PSERC

The Evaluation of Stochastic Available Transfer Capability for Transmission Expansion

Intermediate Project Report

Power Systems Engineering Research Center

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The Evaluation of Stochastic Available Transfer Capability for Transmission Expansion

Intermediate Report
for the PSERC Project
Uncertain Power Flows and Transmission Expansion Planning

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Power Systems Engineering Research Center

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Preface

This is an intermediate report of progress on a PSERC project entitled *Uncertain Power Flows and Transmission Expansion Planning*. Dr. Gerald Sheblé at Iowa State University is the project leader. The project began in 2004 and is expected to be completed in 2006.

Executive Summary

The major information problem for smooth transmission expansion is the balancing of expansion costs with the cost of congestion and reliability during the operation of the energy system multiple markets over a repeated number of years. The risk of delivery failure due to transmission limitations, the risk of market power due to insufficient market reach to another supplier, and the operational risk due to maintenance schedules are known risks that must be mitigated by transmission expansion over some long-term future time horizon. This project assumes that the transmission owners are regulated entities that are compensated based on total costs each year plus a predetermined rate of return based on quality of service (performance based rates).

This report relates to available transfer capability (ATC) between two points in a power network. The ATC is a limit to the amount of power that can be exchanged between two points. The calculation of ATC has traditionally been a deterministic calculation. However, loads and line status are stochastic phenomena that possess random elements. In this report, the ATC calculation is treated as a stochastic process and the calculation is made using stochastic power flow technology. In doing this, the probability density of the ATC is obtained. It is then possible to calculate the expected level of ATC (and therefore the expected level of revenue that can be obtained from transmission resource). In addition to expected values, the standard deviation and other statistical data can be calculated. Some simple examples are produced to illustrate the concept.

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Nomenclature

ANN	artificial neural networks
ATC	available transfer capability
ATC^{A-B}	available transfer capability between bus A and bus B
c_j	*j*th constant in the Gram Charlier Type A series
CBM $col_A(\Lambda)$	capacity benefit margin the column A of the matrix of the argument
CPF $diag(\Lambda)$	continuation power flow a diagonal matrix of the values of the argument
$DF_{ij,k}$	distribution factor of line from bus *i* to bus *j* with respect to a power injection at bus *k*
$E[\Lambda]$	expectation operator
exp	exponential
$\Im\{\Lambda\}$	Fourier transform
f(x)	probability density function
FERC	Federal Energy Regulatory Commission
G(x)	characteristic Gaussian function
$H_j(x)$	*j*th Hermite polynomial
ISO	independent system operator
k_n	*n*th cumulant
LATC	linear available transfer capability
lse	limiting system element for a particular ATC transfer

Nomenclature (continued)	
$m_x^{(k)}$	*k*th raw moment of *x*
M_x	vector of means of variable *x*
$M_{1,lse}^{Line}$	matrix of the means of line *lse*
$m_{lse}^{(k)}$	*k*th raw moment of line flows of line *lse*
$m_{ATC}^{(k)}$ $\min_{\forall\ rows}(\Lambda)$	*k*th raw moment of the ATC the minimum of all rows of its vector argument
MW	megawatts
NERC	North American Electric Reliability Counsel
OASIS	open access same-time information system
P	active power
P_{ij}^{Rating}	rating of the transmission line connected between buses *i* and *j*
P_{ij}^{Line}	load flower for the transmission line between bus *i* and *j*
PDF	probability density function
Pr_{ATC}	general price function
Pr_{ATC}^{14}	example price function for the 14 bus system
Pr_{ATC-1}^{4}	4 bus price function 1
Pr_{ATC-2}^{4}	4 bus price function 2
Q	reactive power
R	resistance

Nomenclature (continued)	
$row_{lse}(\Lambda)$	a scalar in the row *lse* of its vector argument
$\overline{S}_{ij}$	transmission line complex power
S_k	complex bus power
TTC	total transfer capability
V	voltage
X	reactance
x	variable
z_{ij}	impedance of line from bus *i* to bus *j*
$(Z_{bus})_{ik}$	*i*th row and *k*th column of the Z_{bus} matrix
$\mu_x^{(k)}$	*k*th central moment of *x*
μ_{GC}	mean used in the evaluation of the Gram Charlier series
$\phi(\omega)$	characteristic function
$\lvert\Lambda\rvert$	absolute value
Σ_x	covariance matrix of variable *x*
σ_{GC}^2	standard deviation of the Gram Charlier series
∞	infinity

1. Stochastic Power Flow Study Applications to Power Marketing

1.1 Overview

The major information problem for smooth transmission expansion is the balancing of expansion costs with the cost of congestion and reliability during the operation of the energy system multiple markets over a repeated number of years. The risk of delivery failure due to transmission limitations, the risk of market power due to insufficient market reach to another supplier, and the operational risk due to maintenance schedules are known risks that must be mitigated by transmission expansion over some long-term future time horizon.

The Available Transfer Capacity (ATC) is the amount of power that can be traded between any two buses a power system. Numerical values for the ATC are commonly used to determine the amount of money that is generated for a trade. The money generated from the ATC can be compared to expansion costs of a system. Theses comparisons can further mitigate the risks of transmission expansion and determine the effect line outages have on the money generated from trading.

Since power is commonly traded in time intervals, the stochastic nature of power systems show that load demands can have a probabilistic behavior. The probabilistic behaviors of the loads indicate that the ATC can also behave probabilistically. Solving a stochastic ATC problem can assist the justification of transmission expansion over some long term-future time horizon.

1.2 Statement of the problem

Currently not much information is known about solving the stochastic ATC problem. For most cases, a stochastic ATC problem can be solved using a Monte Carlo analysis. In order to get the best results from a Monte Carlo analysis, a large number of trials have to be solved which can have a large computational time. A stochastic analysis of ATC can have a significant decrease in the computational time utilizing an alternative method incorporating a stochastic load flow algorithm.

1.3 Available transfer capability literature review

The Federal Energy Regulatory Commission (FERC), in response to the devolvement of the Federal Power Act in 1992, issued a series of proposed rules that provided hints to the direction that FERC was headed and sought industry comment. After gathering the wide number of industrial comments, FERC issued the Orders No. 888 [7] and 889 [8] in 1996. These orders established certain guidelines that energy markets have to follow which played key roles in opening the US energy market to competition.

FERC Order 888 further opens up access to existing electric power transmission networks and allows for a better customer choice. Order 888 mandated the separation of electrical services and marketing functions which required utilities to provide open access to their tariffs, and gave existing utilities the right to recover stranded costs from energy customers from investments based on the older regulations.

FERC Order 889 mandated the information of energy market indicators such as Available Transfer Capability (ATC) and Total Transfer Capability (TTC) available to potential competitors, and posting of the energy market indicators on the Open Access

Same-time Information System (OASIS) [8]. ATC is defined by the FERC as the measure of transfer capability remaining in the physical transmission network over committed uses. Also, the TTC is defined as the total amount of power that can be sent in a reliable manner. The purpose of calculation and posting of market indicators such as ATC and TTC to OASIS is to further the open access of the bulk transmission system by providing a market signal of the capability of a transmission system to deliver energy, which would spur competitive bidding in the energy market.

The NERC formed the official definition and proposed a numerical approximation of the ATC in 1995 and 1996 [9, 10]. The documents by NERC defined ATC as a measure of transfer capability which gives how much available transfer room that is left in the physical transmission network above already committed uses. The NERC further defines ATC as a function of increases in power transfers between different systems where as the transfers increase, the flows in transmission lines increase. The TTC is the largest flow in the selected transmission system for which there are no reliability concerns such as thermal overloads, voltage limit violations, voltage collapse and any other system reliability problems. The TTC minus the base case flow and appropriate transmission margin is the ATC for the selected transmission system. Figure 1.1 shows an NERC depiction of ATC. The top line of the figure illustrates the TTC. The changes in TTC depict changing reliability concerns, temperature and other factors that would cause change in the maximum amount of power that can be transferred. The recallable and non-recallable ATC scheduled are the base case load flows which already exist in the system, with the recallable scheduled loads the ISO an option to disconnect loads that affect the ATC transfer. The Transmission Reliability Margin (TRM) show a how much available “room” is left in the system. The TRM is a small amount of reserved available power that insures a margin of stability in the system if unscheduled faults or interruptions occur.

