. Integer Programming is used, simulations are carried out with and without considering zero injections. Results are given in Table 3.6.

Table 3.6 Simulation results for 30-bus system considering single PMU loss

Ignore zero injections		Consider zero injections			
# of PMUs	Loc. (bus #)	Non-linear constraints		Topology transformation	
		# of PMUs	Loc. (bus #)	# of PMUs	Loc. (bus #)
22	2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 21, 23, 25, 26, 27, 29	17	2, 3, 4, 5, 6, 10, 12, 13, 15, 17, 18, 19, 21, 23, 24, 27, 29	18	1, 2, 3, 4, 7, 8, 9, 13, 14, 15, 16, 19, 20, 21, 23, 24, 28, 29

3.3 IEEE 57-bus system

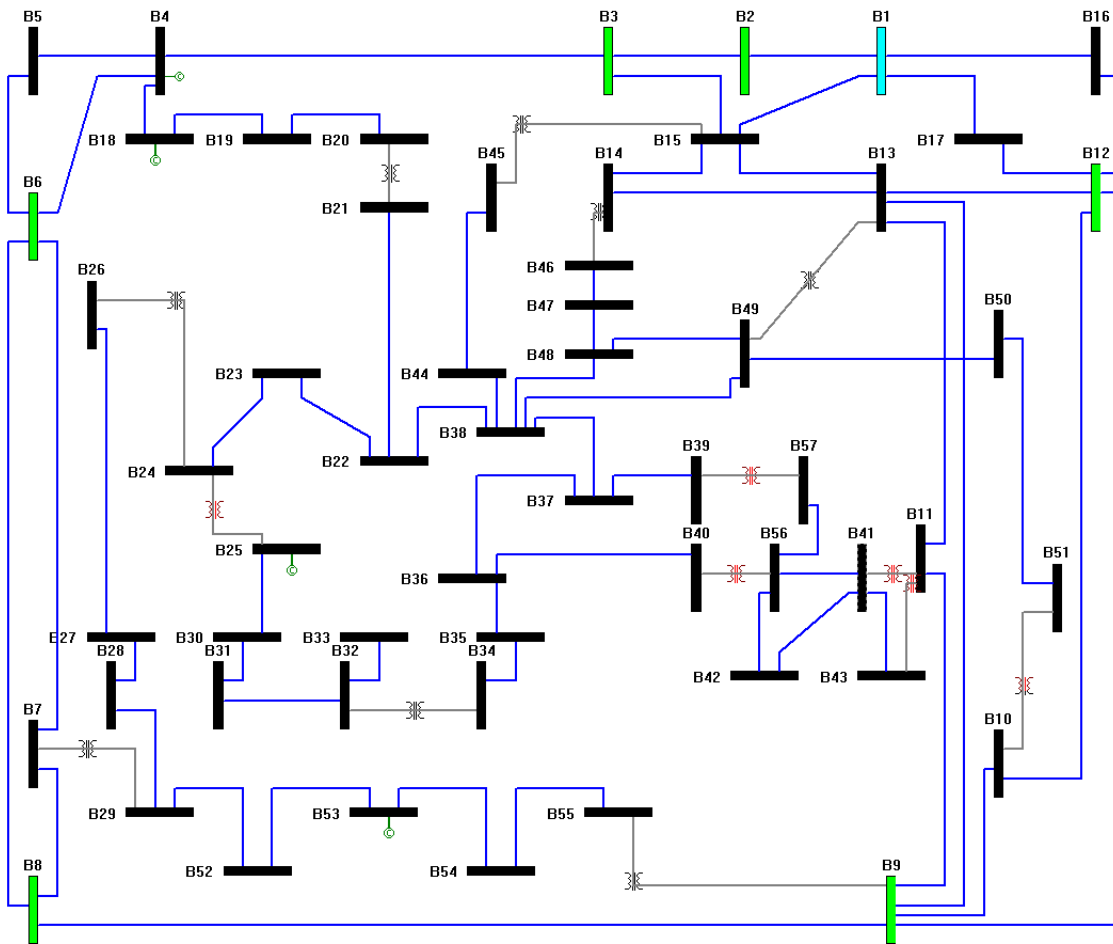


Figure 3.3. IEEE 57-bus system

IEEE 57-bus system is shown in Figure 3.3. The Information of the system and zero injections are given in the Table 3.7.

Table 3.7 System information of IEEE 57-bus system

System	# of branches	# of zero injections	Zero injection buses
IEEE 57-bus	78	15	4, 7, 11, 21, 22, 24, 26, 34, 36, 37, 39, 40, 45, 46, 48

Case 1. Effect of considering zero injection

In this case, Integer Programming method is used to solve the optimal PMU placement problem without considering the loss of single PMU. Simulations are carried out with and without considering zero injections. Results are given in Table 3.8.

Table 3.8 Simulation results for 57-bus system without considering single PMU loss

Ignore zero injections		Consider zero injections			
		Non-linear constraints		Topology transformation	
# of PMUs	Loc. (bus #)	# of PMUs	Loc. (bus #)	# of PMUs	Loc. (bus #)
17	1, 4, 7, 9, 15, 20, 24, 25, 27, 32, 36, 38, 39, 41, 46, 50, 53	13	1, 6, 9, 15, 20, 25, 27, 32, 38, 47, 50, 53, 56	12	1, 5, 9, 14, 15, 20, 25, 28, 32, 50, 53, 56

Case 2. Considering single PMU loss

In this case, single PMU loss is considered. Integer Programming is used, simulations are carried out with and without considering zero injections. Results are given in Table 3.9.