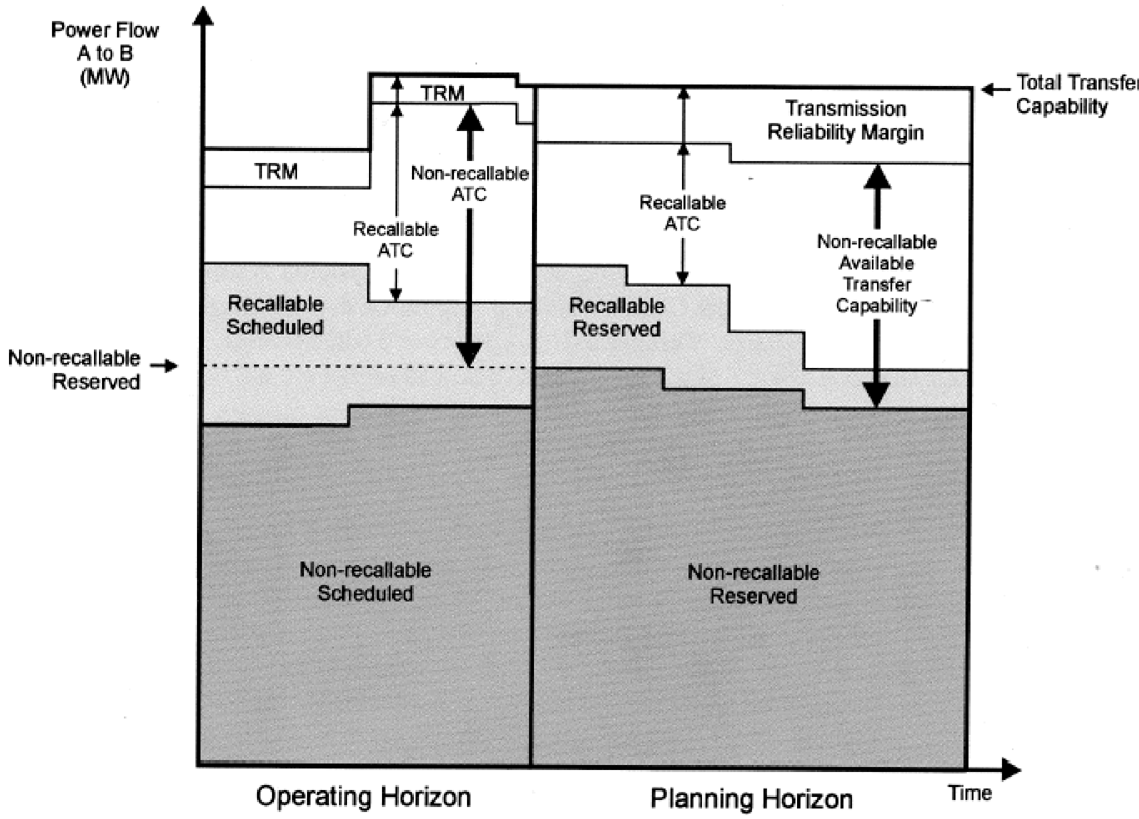

Figure 1.1 TTC, ATC, and related terms in a transmission service reservation system (taken directly from [9]).

Ejebe [14] implemented a program for ATC calculations based on a full AC power flow solution. The features of the program determine the reactive power flows, voltage limits and voltage collapse, as well as thermal loading effects. A continuation power flow (CPF) is used that is based on the Newton power flow algorithm with adaptive localization. Due to the large number of contingencies that are needed to determine ATC for each transfer, a large computation time results.

Gravener [11] saw from a test case of the Pennsylvania-Jersey-Maryland Interconnection that ATC needed to account for the uncertainties that exist in the physical structure of the power system. An ATC calculator is developed which included both linear and nonlinear analysis. Results showed that when the transfer step sizes in the ATC search pattern were varied, ATC values varied as much as 8%. The variation of ATC values in the calculator has an enormous impact on the energy market and improvement on the search algorithms to decrease the variation needs to be implemented.

Ejebe [12] implemented a linear ATC (LATC) calculator based on the linear incremental power flow approximation. The LATC calculator provides a reasonable accurate approximation of the ATC much faster than exact methods. The thermal limits of the line flows and calculation of time independent generating and line outage contingencies before LATC make the calculation of the ATC faster. Tests were done on two large practical systems that provided a close approximation to the exact value of ATC with a faster speed then the continuation power flow method.

Sauer [13] presented some initial concepts on including reactive power in linear methods for computing ATC. The reactive power flows are first determined. The ATC is then attained using active power distribution factors. Results show that inclusion of reactive power in linear ATC can reduce errors in the estimation of the maximum transaction over a transmission system. The computation method can be efficiently implemented in linear ATC programs, without considerable increase in computer requirements. Sauer’s consideration of reactive power in the ATC calculation is only applied to small systems and needs to be tested in large systems.

1.4 Stochastic load flow literature review

Power flow studies have been a major part of the analysis and design of power systems. Throughout the history of load flow techniques, the data that are provided in order to calculate the load flow variables are generally considered to be constant and deterministic. The inputs to a load flow analysis are, in most cases, a snapshot value of a wide range of data. This input technique gives a picture on what happens in the system, but engineers often need to know the effect of a range of operational load values. Since the 1970s, researchers have been studying data ranging. Probabilistic or stochastic load flow methods are an approach to accommodate and model the *random* nature of the operational load and generation data.

Throughout the 1970s and 1980s a number of papers addressed different techniques involved to analyze a stochastic load flow problem. Vorsic, Muzek, and Skerbinek [1] summarized the different stochastic load flow techniques in three basic methods: Monte Carlo, convolution, and the method of statistical moments. The Monte Carlo method is the first known method to obtain the solution of the stochastic load flow problem. This method utilizes repeated trials the deterministic load flow technique to determine the probability distributions of the nodal powers, line flows and losses. Since the accuracy of the probability distribution of line flows, voltages and losses is presumed to be better when modeling all stochastic inputs over a large number of trials, the Monte Carlo method is often characterized by a large computation time. Nonetheless, the Monte Carlo method has been used in many general engineering applications [17-25]. This method has an appeal that a wide range of stochastic phenomena can be modeled, thus suggesting "accuracy" in the results. But computational burden is a clear disadvantage and researchers have sought faster methods to calculate the probability distributions. The Monte Carlo method can be used to verify these faster methods.

Returning to the stochastic power flow studies, the convolution method was perhaps first researched by Borkowska [2]. Prior to her paper, there was little research done in determining the effect of the uncertainty of load data to the uncertainty of bus voltage and line power flow. Borkowska's work concentrated on how probability density functions of the input bus variables calculated through load flow equations yield density functions of branch power flows. She assumed that active and reactive power flows are independent of each other. The random processes associated with the input bus variables are assumed independent and have a general statistical distribution which is calculated though linear load flow analysis using a convolution technique. The resulting density function gives a practical view of the probability of exceeding the capacity limit in branch flows and the practical and probable range of branch load values. Some problems in her assumptions are the nonlinear relation between node loads and branch flows, and proper balance of generation and loads.

Dopazo, Klitin, and Sasson [3] addressed the stochastic load flow problem approximately at the same time as Borkowska. Their objective was similar to Borkowska's convolution method but with a different approach. Employing a method that assumed normally distributed variables of load bus P and Q and normally distributed generator bus variables P and V, Dopazo, Klitin, and Sasson calculated load flows using classical methods. A covariance matrix of the load flow data is formed. Using the variances assumed for the input data, another covariance matrix is formed. The two covariance matrices are then used to obtain confidence limits, or ranges of which the true value has a given probability of existence (e.g., a 99% confidence interval). Note that normally distributed complex random variables have their difficulties as described in [26-32].

The method of statistical moments, used by Sauer and Heydt [4], denotes a density function of random variables as an infinite series each of whose terms depend on statistical moments. The *kth* raw and central statistical moments of random variable *x* are

$$m_x^{(k)} = E(x^k)$$

$$\mu_x^{(k)} = E((x - m_x^{(1)})^k)$$

where $E(\Lambda)$ denotes expectation. Following linearization of the system equations, the statistical moments of voltages and load flows are used to determine the distribution of voltages and load flows. The method works well solving the stochastic load flow equations with errors in the output data being smaller than other methods [2, 3]. Note that the probability density function of a random variable is not determined by its statistical moments in general, but in practical engineering applications one may assume that the density and distribution are uniquely determined by the statistical moments. [26-32].

A simpler solution of the Sauer and Heydt's approach was done by Patra and Misera [5]. A proposed method of cumulants by Sanabria and Dhillon [6] is the basis of the work done by Patra and Misra. The method, like [4], uses the method of statistical moments to analyze the complex random variables while using the Gram-Charlier expansion [27] to approximate the probability density functions of the output bus voltages and line flows. The cumulants k_n of a probability distribution are found from the statistical moments of the distribution. In terms of the first four raw moments of random variable *x*, the first four cumulants of *x* are

$$
\begin{aligned}
k_1 &= m_x^{(1)} \\
k_2 &= m_x^{(2)} - (m_x^{(1)})^2 \\
k_3 &= m_x^{(3)} - 3m_x^{(2)} m_x^{(1)} + 2(m_x^{(1)})^3 \\
k_4 &= m_x^{(4)} - 4m_x^{(3)} m_x^{(1)} - 3(m_x^{(2)})^2 + 12m_x^{(2)} (m_x^{(1)})^2 - 6(m_x^{(1)})^4
\end{aligned}
$$

where $m_x^{(n)}$ with is the nth raw moment of the the random variable *x*. The cumulants are used in the Gram-Charlier expansion where the bus voltages and line flows are obtained from both the statistical moments and cumulants. Conclusions of Patra and Misera's work show that their technique has better computational advantages compared to existing methods using complex random variables for the load flow. However, Patra and Misera's method seems to give higher errors than in [4].

Zhang and Lee [15] proposed a method of computing a probabilistic load flow study in large power systems. The method can is proposed to be a quick screening tool to analyze major investments on improving transmission system inadequacy. Zhang and Lee's scheme combines the determination of line flow cumulants and the Gram Charlier expansion to obtain probabilistic distribution functions of the transmission line flows. Their conclusions show that their method is able to accurately approximate the cumulative distribution function of transmission line flows for small systems. They state that future research will consider applying their method to larger size systems.

1.5 Probability density fitting

An important step in a stochastic load flow problem is the representation of the line data, or line statistical moments into a probability density function. Sauer [4] implemented the Gram Charlier Type A [27,33] series in the stochastic load flow problem. The general concept of construction of a probability density function has occupied considerable attention in the classical probability literature. Alternatives are illustrated here with the implication that the probability density function of ATC might be better constructed using one of these alternatives. Three different fitting theories based on the works of Karl Pearson, H. L. Gram, C. V. Charlier, and F. Y. Edgeworth are well documented and have many distinguishing qualities.

Pearson's distributions [27] provide a systematic approach to fit statistical moments to one of twelve specific closed form density functions. Three are well known normal, beta, and gamma density functions. The criteria for determining which of the twelve densities to use are given in [27], and the process is quite lengthy. The process for determining the distribution as well as the criteria has a significant disadvantage of the required data due to the large size as well as the lengthy computation time needed to run in a computer program.

The Gram-Charlier and Edgeworth series [27,33-36] are independently derived representations a density function as an infinite series involving statistical moments and cumulants. The methods generally stem from the uniqueness of the characteristic function

$$\phi_x(\omega) = E[e^{j\omega x}] = \int_{-\infty}^{\infty} e^{j\omega x} f(x)dx\,.$$

Three of the Gram-Charlier series have been proposed based on an infinite series of functions of Hermite polynomials and statistical moments. The Type A series is the most commonly used type and is based on derivatives of the characteristic function of the normal distribution. The characteristic function of a probability density function is its Fourier transform,

$$\Im\left\{f_x(x)\right\} = \phi_x(\omega).$$

The Gram Charlier series Type A has been widely applied and can accurately represent many density functions, but in some cases, can produce negative frequencies. The Type B and C series [27,33] are based on derivatives of the Poisson density and the gamma and beta densities. The Type B series has certain mathematical advantages and this series has been used to accommodate discontinuous variation resembling the Poisson distribution. The Type B series is rarely used. The Type C series, based on derivatives of the gamma and beta densities, was proposed by Charlier to avoid negative frequencies but never has had the appeal of the Type A series.

The Edgeworth series [34-36] , which resembles the Gram-Charlier series, utilizes cumulants, instead of statistical moments in an infinite series based on the derivates of the same normal, Poisson, gamma and beta densities.

2. Probabilistic Available Transfer Capability

2.1 Introduction

In order to open access to electric power transmission networks and promote generation competition and customer choice, the Federal Energy Regulatory Commission requires that available transfer capability be made available on a publicly accessible Open Access Same-time Information System. ATC is defined as a measure of the transfer capability, or available "room" in the physical transmission network for transfers of power for further commercial activity, over and above already committed uses.

As an illustration, consider Figure 2.1 in which 1000 MW is transmitted from bus A1 to bus B1. If the line rating is 1500 MW and if no parallel paths exist from A to B, the ATC from A1 to B1 is 500 MW.

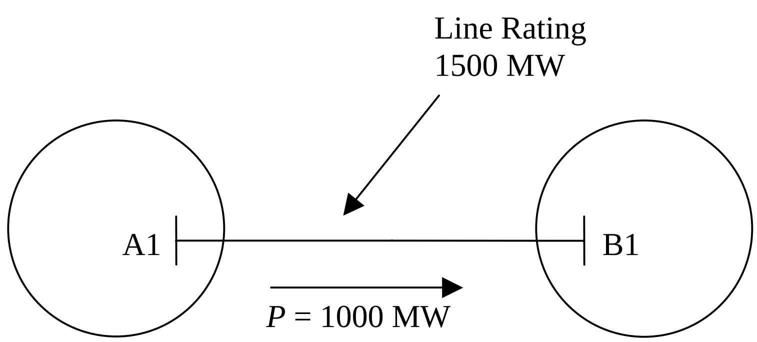

Figure 2.1 Illustration of ATC

Due to uncertainty in bus loading, the power flows within a power network also become uncertain. By applying a statistical method of moments, the statistical moments of line power flows within a system can be found. Although difficult to find analytically, the statistical moments of the line power flows can be found by applying a Monte Carlo simulation to an existing power flow algorithm with bus loads being pseudorandom.

The probabilistic approach to the determination of ATC analyzes market conditions in which future scenarios are tested in an auction process where load forecasting techniques can be applied. The uncertainty of the future loads implies that the power flow is probabilistic. Thus, a stochastic load flow study. The stochastic load flow study yields a series of statistical moments describing the behavior of the line power flows. This uncertain behavior of line power flows affects ATC and causes ATC to be probabilistic. The proposed algorithm described in this chapter is for the utilization of stochastic methods for ATC calculation and power marketing. Figure 2.2 shows the general concept. The Gram Charlier series is proposed as a statistical model of the ATC.

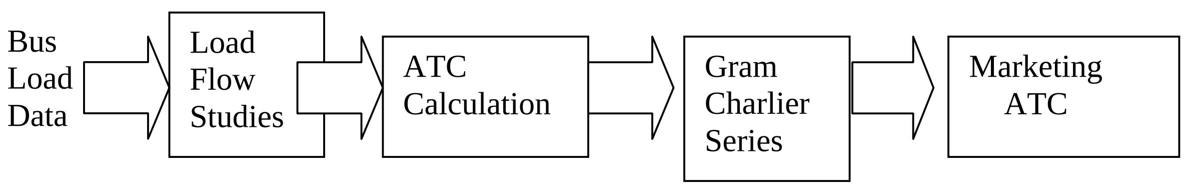

Figure 2.2 The general concept of the proposed algorithm

2.2 Statistical moments of line flows

The Monte Carlo power flow algorithm utilizes a method of multiple trials to obtain selected statistical results. In stochastic power flow evaluations, the Monte Carlo method is useful for checking theoretical formula based results. In this section, the statistical moments of line active power flows in a power system are considered. Consider Figure 2.2 as an example of the Monte Carlo method of finding the line flows and the statistical moments of those flows.

The idea behind the Monte Carlo power flow study is a large number of repeated trials of the Newton-Raphson power flow study. For each iteration of the Monte Carlo power flow study, a set of bus loads are pseudorandomly generated and analyzed in a Newton-Raphson power flow study. The result is a large number of line power flows, in which statistical data for the line power flows can now be found. The line statistical raw moments are then calculated using the definition of finding sample raw moments,

$$m_x^{(k)} = \frac{\sum_{i=1}^{n} x_i^k}{n}, \tag{2.1}$$

where k is the moment order, and n is the number of samples and x_i is the datum in sample i.

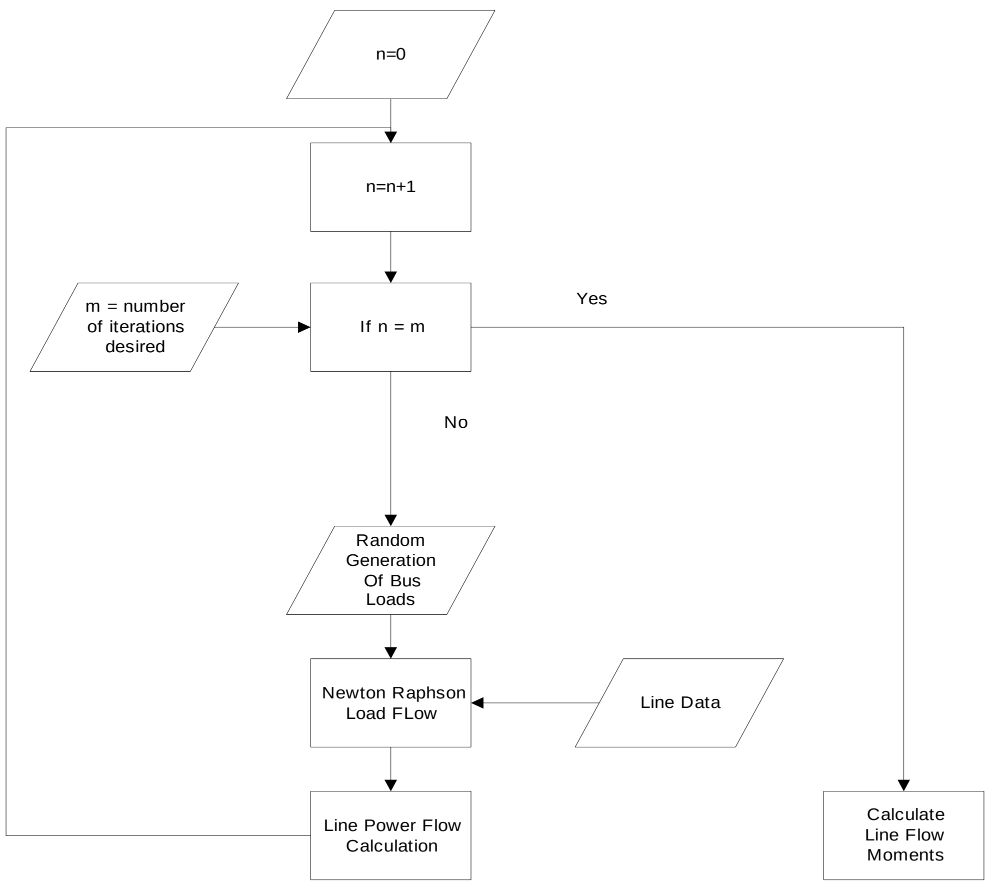

Figure 2.3 Probabilistic power flow using the Monte Carlo method

Pseudorandomly generating bus loads for the Monte Carlo trials depends on the specific probability distribution that best characterizes the bus loads. In most cases the probability density function of the bus loads are:

- Non stationary (statistical moments vary with time)
- Correlated among themselves
- Complex
- Not any recognized distribution.

However, the normal uncorrelated density is commonly used as a “first cut” approximation [1-6]. Arbitrary probability densities can be accommodated by working with the statistical moments of the load active power demands.