Table 3.9 Simulation results for 57-bus system considering single PMU loss

Ignore zero injections		Consider zero injections			
		Non-linear constraints		Topology transformation	
# of PMUs	Loc. (bus #)	# of PMUs	Loc. (bus #)	# of PMUs	Loc. (bus #)
35	1, 2, 4, 6, 7, 9, 11, 12, 13, 15, 19, 20, 22, 24, 25, 26, 27, 29, 30, 32, 33, 34, 36, 37, 38, 39, 41, 44, 46, 47, 50, 51, 53, 54, 56	30	1, 2, 6, 7, 9, 10, 12, 14, 15, 18, 20, 21, 24, 25, 27, 29, 31, 32, 33, 34, 37, 38, 41, 44, 47, 49, 50, 53, 54, 56	26	1, 2, 4, 5, 9, 12, 13, 14, 15, 18, 20, 23, 25, 28, 29, 30, 32, 33, 35, 38, 41, 50, 51, 53, 54, 56

3.4 IEEE 118-bus system

IEEE 118-bus system is shown in Figure 3.4. The Information of the system and zero injections are given in the Table 3.10.

Table 3.10 System information of IEEE 118-bus system

System	# of branches	# of zero injections	Zero injection buses
IEEE 118-bus	179	10	5, 9, 30, 37, 38, 63, 64, 68, 71, 81

Case 1. Effect of considering zero injection

In this case, Integer Programming method is used to solve the optimal PMU placement problem without considering the loss of single PMU. Simulations are carried out with and without considering zero injections. Results are given in Table 3.11.

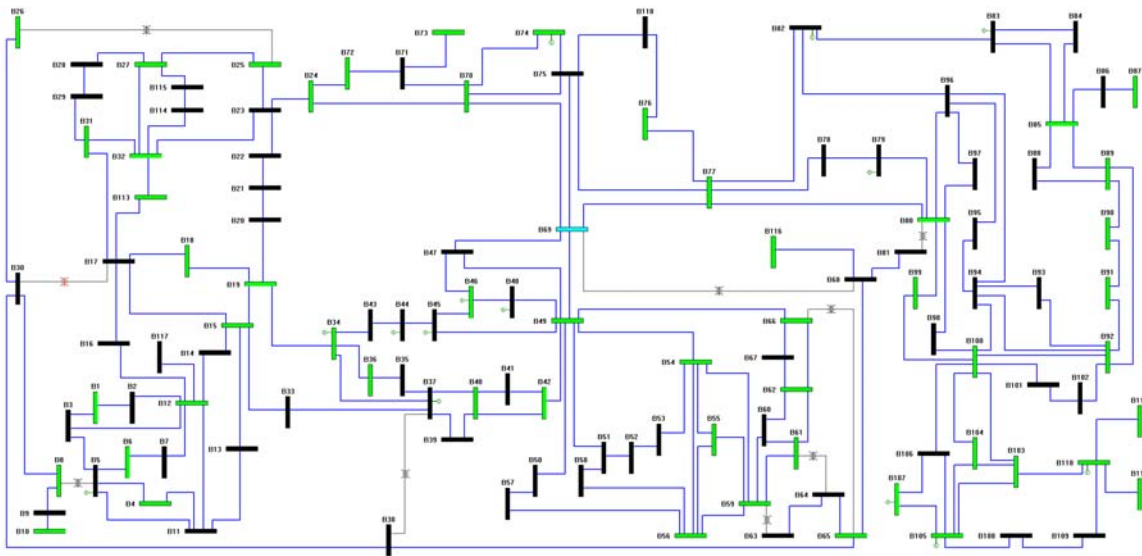


Figure 3.4. IEEE 118-bus system

Table 3.11 Simulation results for 118-bus system without considering single PMU loss

Ignore zero injections		Consider zero injections			
		Non-linear constraints		Topology transformation	
# of PMUs	Loc. (bus #)	# of PMUs	Loc. (bus #)	# of PMUs	Loc. (bus #)
32	2, 5, 9, 11, 12, 17, 21, 24, 25, 28, 34, 37, 40, 45, 49, 52, 56, 62, 63, 68, 73, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110, 114	29	2, 8, 11, 12, 15, 19, 21, 27, 31, 32, 34, 40, 45, 49, 52, 56, 62, 65, 72, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110	28	2, 8, 11, 12, 17, 21, 25, 28, 33, 34, 40, 45, 49, 52, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110, 114

Case 2. Considering single PMU loss

In this case, single PMU loss is considered. Integer Programming is used, simulations are carried out with and without considering zero injections. Results are given in Table 3.12.

Table 3.12 Simulation results for 118-bus system considering single PMU loss

Ignore zero injections		Consider zero injections			
		Non-linear constraints		Topology transformation	
# of PMUs	Loc. (bus #)	# of PMUs	Loc. (bus #)	# of PMUs	Loc. (bus #)
72	1, 2, 4, 5, 6, 9, 10, 11, 12, 15, 16, 17, 19, 21, 22, 24, 25, 27, 28, 29, 30, 32, 34, 35, 37, 39, 40, 41, 43, 45, 46, 49, 50, 51, 52, 54, 56, 59, 62, 63, 64, 66, 68, 70, 71, 73, 75, 76, 77, 78, 80, 81, 83, 85, 86, 87, 89, 90, 92, 94, 96, 100, 101, 105, 106, 108, 110, 111, 112, 114, 116, 117	65	1, 2, 6, 8, 9, 11, 12, 13, 14, 15, 17, 19, 20, 21, 23, 24, 27, 28, 31, 32, 34, 35, 37, 40, 41, 43, 45, 46, 49, 51, 52, 54, 56, 57, 59, 62, 65, 67, 68, 70, 72, 75, 76, 77, 78, 80, 83, 85, 86, 87, 89, 90, 92, 94, 96, 100, 101, 105, 106, 108, 110, 111, 112, 114, 117	65	1, 2, 5, 6, 8, 10, 11, 12, 15, 16, 17, 19, 21, 22, 24, 25, 27, 28, 31, 32, 33, 34, 35, 39, 40, 41, 43, 45, 46, 49, 50, 51, 52, 53, 56, 59, 62, 65, 66, 70, 72, 75, 76, 77, 78, 80, 83, 85, 86, 87, 89, 90, 92, 94, 96, 100, 101, 105, 106, 108, 110, 111, 112, 114, 117

Case 3. System having several conventional measurements

In this case, several conventional measurements are introduced into IEEE 118-bus system and several observable islands are formed. The list of flow and injection measurements is given in Table 3.13. Zero injections shown in Table 3.10 are also considered and treated the same as injection measurements. Topology based method to merge observable islands is used and results are shown in Table 3.14. Loss of single PMU is not considered in this case.

Table 3.13 Measurements information for IEEE 118-bus system

Flow measurements		Injection measurements		# of observable islands
No.	Locations	No.	Loc. (bus #)	
49	6-7, 5-6, 1-3, 3-12, 3-5, 8-30, 8-5, 8-9, 26-25, 25-27, 29-31, 28-29, 23-32, 32-114, 27-32, 70-74, 74-75, 47-69, 46-47, 82-83, 83-84, 93-94, 92-94, 94-100, 99-100, 98-100, 106-107, 105-107, 51-52, 51-58, 55-59, 54-59, 59-60, 15-19, 19-20, 19-34, 12-16, 12-117, 35-37, 34-37, 35-36, 38-37, 43-44, 49-50, 65-68, 68-116, 68-69, 110-111, 110-112	29	5, 9, 12, 19, 21, 27, 28, 30, 32, 37, 38, 41, 44, 47, 50, 53, 59, 62, 63, 64, 68, 71, 81, 83, 86, 94, 96, 108, 110	58

Table 3.14 Simulation results for 118-bus system considering conventional measurements

# of PMUs	19
Loc.(bus #)	2, 11, 17, 21, 24, 40, 49, 56, 62, 71, 77, 80, 86, 89, 91, 100, 102, 108, 118

4. TVA System Case Studies

4.1 TVA System (version 1)

a. System information – version 1

At the beginning of the project, TVA network data, along with a list of buses where PMUs were installed or planned to be installed, are received from TVA. The data also revealed several buses where the net injected power was zero. Such buses, referred as zero injection buses, allow further reductions in the required number of PMUs in making the overall system observable. Initial PMU placement algorithm is then modified to account for these zero injection buses, as well as for those already installed PMUs.

The developed algorithm is applied to TVA data, based on various assumptions and PMU placement results are obtained. TVA data which are received in August 2004 from TVA contain 1375 buses out of which 330 are zero injection buses. Information about the already installed and scheduled to be installed PMUs is provided as a list of bus names. For several of the bus names in this list, it is found that more than one bus number (of different voltage levels) correspond to the same bus name in the TVA data file. Thus, it is assumed that all such buses have PMUs. Also, it is noted that some of these buses have zero injections, which allow further reductions in the required PMUs to be placed in the system. There are a total of 11 PMUs installed in this system and 32 PMUs scheduled to be installed. A brief information of TVA system is listed in Table 4.1. The lists of installed and scheduled to be installed PMUs are shown in Tables 4.2 and 4.3 respectively.

Table 4.1 Brief information of TVA system (version 1)

Number of buses	1375
Number of branches	1735
Number of zero injections	330

Table 4.2 Bus names and numbers with installed PMUs

Bus Name	Number of PMUs	Bus Number
Cordova	3	1194, 1195, 1196
Shelby	3	1381, 1382, 1383
Freeport	2	1217, 1218
Volunteer	2	1400, 1401
Summer Shade	1	1008
Total PMUs Installed	11	
* 8 of them are zero injections buses		

Table 4.3 Bus names and numbers where PMUs are scheduled to be installed

Bus Name	Number of PMUs	Bus Number
Paradise	2	1322, 1323
Shawnee	6	1364, 1374, 1375, 1376, 1377, 1379
Marshall	2	1315, 1316
Widows Creek	3	1408, 1409, 1410
Davidson	2	1200, 1201
Weakley	2	1406, 1407
Cumberland	2	402, 1197
Sullivan	2	1388, 1389
Bull Run	2	1183, 1184
Lowndes	2	1299, 1303, (1300, 1301, 1302)
West Point	2	1402, 1403
Johnsonville	5	1229, 1230, 1231, 1232, 1233
PMU Scheduled:	32	
* 24 of them are zero injection buses		

b. Studied cases – version 1

All simulations are carried out using the Integer Programming method. TOMLAB/MINLP [4] software package is used to solve the Integer Nonlinear Programming problem. Four cases that reflect different possibilities are investigated.