In order to find the statistical moments of line power flows in a stochastic load flow problem, the bus loads have to be represented by probabilistic models. If data for the bus loads are known, raw statistical moments are generated from the data using Equation 2.1. If bus data are not known, statistical moment generating functions available in [27] can generate statistical moments for a given probability density.

Many different stochastic power flow algorithms exist as documented by [1-6,16]. Their computational time compared to the Monte Carlo approach may be significantly less, but at the cost of complex mathematical structure. Figure 2.3 illustrates the stochastic load flow study process. Inputs into the stochastic load flow study calculation are the bus load statistical moments and power system information (e.g., line impedances, PV bus information, transformer impedances, and shunt capacitor impedances).

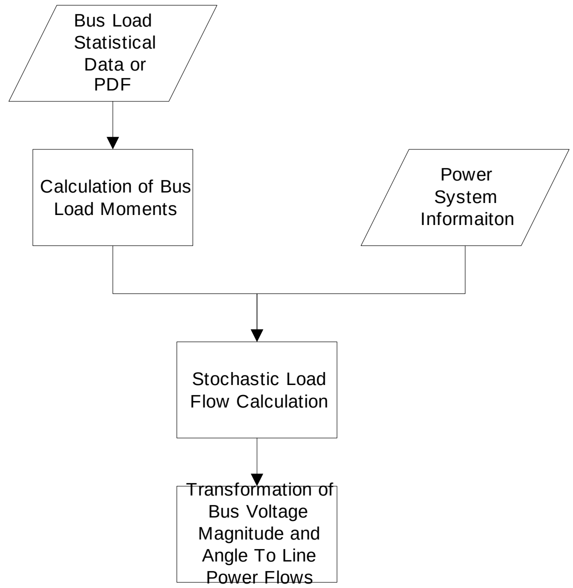

Figure 2.4 Stochastic load flow study process

2.3 ATC calculation

Increasing the transfer power increases the loading in the network. At some point operational or physical limits to various elements are reached which prevent further increase. The largest value of transfer that causes no limit violations is used to compute the Total Transfer Capability (TTC) and ATC.

Within a particular system, for power transfers from bus A to bus B, a single transmission line will often become the limiting factor whenever determining the ATC. But double circuits, transformer limits, and similar transmission path limits may occur. These limits are affected by the power injections at both buses A and B. This effect can be found analytically by finding the distribution factors [16] of the lines and other components. In this chapter, “distribution factor” refers to the power transfer distribution factor for line *i*-*j* and bus *k*. The distribution factor $DF_{ij,k}$ is defined as

$$DF_{ij,k} = \frac{\partial \overline{S}_{ij}}{\partial S_k},$$

where $\overline{S}_{ij}$ is the complex power flow in the line (or component) *ij* and S_k is the complex power injected into bus B. To find $DF_{ij,k}$:

1. Find bus impedance matrix of the system referenced to a solidly grounded swing bus.

2. $DF_{ij,k} = \left(\dfrac{(Z_{bus})_{ik} - (Z_{bus})_{jk}}{z_{ij}} \right)^*$

   where:

   ij is the transmission line from bus *i* to bus *j*

   k is the injected bus

   z_{ij} is the primitive impedance of line *ij*

   Z_{bus} is the bus impedance matrix referenced to the swing bus [16].

3. If *k* is the swing bus, the value for Z_{bus} is zero.

The distribution factor $DF_{ij,k}$ is used to approximate the change in line flow $\Delta\overline{S}_{ij}$ when there is a change in injected power ΔS_k at bus *k*,

$$\Delta\overline{S}_{ij} = DF_{ij,k}\Delta S_k \ .$$

Because this approximation is linear and $DF_{ij,k}$ is approximately constant, the results of many bus injection changes $\Delta\overline{S}_{ij}$ can be found using superposition.

ATC is defined by the NERC [9] as the Total Transfer Capability minus the Transmission Reliability Margin, Capacity Benefit Margin and existing power flows.

$$ATC = TTC - TRM - CBM - \text{Existing power flows} \qquad (2.2)$$

where the Total Transfer Capability *(TTC*) is the total amount of power that can be sent from bus A to bus B within a power network in a reliable manner, the Transmission

Reliability Margin (*TRM*) is the amount of transmission transfer capability necessary to ensure network security under uncertainties, and the Capacity Benefit Margin (*CBM*) is the amount of *TTC* reserved by load serving entities to ensure access to generation from interconnected systems to meet generation reliability requirements.

For purposes of this study, the *TRM* and *CBM* are ignored in the calculations, but can be easily implemented. Neglecting TRM and CBM reduces the computation of *ATC* to

$$ATC = TTC - \text{Existing power flows.} \tag{2.3}$$

A basic determination of *TTC* [13] is found using line ratings and distribution factors for injection at both buses

$$TTC_{ij} = \frac{P_{ij}^{Rating}}{DF_{ij,k}} \tag{2.4}$$

where *ij* is the transmission line from bus *i* to bus *j* and *k* is the injected buses A and B.

The *TTC* has to be found for injections for each bus for which the transfer is made. For electrically distant injection bus/transmission line pairs, the corresponding $\left|DF_{ij,k}\right|$ maybe small. At this point, note that $DF_{ij,k}$ is a complex quantity. For a lossless system, $DF_{ij,k}$ is purely real. Considerable attention is given to active power flows, and in the subsequent discussion, only real $DF_{ij,k}$ are considered. This is not to imply that reactive power flow is unimportant, but rather, that an analysis of the role of reactive power flow in marketing active power is relegated to "future work".

The *ATC* is the maximum amount of power that can be transferred between two buses considering the base case power flows. Equation 2.3 gives a deterministic value for a given transfer, but modifications to the equation can result in a probabilistic determination of the ATC. If the line ratings and line flows are known, the difference between the two is the additional power that can be sent. Using distribution factors, the ATC between A and B, ATC^{A-B}, is approximately

$$ATC^{A-B} = \min\left\{\left|\frac{P_{ij}^{Rating} - P_{ij}^{Line}}{DF_{ij,A}}\right|, \left|\frac{P_{ij}^{Rating} - P_{ij}^{Line}}{DF_{ij,B}}\right|\right\} \tag{2.5}$$

where *ij* is the transmission line from bus *i* to bus *j*, and *A*,*B* are the sending and receiving buses. Note that $DF_{ij,A}$ and $DF_{ij,B}$ are considered to be purely real for this simplified analysis. In actual applications, these distribution factors are complex.

The distribution factors for each line show the direction and percentage of line flows that result from injections at each bus. It is possible to find the distribution factors for the whole transfer by taking into account both the distribution factors of the sending and receiving bus. Since the receiving bus B of the transfer is actually a negative injection of active power, the distribution factors for the transfer are

$$DF_{ij}^{A-B} = DF_{ij,A} - DF_{ij,B}\,. \tag{2.6}$$

This determination of the distribution factors yields a direct solution to the Monte Carlo simulated ATC of each transmission line, where the minimum value of ATC of the set of lines is the ATC for the whole transfer,

$$P_{ij}^{Rating} = P_{ij}^{Line} + (DF_{ij}^{A-B})ATC^{A-B} \tag{2.7}$$

$$P_{ij}^{Rating} = P_{ij}^{Line} + (DF_{ij,A} - DF_{ij,B})ATC^{A-B} \tag{2.8}$$

$$ATC^{A-B} = \min_{\forall\ rows}\left\{(P^{Rating} - P^{Line})./(col_A(DF) - col_B(DF)\right\} \tag{2.9}$$

where ATC^{A-B} is the scalar ATC from bus A to bus B; $\min\limits_{\forall \ rows}(\Lambda)$ refers to the minimum entry of a vector argument over all rows (e.g., $\min\limits_{\forall \ rows}\begin{pmatrix}-1\\2\\3\end{pmatrix} = -1$); P^{Rating} and P^{Line} are vectors of line ratings and line flows; "./" is the term by term division of two vectors (e.g., $\begin{pmatrix}1\\2\\3\end{pmatrix}./\begin{pmatrix}4\\5\\6\end{pmatrix}$ is $\begin{pmatrix}1/4\\2/5\\3/6\end{pmatrix}$) ; $col_A(\Lambda)$ and $col_B(\mathrm{K})$ are columns *A* and *B* if the matrix argument (e.g., $col_3\begin{pmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{pmatrix} = \begin{pmatrix}3\\6\\9\end{pmatrix}$); and *DF* is the rectangular matrix of distribution factors,

$$DF = \begin{matrix} \uparrow \\ lines \\ \downarrow \end{matrix} \ \overset{\leftarrow \ buses \ \rightarrow}{\begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}} .$$

The limiting transmission line can be found directly by using the first raw moments of the all the line power flows from Section 2.2. The $ATC^{A \to B}$ is evaluated using the statistical moments for each line. The row number for which the mean ATC is minimum is the limiting transmission element. This row number is denoted *lse*,

$$\begin{aligned} & row_{lse}\left\{(P^{Rating} - m^{(1)})./(col_A(DF) - col_B(DF))\right\} = \\ & \qquad \min_{\forall \ rows}\left\{(P^{Rating} - m^{(1)})./(col_A(DF) - col_B(DF)\right\} \end{aligned} \tag{2.10}$$

In (2.10), $m^{(1)}$ is the mean of vector P^{Line}, and $row_{lse}(\Lambda)$ is a scalar in the row *lse* of its vector argument (e.g., $row_3\begin{pmatrix}10\\20\\30\\40\end{pmatrix} = 30$). Finding *lse* now yields a direct solution to the ATC,

$$ATC^{A-B} = \frac{P_{lse}^{Rating} - m_{lse}^{(1)}}{DF_{lse,A} - DF_{lse,B}}. \qquad (2.11)$$

2.4 Statistical moments of ATC

From Equation (2.11), the ATC is a function of the line rating, distribution factors, and line power flows of a particular limiting line *lse* within a power system for power transfers between bus A and bus B. In order to form a probabilistic evaluation of ATC, the deterministic expression in Equation (2.11) has to be recast in terms of statistical moments. The only statistical data that is known for terms in Equation (2.11) are the statistical moments of the Gaussian distributed vector P_{lse}^{Line}. Therefore ATC is only a linear transformation of P_{lse}^{Line}.

The computation of ATC is only a function of one random variable, therefore the statistical moments of ATC can easily be found from the statistical moments of P_{lse}^{Line}. From Section 2.2, it is stated that the statistical data for line flows are often assumed to be Gaussian. For a given linear transformation,

$$Y = AX \qquad (2.12)$$

let X be a Gaussian distributed vector with mean vector M_x and covariance matrix Σ_x. The resulting mean and variance of Y are

$$M_Y = AM_X \tag{2.13}$$

$$\Sigma_X = A\Sigma_X A^t . \tag{2.14}$$

To obtain ATC as a probabilistic quantity, Equation (2.11) is compared to the transformation in Equations (2.12-14). Comparing Equations (2.11) and (2.12), the resulting variables for the Gaussian transformation are,

$$X = P_{lse}^{Rating} - P_{lse}^{Line} \tag{2.15}$$

$$m_{lse}^{(1)} = E(X)$$

$$\Sigma_X = m_{lse}^{(2)}$$

$$A = diag\left(\frac{1}{DF_{lse,A} - DF_{lse,B}}\right) \tag{2.16}$$

$$Y = ATC^{A-B} \tag{2.17}$$

where X is a vector of the difference of the line rating of line *lse* and pseudorandom variables of the real power flows for line *lse*, A is a diagonal matrix, $diag(\Lambda)$ is the construction of a matrix of diagonal elements consistent with the dimension of X, (e.g. dimension of X =3, $diag(1.1) = \begin{pmatrix} 1.1 & 0 & 0 \\ 0 & 1.1 & 0 \\ 0 & 0 & 1.1 \end{pmatrix}$), and Y is the vector of the transformed pseudorandom vector X.

The transformation of the Gaussian distributed variable P_{lse}^{Line} yields the mean and variance of the ATC,

$$m_{ATC}^{(1)} = \left(\frac{1}{DF_{lse,A} - DF_{lse,B}}\right)\left(P_{lse}^{Rating} - m_{lse}^{(1)}\right) \tag{2.18}$$

$$m_{ATC}^{(2)} = \left(\frac{1}{DF_{lse,A} - DF_{lse,B}} \right)^2 \left(m_{lse}^{(2)} \right) \tag{2.19}$$

where $DF_{lse,A}$ is the distribution factor of line *lse* with respect to injections at bus *A*, $DF_{lse,B}$ is the distribution factor of line *lse* with respect to injections at bus *B*, and $m_{lse}^{(L)}$ is the L^{th} order statistical moment of the power in line *lse*.

The mean and covariance of a vector fully determine the probability density of that vector if the vector is normally distributed. If the vector is not normally distributed, higher order statistical moments may be needed. Probability theory provides the well known fact that statistical moments do not, in general, describe a probability density function. For practical, well-behaved densities, however, the statistical moments do describe the probability density function. Since ATC is only a probabilistic function of the line power flow raw moments of line *lse*, the higher order raw become

$$m_{ATC}^{(k)} = \left(\frac{1}{DF_{lse,A} - DF_{lse,B}} \right)^k \left(m_{lse}^{(k)} \right), \quad k>1. \tag{2.20}$$

2.5 Fitting of the Gram Charlier series

The mean, variance, and other raw moments of an ATC transfer can be used to obtain an approximation of the probability density function. If higher order statistical moments of the ATC are known, the Gram Charlier Type A series can yield an approximate probability density function of the ATC. The subroutine written in Matlab is shown in Appendix A.

The Gram Charlier Type A series is a standard frequency function, denoted as *f(x),* with the mean $\mu_{GC} = 0$ and standard deviation $\sigma_{GC}^2 = 1$ expanded in a series of derivatives of the normal (Gaussian) function

$$G(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}. \tag{2.21}$$

The derivatives of Equation (2.23) result in a Taylor series representation, as shown in [27]. This Taylor series is the Gram Charlier Type A series

$$f(x) = \sum_{j=0}^{\infty} c_j H_j(x) G(x), \tag{2.22}$$

where c_j are constants based on a function of standardized moments, $H_j(x)$ are Hermite polynomials, and *G(x)* is the characteristic Gaussian function. The first five Hermite polynomials are

$$\begin{aligned} H_0(x) &= 1 \\ H_1(x) &= x \\ H_2(x) &= x^2 - 1 \\ H_3(x) &= x^3 - 3x \\ H_4(x) &= x^4 - 6x^2 + 3 \\ H_5(x) &= x^5 - 10x^3 + 15, \end{aligned}$$

where *x* is a scalar. For *l* greater than 1 the Hermite polynomials are,

$$H_l(x) = x^l + \sum_{k=2}^{t} (-1)^{k-1} x^{l-2(k-1)} \binom{l}{2(k-1)} \prod_{r=1,3,5}^{2k-3} r, \tag{2.23}$$

where

$$\binom{a}{b} = \frac{a!}{b!(a-b)!}, \quad t = \frac{l+1}{2} \quad \text{for odd } l \text{ and } t = \frac{l+2}{2} \quad \text{for even } l.$$

The constant c_j in the Gram Charlier Type A series is found using standardized moments,

$$c_n = \frac{1}{n!}\left[\frac{m_{ATC}^{(n)}}{(m_{ATC}^{(2)})^{n/2}} + \sum_{k=2}^{t}(-1)^{k-1}\frac{m_{ATC}^{(n-2(k-1))}}{(m_{ATC}^{(2)})^{\frac{n-2(k-1)}{2}}}\binom{l}{2(k-1)}\prod_{v=1,3,5}^{2k-3} v\right]. \qquad (2.24)$$

For central moments, the first five values for c_n become

$$\begin{aligned}
c_0 &= 1 \\
c_1 &= 0 \\
c_2 &= \frac{1}{2}(\frac{m_{ATC}^{(2)}}{m_{ATC}^{(2)}} - 1) \\
c_3 &= \frac{1}{6}\frac{m_{ATC}^{(3)}}{m_{ATC}^{(2)}} \\
c_4 &= \frac{1}{24}(\frac{m_{ATC}^{(4)}}{(m_{ATC}^{(2)})^{3/2}} - 6\frac{m_{ATC}^{(2)}}{m_{ATC}^{(2)}} + 3) \\
c_5 &= \frac{1}{120}(\frac{m_{ATC}^{(5)}}{(m_{ATC}^{(2)})^{5/2}} - 10\frac{m_{ATC}^{(3)}}{(m_{ATC}^{(2)})^{3/2}})
\end{aligned}$$

Using the Hermite polynomials, $H_j(x)$, and Gram Charlier constants, c_j, a formal expression of *f(x)* is

$$f(x) = G(x)\left[1 + \frac{1}{2}(\frac{m_{ATC}^{(2)}}{\sigma^2} - 1)H_2 + \frac{1}{6}\frac{m_{ATC}^{(3)}}{\sigma^3}H_3 + \frac{1}{24}(\frac{m_{ATC}^{(4)}}{\sigma^4} - 6\frac{m_{ATC}^{(2)}}{\sigma^2} + 3)H_4 + \ldots\right]. \qquad (2.25)$$

Since *f(x)* is found using a standard form of $\mu_{GC} = 0$ and $\sigma_{GC}^2 = 1$, *f(x)* needs to be transformed into a function *f(y)* so that the representing density function for *ATC* has a $\mu_{GC} = \mu_{ATC}$ and $\sigma_{GC}^2 = \sigma_{ATC}^2$. The standardizing equation used for the Gram Charlier Type A series is

$$x = \frac{y - \mu_{ATC}}{\sigma_{ATC}},$$

where *x* are the standardized variables of the ATC, *y* are the real variables of the ATC, μ_{ATC} is the mean of the ATC, and σ_{ATC} is the standard deviation of the ATC. The new Gaussian function now becomes,

$$A(y) = \frac{1}{\sigma_{ATC}\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{y-\mu_{ATC}}{\sigma_{ATC}})^2} = \frac{G(y)}{\sigma_{ATC}}. \tag{2.26}$$

The resulting probability density function *f(y)* is the actual distribution of *ATC*

$$f(y) = \frac{1}{\sigma_{ATC}} \sum_{j=0}^{\infty} c_j H_j(y) G(y). \tag{2.27}$$

In power marketing, the ATC has a commercial value and is termed "the price of ATC" for this report. Once the probability density function *f(y)* is known for the *ATC*, the price of the transfer can be evaluated using *f(y)* and a price function Pr_{ATC}. The quantity Pr_{ATC} represents the price of the power in *$/MW* for current market conditions. The reader is cautioned to note that Pr_{ATC} refers to *price*, not probability. But Pr_{ATC} is a random variable in this probabilistic formulation. An engineer may want to know the expected price for a future transfer and this price is determined by

$$E[\mathrm{Pr}_{ATC}] = \int_{-\infty}^{\infty} p(x) F(y) dx, \quad y = x\sigma_{ATC} + \mu_{ATC}. \tag{2.28}$$

2.6 Summary of the proposed algorithm

Figure 2.3 illustrates the algorithm used to determine stochastic ATC. A Monte Carlo power flow algorithm solves the line power flow variables. The resulting variables are a set of line flows for a large number of cases. For example, if there are 10 power lines within a system, and 1000 different Monte Carlo power flow studies are done, there are 1000 different sets of line power flows for each of the 10 lines. The mean of the real power of each line is found and used to find the limiting line of a power transfer from bus A to bus B. The limiting line is the line used to evaluate the problem.