Case 1: This case does not involve any assumptions and it does not try to take advantage of any existing zero injection buses. Hence, this can be considered as the worst case scenario. In this case, out of the total 1375 system buses, 377 of them are chosen for PMU installations. When these suggested locations are checked against those listed in Table 4.2, 17 of them match the chosen bus numbers as shown in Table 4.4. This result ought to be evaluated by keeping in mind the fact that the developed PMU placement algorithm does not yield a unique answer. Depending on the initialization, several equally good (yielding the same number of PMUs) solutions can be obtained. The one presented here happens to be one of them.

Case 2: This case is the same as case 1, except the solution is forced to include those already installed PMUs from Table 4.2. Note that, compared to case 1, the total number of PMUs is increased by one as indicated in Table 4.4. This implies that those already installed PMUs are located at fairly optimal locations, requiring only one extra PMU with respect to the optimal case.

Case 3: This case follows case 2, where in addition to the already installed PMUs, those, which are scheduled to be installed, are also forced into the solution. In this case, as given in Table 4.4, the total number of required PMUs jumped to 390, a significant deviation from the optimal value of 377. Based on this information, the simple conclusion will be that the scheduled PMU installations are not the best from the point of view of observability. Naturally, this conclusion carries a lot of uncertainty due to the fact that our interpretation of Table 4.2 for the scheduled PMU locations may not be correct. Furthermore, observability may not be the primary concern behind the decisions for the scheduled locations. They may be planned to be placed for other important concerns such as angle or voltage stability monitoring.

Case 4: This case is similar to case 1, except for the fact that all buses with zero injections are assumed passive (no generation and no load) and PMUs are placed accordingly. As shown in Table 4.4, including these constraints (330 buses are identified as zero injection buses) reduced the optimal PMU locations significantly down to 295. It should be noted that the solution found in this case matches 8 buses from Table 4.2.

Table 4.4 Results of case studies for the TVA data (version 1)

Ignore zero-inj.	No PMUs	Number of PMUs:	377
		Matched Locations:(17)	1195, 1217, 1322, 1323, 1376, 1377,1316, 1409, 1410, 1407, 402, 1197, 1389,1184, 1303, 1403, 1233
	Installed PMUs	Total PMUs:	378
		New PMUs	366
	Installed & Scheduled PMUs	Total PMUs:	390
		New PMUs	347
Consider zero-inj.	No PMUs	Number of PMUs:	295
		Matched Locations:(8)	1323, 1409, 1410, 1201, 402, 1184,1403, 1233

Detailed list of bus numbers indicating the locations for PMUs for the four cases described above, are given in Tables 4.5 through 4.8 below.

Table 4.5 Detailed results for case 1

Total No. of PMUs	377
Loc. (bus #)	1, 2, 13, 16, 19, 21, 25, 26, 31, 33, 39, 43, 45, 47, 57, 65, 67, 70, 73, 77, 85, 88, 90, 97, 99, 102, 105, 109, 111, 112, 117, 120, 130, 133, 136, 141, 142, 144, 157, 160, 162, 163, 164, 171, 174, 175, 177, 189, 190, 191, 192, 198, 203, 207, 212, 214, 215, 216, 222, 224, 225, 227, 230, 232, 234, 235, 239, 242, 256, 260, 264, 270, 271, 272, 275, 276, 280, 281, 282, 283, 285, 292, 298, 300, 302, 304, 307, 310, 314, 315, 319, 326, 328, 333, 334, 335, 342, 346, 350, 352, 359, 373, 374, 375, 376, 377, 384, 386, 390, 391, 392, 393, 394, 397, 399, 400, 401, 402, 407, 409, 414, 415, 416, 421, 429, 430, 436, 437, 438, 445, 447, 449, 451, 452, 458, 461, 466, 476, 477, 479, 481, 488, 489, 493, 502, 504, 510, 511, 525, 526, 527, 528, 538, 541, 543, 545, 546, 550, 558, 563, 565, 568, 569, 577, 578, 579, 582, 583, 587, 590, 594, 599, 601, 603, 605, 609, 616, 619, 620, 623, 624, 645, 646, 652, 653, 654, 655, 670, 675, 676, 677, 681, 683, 687, 688, 695, 696, 699, 700, 705, 708, 718, 725, 731, 733, 734, 735, 738, 741, 748, 749, 756, 759, 767, 771, 772, 776, 777, 780, 787, 789, 793, 794, 797, 798, 799, 800, 801, 804, 805, 816, 817, 822, 823, 827, 830, 835, 839, 853, 860, 870, 871, 881, 883, 885, 889, 893, 894, 899, 902, 905, 909, 914, 928, 938, 939, 940, 943, 944, 948, 951, 953, 956, 958, 964, 968, 972, 973, 979, 980, 981, 987, 995, 1000, 1006, 1018, 1022, 1024, 1028, 1034, 1035, 1043, 1044, 1045, 1046, 1047, 1055, 1059, 1060, 1065, 1073, 1080, 1086, 1092, 1095, 1119, 1126, 1132, 1133, 1136, 1137, 1138, 1139, 1147, 1149, 1150, 1154, 1160, 1163, 1164, 1165, 1166, 1173, 1176, 1180, 1184, 1186, 1195, 1197, 1202, 1204, 1207, 1212, 1216, 1217, 1219, 1225, 1226, 1228, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1273, 1279, 1280, 1281, 1282, 1283, 1284, 1303, 1316, 1318, 1321, 1322, 1323, 1327, 1329, 1351, 1356, 1362, 1363, 1366, 1367, 1369, 1376, 1377, 1378, 1384, 1385, 1389, 1390, 1403, 1405, 1407, 1409, 1410, 1421, 1426