The stochastic ATC problem uses the set of line power flow variables to form a set of line power flow statistical moments. The limiting line power flow statistical moments are then transformed into the ATC for a transfer from bus A to bus B. The ATC statistical moments are then used to find the Gram Charlier Type A series. Plots of the Gram Charlier series can be formed to see the probability distribution of the ATC. The Gram Charlier series can also be used, in conjunction with a price function, to find the expected price of the transfer.

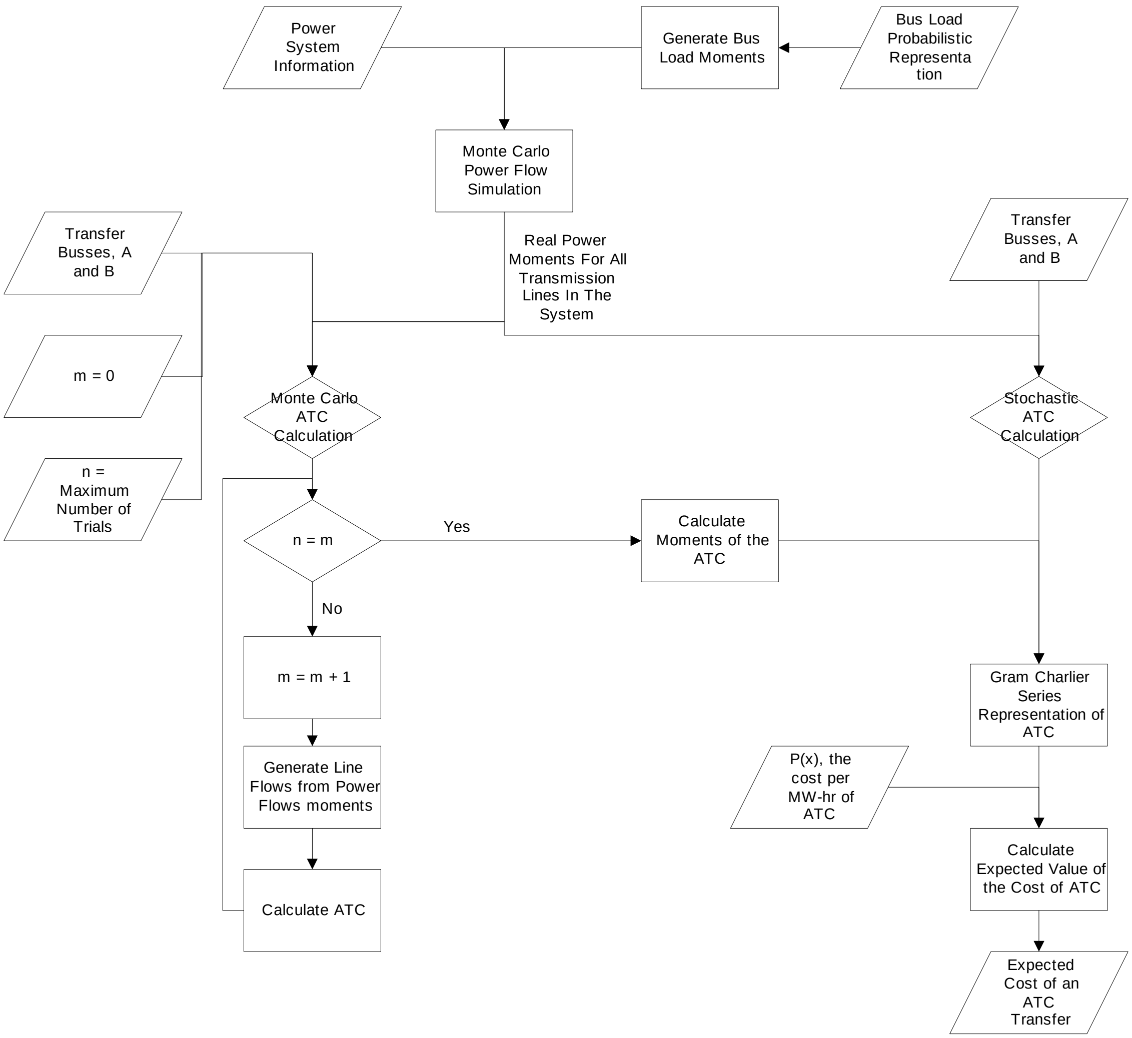

Figure 2.5 Probabilistic ATC determination using stochastic analysis and Monte Carlo methods

3. Stochastic Available Transfer Capability Calculations

3.1 Description of the test beds and examples

In this chapter, illustrations of stochastic ATC are given. For this purpose, two test bed systems are used: a 4 bus system, and a 14 bus system. Both test beds are intended to give an understanding of how the stochastic ATC problem is formed and determined. The 4 bus test bed is a small system which represents a minimum number of elements in a power system and is only used as a quick and simple illustration of the stochastic ATC problem. The 14 bus test bed analyzes the approach for a larger power system which includes additional elements: transformers, synchronous condensers, and multiple generators. Appendix C lists all the examples and their corresponding conditions.

3.2 The 4 bus test bed and examples

The 4 bus test bed is a small power system which contains 5 lines as shown in Figure 3.1. The test bed is used as a simple illustration of the stochastic ATC problem. Examples which contain the 4 bus test bed are shown in Appendix C. As commonly used in stochastic load flow examples, the active power loads are represented by normally distributed pseudorandom variables. In the examples, the bus demands are considered to be uncorrelated. Two techniques will be illustrated as examples of the calculation of ATC. The first is Monte Carlo based and this example is denominated as Example 4MC. The second method is a stochastic analysis, and this example is denominated as Example 4S. For Examples 4MC and 4S, the bus loads are considered to be normally distributed with statistics shown in Table 3.1. Note that reactive power is neglected.

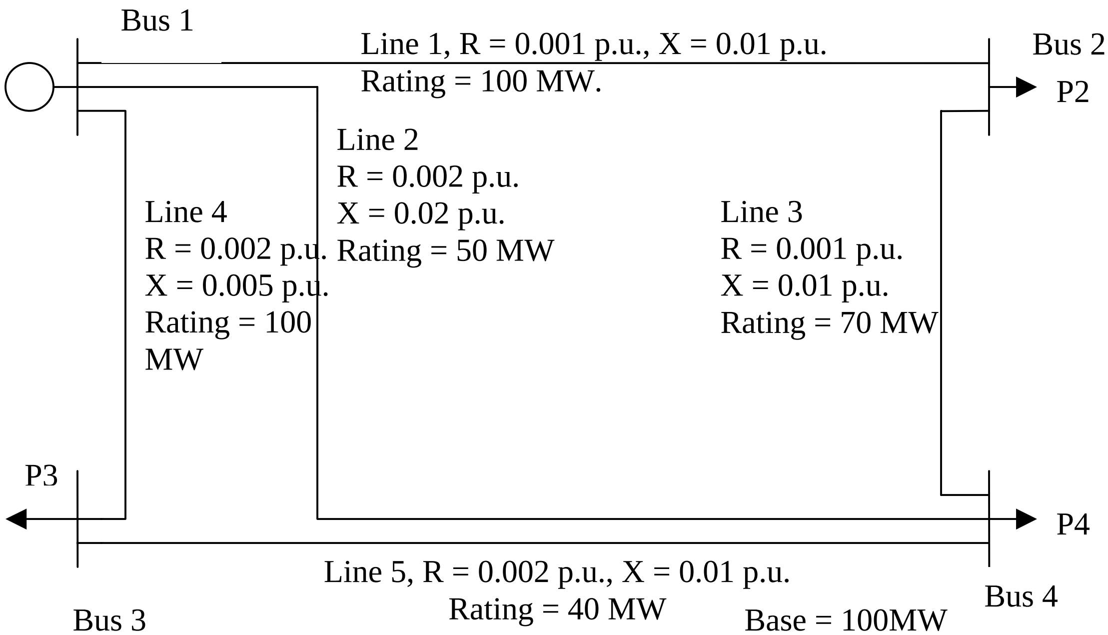

Figure 3.1 The 4 bus test bed, Examples 4MC and 4S

Table 3.1 The 4 bus test bed bus data in Examples 4MC and 4S

Demand	Mean (MW)	Variance (MW^2)
P2	10	1
P3	20	4
P4	10	1

Section 3.4 contains Examples 4S and 4MC. The examples compare the stochastic analysis (S) and Monte Carlo (MC) approaches to the stochastic ATC problem. This comparison is done for active power transfers between each bus pairs of the system. The agreement between the two approaches is shown for the 4 bus system. This agreement as well as other testing not shown here, suggests that it is possible to use the stochastic analysis method in further examples.

In Example 4MC, shown as the left hand portion of Figure 3.2, the statistical moments of ATC are calculated using the Monte Carlo method. The output of the Monte

Carlo simulated load flow study, for this example, is a matrix of line power flows in which the rows represent each line in the system, and the columns represent the trial number; 1000 trials are used. Once the ATC is calculated for each trial, the statistical moments of ATC are then determined. The Gram Charlier series is formed using the first five statistical moments and then plotted to show the probability density function of the ATC.

In Example 4S, shown as the right hand portion of Figure 3.2, the output of the Monte Carlo simulated load flow study is the statistical moments of the line power flows. The stochastic analysis method is then used to calculate the statistical moments of ATC. The Gram Charlier series is found using the five statistical moments and then plotted to show the probability density function of the ATC.

Using the resulting probability density functions for each transfer from Example 4S, Section 3.5 shows the results of the expected price for each transfer. The comparison is done using two example price functions are Pr^4_{ATC-1} and Pr^4_{ATC-2}

$$\mathrm{Pr}^4_{ATC-1}(x) = 100x \quad \$/MW \tag{3.1}$$

$$\mathrm{Pr}^4_{ATC-2}(x) = x^2 + 100x \quad \$/MW. \tag{3.2}$$

The superscript ‘4’ in (3.1) and (3.20 refer to the bus system in Example 4S. The subscripts 1 and 2 refer to pricing scenarios 1 and 2. For both cases, the probability density functions used are the resulting Gram Charlier Type A series from the stochastic analysis method.

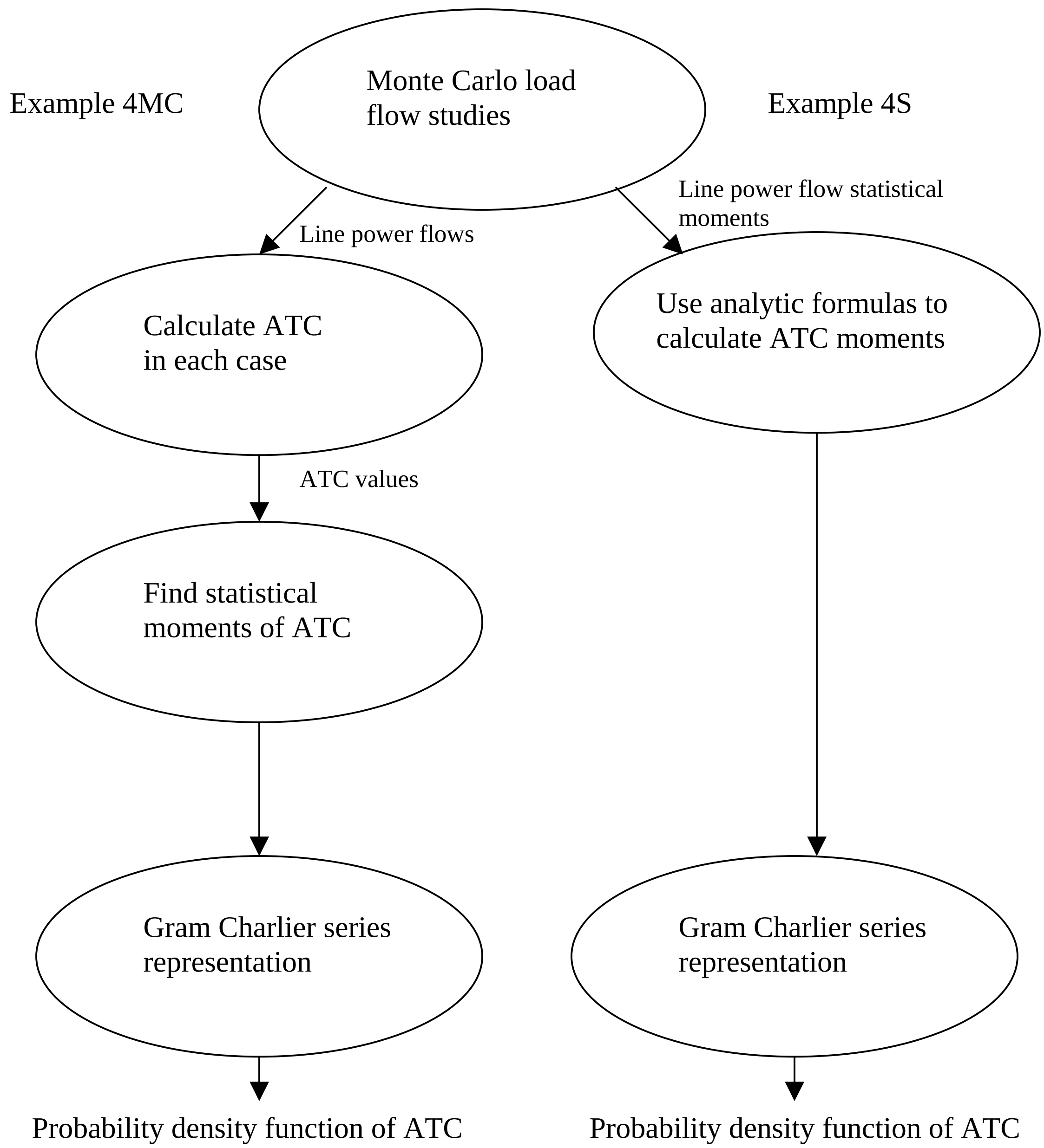

Figure 3.2 Two solution approaches for the 4 bus example

3.3 The 14 bus test bed and examples

The 14 bus test bed [37] consists of 20 power lines as shown in Figure 3.3. The system data are shown in Table 3.2. Examples which contain the 14 bus test bed are shown in Appendix C. This system is a representation consisting of transformers, synchronous condensers and complex power loads. All bus loads are once again represented by normally distributed pseudorandom variables with means and variances given in Table 3.3. The ATC calculation for this system is illustrated by stochastic analysis. The example offered is denominated as 14S.

Table 3.2 The 14 bus test bed system data *

Line No.	Between buses		*R* (p.u.)	*X* (p.u.)	Line charging susceptance in per unit	Tap setting	Rating (MW)
1	1	2	0.01938	0.05917	0.0528	0	200
2	1	5	0.05403	0.22304	0.0492	0	110
3	2	3	0.04699	0.19797	0.0438	0	110
4	2	4	0.05811	0.17632	0.034	0	80
5	2	5	0.05695	0.17388	0.0346	0	70
6	3	4	0.06701	0.17103	0.0128	0	50
7	4	5	0.01335	0.04211	0	0	100
8	4	7	0	0.20912	0	0.978	50
9	4	9	0	0.55618	0	0.969	50
10	5	6	0	0.25202	0	0.932	70
11	6	11	0.09498	0.1989	0	0	30
12	6	12	0.12291	0.25581	0	0	30
13	6	13	0.06615	0.13027	0	0	50
14	7	8	0	0.17615	0	0	60
15	7	9	0	0.11001	0	0	60
16	9	10	0.03181	0.0845	0	0	50
17	9	14	0.12711	0.27038	0	0	50
18	10	11	0.08205	0.19207	0	0	50
19	12	13	0.22092	0.19988	0	0	50
20	13	14	0.17093	0.34802	0	0	50

* On a 100 MVA base

Table 3.3 The14 bus test bed bus data

Bus	Active Power		Reactive Power	Generation (MW)
	Mean (MW)	Variance (MW^2)	Fixed Q (MVAr)	
1	22	2.2	0	-
2	94	9.4	12.7	40
3	48	4.8	19	0
4	8	0.8	-3.9	0
5	12	1.2	1.6	0
6	30	3	7.5	0
7	0	0	0	0
8	0	0	0	0
9	9	0.9	16.6	0
10	0	0	5.8	0
11	3.5	0.35	1.8	0
12	6	0.6	1.6	0
13	14	1.4	5.8	0
14	15	1.5	5	0

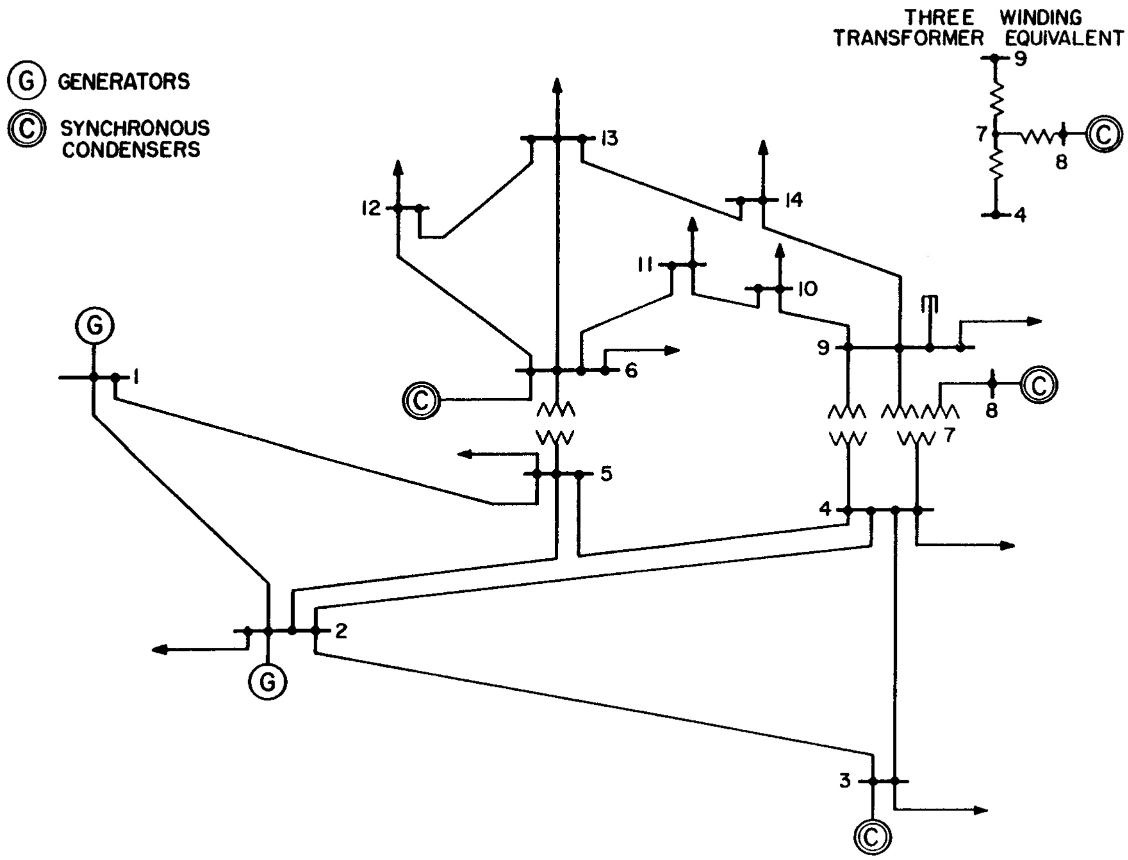

Figure 3.3 The 14 bus test bed (taken directly from [37])