Table 4.6 Detailed results for case 2

Total No. of PMUs		378
Installed PMUs	Number of PMUs	Locations (Bus No.)
	11	1008, 1194, 1195, 1196, 1217, 1218, 1381, 1382, 1383, 1400, 1401
Newly Selected Buses	367	1, 2, 16, 17, 19, 21, 22, 26, 31, 39, 43, 45, 47, 56, 57, 62, 65, 70, 73, 81, 85, 88, 90, 97, 99, 102, 105, 109, 112, 117, 120, 127, 128, 130, 133, 136, 141, 142, 144, 146, 157, 162, 163, 164, 172, 174, 175, 177, 189, 190, 191, 192, 198, 201, 203, 207, 212, 214, 215, 216, 222, 224, 225, 227, 230, 232, 234, 235, 239, 241, 242, 252, 255, 256, 260, 264, 269, 271, 275, 276, 280, 281, 285, 292, 298, 300, 304, 310, 315, 318, 319, 326, 328, 333, 334, 335, 342, 346, 350, 352, 356, 359, 373, 374, 375, 376, 377, 384, 386, 391, 392, 397, 399, 400, 401, 402, 403, 409, 410, 411, 413, 416, 421, 429, 430, 436, 437, 438, 445, 447, 449, 451, 458, 466, 476, 477, 479, 481, 488, 493, 502, 504, 508, 510, 511, 525, 526, 527, 528, 529, 538, 541, 543, 545, 546, 550, 563, 565, 568, 569, 579, 582, 583, 587, 590, 594, 599, 601, 603, 605, 609, 616, 620, 623, 624, 635, 645, 646, 653, 654, 655, 667, 670, 675, 676, 677, 681, 683, 684, 687, 688, 695, 696, 699, 700, 705, 718, 721, 725, 731, 734, 735, 738, 741, 748, 749, 750, 759, 767, 771, 776, 777, 780, 782, 787, 789, 793, 794, 796, 797, 798, 799, 800, 801, 805, 814, 817, 822, 823, 827, 830, 835, 839, 853, 856, 860, 870, 871, 881, 883, 889, 893, 894, 899, 902, 904, 909, 928, 938, 939, 940, 942, 943, 944, 948, 951, 953, 956, 958, 964, 972, 979, 980, 981, 987, 995, 1000, 1006, 1018, 1022, 1024, 1028, 1034, 1035, 1043, 1045, 1047, 1048, 1055, 1059, 1060, 1065, 1073, 1080, 1086, 1092, 1095, 1115, 1119, 1132, 1133, 1136, 1137, 1138, 1139, 1147, 1149, 1150, 1154, 1160, 1163, 1164, 1165, 1166, 1173, 1176, 1180, 1184, 1186, 1197, 1201, 1204, 1207, 1212, 1216, 1225, 1226, 1228, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1273, 1279, 1280, 1281, 1282, 1283, 1284, 1303, 1314, 1316, 1318, 1321, 1322, 1323, 1327, 1330, 1351, 1356, 1362, 1363, 1366, 1367, 1369, 1376, 1377, 1378, 1384, 1385, 1389, 1390, 1403, 1404, 1407, 1409, 1410, 1421, 1426

Table 4.7 Detailed results for case 3

Total No. of PMUs		390
Installed PMUs	Number of PMUs	Locations (Bus No.)
	11	1008, 1194, 1195, 1196, 1217, 1218, 1381, 1382, 1383, 1400, 1401
Scheduled Pmus	32	402, 1183, 1184, 1197, 1200, 1201, 1229, 1230, 1231, 1232, 1233, 1299, 1303, 1315, 1316, 1322, 1323, 1364, 1374, 1375, 1376, 1377, 1379, 1388, 1389, 1402, 1403, 1406, 1407, 1408, 1409, 1410
Newly Selected Buses	347	2, 11, 13, 16, 19, 21, 22, 26, 39, 43, 45, 47, 56, 57, 65, 66, 67, 69, 70, 73, 77, 85, 88, 90, 97, 99, 102, 105, 111, 112, 117, 120, 128, 130, 133, 136, 138, 141, 142, 144, 146, 157, 162, 163, 164, 174, 175, 177, 189, 190, 191, 192, 198, 201, 203, 207, 212, 214, 215, 216, 222, 225, 227, 230, 232, 234, 235, 239, 241, 242, 252, 255, 256, 260, 264, 269, 271, 272, 275, 276, 280, 281, 282, 283, 285, 292, 298, 300, 304, 305, 310, 315, 318, 319, 326, 328, 333, 334, 335, 342, 346, 350, 352, 356, 359, 373, 374, 375, 376, 377, 384, 386, 390, 391, 392, 397, 399, 400, 401, 403, 409, 411, 413, 416, 421, 429, 430, 436, 437, 438, 445, 447, 449, 451, 452, 458, 466, 477, 479, 481, 488, 493, 502, 504, 508, 510, 511, 525, 526, 527, 528, 529, 538, 541, 543, 545, 546, 550, 558, 563, 565, 568, 569, 579, 582, 583, 587, 590, 594, 599, 601, 603, 605, 609, 613, 616, 620, 623, 624, 635, 637, 645, 646, 653, 654, 655, 667, 670, 675, 676, 677, 681, 683, 687, 688, 695, 696, 699, 700, 705, 708, 711, 718, 725, 731, 734, 735, 737, 740, 741, 748, 749, 759, 767, 771, 776, 777, 780, 787, 789, 793, 794, 796, 797, 798, 799, 800, 801, 805, 817, 822, 823, 827, 835, 839, 853, 858, 860, 870, 871, 881, 883, 889, 893, 894, 899, 902, 905, 909, 938, 939, 940, 942, 943, 944, 948, 951, 953, 958, 964, 972, 973, 979, 980, 981, 987, 995, 1000, 1006, 1013, 1018, 1022, 1024, 1028, 1034, 1035, 1043, 1045, 1047, 1048, 1055, 1059, 1060, 1065, 1073, 1080, 1086, 1092, 1095, 1111, 1119, 1132, 1133, 1136, 1137, 1138, 1139, 1147, 1149, 1150, 1154, 1160, 1163, 1164, 1165, 1166, 1173, 1176, 1180, 1186, 1204, 1207, 1212, 1225, 1226, 1228, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1273, 1279, 1280, 1281, 1282, 1283, 1284, 1318, 1321, 1330, 1351, 1356, 1362, 1378, 1384, 1385, 1390, 1404, 1421, 1426

Table 4.8 Detailed results for case 4

Total No. of PMUs	295
Loc. (bus #)	1, 5, 13, 23, 27, 28, 31, 33, 43, 46, 48, 57, 67, 70, 81, 86, 90, 95, 100, 105, 109, 113, 115, 117, 131, 133, 141, 142, 143, 146, 150, 160, 162, 163, 166, 168, 169, 171, 174, 176, 177, 181, 183, 185, 187, 199, 205, 207, 208, 210, 217, 219, 225, 230, 234, 237, 239, 241, 242, 243, 247, 259, 263, 264, 265, 268, 272, 273, 274, 275, 276, 280, 282, 284, 285, 290, 293, 298, 300, 304, 305, 307, 311, 314, 317, 318, 320, 327, 330, 338, 340, 343, 346, 351, 359, 365, 367, 369, 371, 384, 385, 386, 390, 391, 392, 399, 402, 407, 410, 414, 417, 419, 425, 427, 432, 434, 447, 449, 450, 452, 453, 458, 460, 466, 472, 481, 486, 494, 505, 508, 510, 511, 517, 519, 521, 523, 529, 538, 541, 542, 547, 552, 557, 558, 559, 565, 579, 580, 582, 587, 591, 594, 595, 607, 609, 616, 619, 621, 624, 627, 632, 633, 635, 641, 643, 653, 661, 663, 670, 671, 673, 680, 681, 692, 695, 696, 700, 705, 711, 715, 731, 733, 734, 735, 742, 748, 749, 750, 759, 767, 771, 776, 780, 784, 791, 793, 799, 800, 828, 831, 835, 838, 839, 843, 844, 852, 860, 866, 872, 881, 885, 888, 893, 904, 914, 918, 932, 938, 940, 943, 948, 950, 951, 956, 964, 968, 972, 981, 1006, 1017, 1018, 1020, 1026, 1028, 1036, 1043, 1045, 1056, 1065, 1087, 1095, 1124, 1126, 1130, 1132, 1140, 1141, 1142, 1150, 1154, 1160, 1167, 1168, 1169, 1177, 1184, 1187, 1198, 1201, 1203, 1221, 1223, 1228, 1233, 1253, 1255, 1257, 1259, 1261, 1263, 1265, 1267, 1269, 1273, 1276, 1285, 1286, 1288, 1290, 1292, 1294, 1323, 1352, 1357, 1365, 1371, 1385, 1391, 1396, 1403, 1409, 1410, 1421, 1425, 1426

4.2 TVA System (version 2)

a. System information – version 2

An new set of network and measurement data are received from TVA via Areva in late June 2005. This data will be referred as version 2 in the report. The information of TVA system (version 2) is shown in Table 4.9. There are 4 PMUs installed in the system. The information of PMUs is given in Table 4.10.

Table 4.9 System information of TVA system (version 2)

Number of buses	1367
Number of branches	1666
Number of zero injections	192
Number of paired inj. meas.	346
Number of paired flow meas.	303

Table 4.10 Information of installed PMUs in TVA system (version 2)

Number of PMUs	4
Bus No. with V-Phasor	Branches with I-Phasor
252	252-174, 252-533, 252-271, 252-180
533	533-252, 533-3115, 533-107
180	180-252
508	508-243, 508-211, 508-505

b. Simulation results of TVA system – version 2

Three case studies are conducted using the new version of TVA data. In all these studies, the problem is formulated and solved using Integer Programming method. TOMLAB/MINLP software package is used to solve the Integer Nonlinear Programming problem. Brief description of these cases are given below followed by their respective simulation results.

Case 1

This case assumes no existing PMUs and disregards all conventional measurements including zero injections. Hence, the results reflect the worst case scenario where the entire system is made observable by newly placed PMUs only.

Case 2

In this case, all existing conventional measurements, namely the bus power injections (zero as well as non-zero) and line power flows are incorporated into the optimization formulation.

Case 3

This case is the same as Case 2, except it also includes already installed PMUs (which are shown in Table 4.10). As can be seen from the optimization results, these do not make any difference in the optimal placement solution with respect to Case 2.

The simulation results showing the number of required PMUs and their locations (bus numbers) are given below in Tables 4.11 – 4.13 for cases 1-3 respectively.

Table 4.11 Simulation results of TVA system (version 2) for Case 1

Total No. of PMUs	369
Loc. (bus #)	6, 14, 17, 19, 23, 29, 31, 35, 39, 40, 41, 47, 51, 55, 56, 58, 75, 76, 77, 78, 82, 83, 84, 85, 86, 101, 102, 103, 104, 105, 106, 120, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 154, 156, 163, 164, 165, 166, 167, 171, 172, 173, 175, 178, 187, 192, 198, 205, 206, 207, 210, 211, 214, 217, 221, 223, 225, 227, 232, 233, 234, 238, 241, 244, 246, 251, 252, 255, 256, 261, 262, 263, 264, 277, 279, 280, 281, 284, 285, 287, 288, 289, 291, 300, 301, 302, 303, 309, 311, 312, 315, 320, 325, 329, 330, 331, 334, 338, 343, 345, 348, 351, 362, 363, 370, 371, 375, 381, 397, 401, 402, 405, 406, 407, 409, 410, 413, 415, 418, 422, 424, 425, 427, 429, 433, 438, 445, 448, 450, 453, 456, 457, 459, 460, 461, 464, 467, 471, 472, 477, 482, 488, 489, 495, 499, 501, 505, 507, 512, 514, 526, 528, 531, 532, 540, 541, 542, 545, 548, 549, 550, 556, 558, 561, 565, 569, 572, 573, 575, 577, 578, 580, 585, 588, 593, 594, 597, 600, 602, 609, 612, 614, 615, 617, 618, 625, 626, 628, 638, 639, 649, 656, 660, 664, 666, 677, 679, 680, 685, 689, 693, 699, 700, 701, 702, 714, 717, 718, 719, 730, 733, 741, 744, 745, 752, 754, 756, 757, 758, 763, 765, 774, 775, 777, 788, 791, 796, 800, 801, 802, 803, 823, 828, 829, 837, 840, 841, 845, 846, 855, 856, 863, 874, 877, 882, 884, 885, 888, 890, 894, 898, 904, 911, 920, 922, 926, 929, 930, 935, 939, 940, 943, 944, 945, 946, 949, 951, 954, 956, 963, 967, 968, 976, 979, 980, 981, 983, 990, 997, 999, 1002, 1004, 1006, 1010, 1012, 1017, 1037, 1039, 1047, 1058, 1060, 1061, 1062, 1063, 1076, 1078, 1079, 1080, 1081, 1095, 1100, 1103, 1110, 1114, 1115, 1120, 1123, 1131, 1136, 1137, 1143, 1148, 1152, 1153, 1162, 1164, 1168, 1182, 1183, 1195, 1196, 1197, 1209, 1222, 1225, 1228, 1232, 1234, 1244, 1251, 1254, 1255, 1262, 1271, 1274, 1287, 1291, 1293, 1295, 1312, 1315, 1316, 1324, 1329, 1333, 1338, 1339, 4840, 4841, 4842, 4844, 4847, 4848, 4849, 4852, 4855, 4856, 4861

Table 4.12 Simulation results of TVA system (version 2) for Case 2

Total No. of PMUs	279
Loc. (bus #)	2, 4, 7, 8, 20, 22, 29, 31, 34, 38, 39, 41, 47, 48, 51, 53, 55, 76, 77, 78, 82, 86, 107, 138, 153, 154, 156, 166, 167, 187, 217, 221, 223, 225, 227, 238, 241, 244, 254, 262, 273, 276, 277, 279, 280, 281, 284, 285, 287, 288, 291, 292, 300, 301, 302, 303, 310, 312, 317, 318, 319, 324, 325, 330, 331, 334, 349, 363, 366, 370, 371, 375, 380, 383, 387, 390, 392, 405, 406, 407, 409, 410, 413, 414, 415, 418, 429, 430, 433, 438, 442, 448, 449, 450, 455, 459, 460, 461, 472, 473, 477, 481, 482, 489, 503, 507, 512, 514, 528, 532, 534, 545, 558, 563, 566, 569, 571, 572, 575, 577, 578, 580, 585, 588, 594, 597, 599, 609, 612, 614, 616, 618, 623, 628, 635, 638, 639, 642, 652, 656, 664, 665, 666, 668, 673, 679, 683, 685, 693, 714, 715, 717, 718, 733, 750, 754, 757, 758, 763, 766, 774, 775, 778, 788, 793, 795, 798, 802, 810, 823, 882, 827, 834, 840, 855, 863, 867, 874, 876, 885, 888, 890, 892, 895, 896, 898, 905, 910, 921, 922, 929, 930, 932, 939, 940, 943, 944, 963, 949, 953, 958, 960, 966, 967, 975, 978, 983, 986, 989, 990, 997, 1004, 1005, 1010, 1013, 1019, 1033, 1039, 1040, 1044, 1052, 1058, 1059, 1077, 1095, 1076, 1102, 1103, 1108, 1110, 1111, 1123, 1125, 1131, 1136, 1137, 1143, 1148, 1153, 1161, 1162, 1167, 1172, 1181, 1183, 1195, 1196, 1208, 1209, 1213, 1215, 1218, 1221, 1222, 1233, 1240, 1262, 1269, 1271, 1276, 1280, 1283, 1287, 1288, 1292, 1295, 1301, 1305, 1321, 1329, 1335, 1344, 4840, 456, 4842, 4845, 4848, 4849, 4851

Table 4.13 Simulation results of TVA system (version 2) for Case 3

Total No. of PMUs	279
Loc. (bus #)	2, 4, 7, 8, 20, 22, 29, 31, 34, 38, 39, 41, 47, 48, 51, 53, 55, 76, 77, 78, 82, 86, 107, 138, 153, 154, 156, 166, 167, 187, 217, 221, 223, 225, 227, 238, 241, 244, 254, 262, 273, 276, 277, 279, 280, 281, 284, 285, 287, 288, 291, 292, 300, 301, 302, 303, 310, 312, 317, 318, 319, 324, 325, 330, 331, 334, 349, 363, 366, 370, 371, 375, 380, 383, 387, 390, 392, 405, 406, 407, 409, 410, 413, 414, 415, 418, 429, 430, 433, 438, 442, 448, 449, 450, 455, 459, 460, 461, 472, 473, 477, 481, 482, 489, 503, 507, 512, 514, 528, 532, 534, 545, 558, 563, 566, 569, 571, 572, 575, 577, 578, 580, 585, 588, 594, 597, 599, 609, 612, 614, 616, 618, 623, 628, 635, 638, 639, 642, 652, 656, 664, 665, 666, 668, 673, 679, 683, 685, 693, 714, 715, 717, 718, 733, 750, 754, 757, 758, 763, 766, 774, 775, 778, 788, 793, 795, 798, 802, 810, 823, 882, 827, 834, 840, 855, 863, 867, 874, 876, 885, 888, 890, 892, 895, 896, 898, 905, 910, 921, 922, 929, 930, 932, 939, 940, 943, 944, 963, 949, 953, 958, 960, 966, 967, 975, 978, 983, 986, 989, 990, 997, 1004, 1005, 1010, 1013, 1019, 1033, 1039, 1040, 1044, 1052, 1058, 1059, 1077, 1095, 1076, 1102, 1103, 1108, 1110, 1111, 1123, 1125, 1131, 1136, 1137, 1143, 1148, 1153, 1161, 1162, 1167, 1172, 1181, 1183, 1195, 1196, 1208, 1209, 1213, 1215, 1218, 1221, 1222, 1233, 1240, 1262, 1269, 1271, 1276, 1280, 1283, 1287, 1288, 1292, 1295, 1301, 1305, 1321, 1329, 1335, 1344, 4840, 456, 4842, 4845, 4848, 4849, 4851

5. Conclusions and Future Work

This project accomplished two objectives. First, two practical methods for determining optimal locations for PMUs are developed. Second, the developed methods are implemented and applied to several test systems, including the internal TVA system.

Placement of PMUs can be carried out using different criteria depending on the objective of the investigator. In this project, the main focus is on the state estimation function and therefore the objective is to make the entire system observable by optimal placement of PMUs. The project considered various scenarios where the system is first assumed to be observed by PMUs only. While this appears impractical today, it may very well be the case in a few years when these devices become standard equipment at substations. Next, the placement problem is considered for a system with existing measurements, some of which may be PMUs. Case studies which are carried out on test systems as well as the TVA's internal system indicate that strategically placing PMUs at roughly one third of the system buses, the entire system can be made observable with only PMUs. Furthermore, zero injections, which can be considered free measurements, can significantly reduce the required number of PMUs for a given system.

PMU placement problem does not have a unique solution. Depending upon the starting point, the developed optimization scheme may yield different sets of optimal solutions, each one providing the same minimum number of PMUs but at different locations. On the other hand, it is not unusual to have additional considerations apart from strict observability criterion, when deciding on the location of PMUs. These considerations can be taken into account by appropriately modifying the optimization problem which is formulated in this project. This can be done as an extension to this project in the future. One of the important functions of state estimators is to detect and eliminate bad measurements in the system. Bad data processing is strongly dependent upon the measurement redundancy as well as accuracy of the measurements used. Even for fully observable systems, strategic placement of few PMUs can significantly improve bad data detection and identification capability. This aspect of PMU placement can also be investigated in the future so that the operation of the existing state estimators can be improved via PMU placement.

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