Section 3.6 contains Example 14S in which the 14 bus power system is analyzed using the stochastic analysis method. Figure 3.4 illustrates the process of finding the statistical moments and the Gram Charlier series of the ATC. The resulting Gram Charlier series is used to calculate the expected price using the price function

$$\Pr_{ATC}^{14}(x) = 100x \quad \$/MW \tag{3.1}$$

and also plotted showing the probability density function of the ATC. Six different bus pair cases are chosen to illustrate the stochastic ATC. All the cases are compared to two different line outage cases illustrated in Chapter 4.

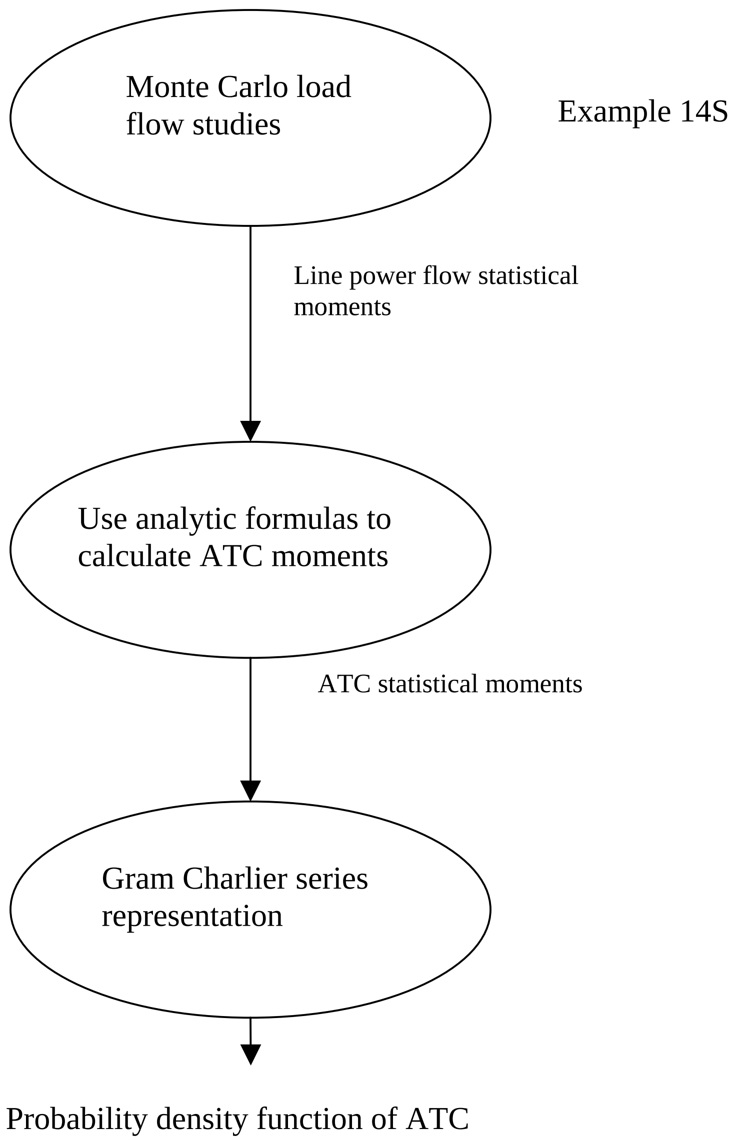

Figure 3.4 The solution of the 14 bus example

3.4 Illustration of the stochastic and Monte Carlo load flow agreement.

Using the 4 bus test bed, a stochastic and Monte Carlo simulated ATC analysis of the system using Matlab will illustrate the agreement of both methods. Even though the Monte Carlo method is an "exact" process of finding the ATC, the computation time for the method is quite large while the stochastic determination of ATC reduces the computation time significantly. The Gram Charlier Type A series appears to be accurately represented using the first 5 statistical moments for each transfer and resulting in a probability of 1.000 when the series is integrated from $-\infty$ to ∞. Figures 3.3-3.14 show plots of the probability density functions for the transfer between each bus pair of the system.

Data from Table 3.4 show the difference between the statistical moments of the ATC_{ij} calculated by two different methods, namely the purely Monte Carlo method and secondly by a stochastic analytical technique. In the case of the latter, the analytical technique employs a calculation using a Gram Charlier series expression for *f(x)* in

$$m_{ATC}^{(k)} = \int_{-\infty}^{\infty} x^k f(x) dx \,, \tag{3.1}$$

where *k* is the moment order and *f (x)* is the probability density function represented by the Gram Charlier Type A series. As shown in Table 3.4, the differences of the first five statistical moments between the Monte Carlo method and stochastic analytical technique have small error. Figures 3.4 – 3.15 show the agreement of the probability density functions.

Table 3.4 First five statistical moments of all ATC transfers (ATC in MW) in Examples 4MC and 4S

Transfer		Monte Carlo Statistical moments				
Sending	Receiving	1	2	3	4	5
1	2	138.7722	1.93E+04	2.67E+06	3.71E+08	5.15E+10
1	3	46.995	2.22E+03	1.05E+05	4.98E+06	2.37E+08
1	4	90.4381	8.20E+03	7.46E+05	6.81E+07	6.23E+09
2	3	63.2527	4.01E+03	2.56E+05	1.63E+07	1.05E+09
2	4	154.4192	2.38E+04	3.68E+06	5.69E+08	8.78E+10
3	4	69.0065	4.76E+03	3.29E+05	2.27E+07	1.57E+09

Transfer		Stochastic Statistical moments				
Sending	Receiving	1	2	3	4	5
1	2	138.7554	1.93E+04	2.67E+06	3.71E+08	5.15E+10
1	3	47.0289	2.22E+03	1.05E+05	4.99E+06	2.38E+08
1	4	90.7325	8.26E+03	7.55E+05	6.93E+07	6.38E+09
2	3	63.3939	4.03E+03	2.57E+05	1.65E+07	1.06E+09
2	4	154.469	2.39E+04	3.69E+06	5.69E+08	8.80E+10
3	4	68.9285	4.75E+03	3.28E+05	2.26E+07	1.56E+09

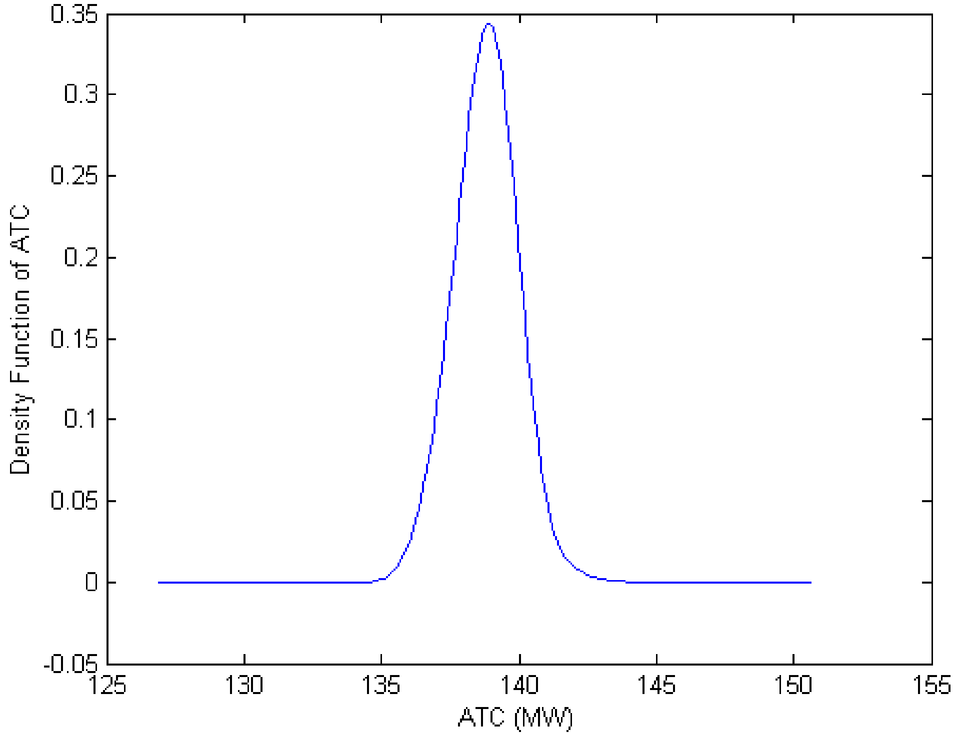

Figure 3.5 PDF of the ATC transfer from 1 to 2 in Example 4MC

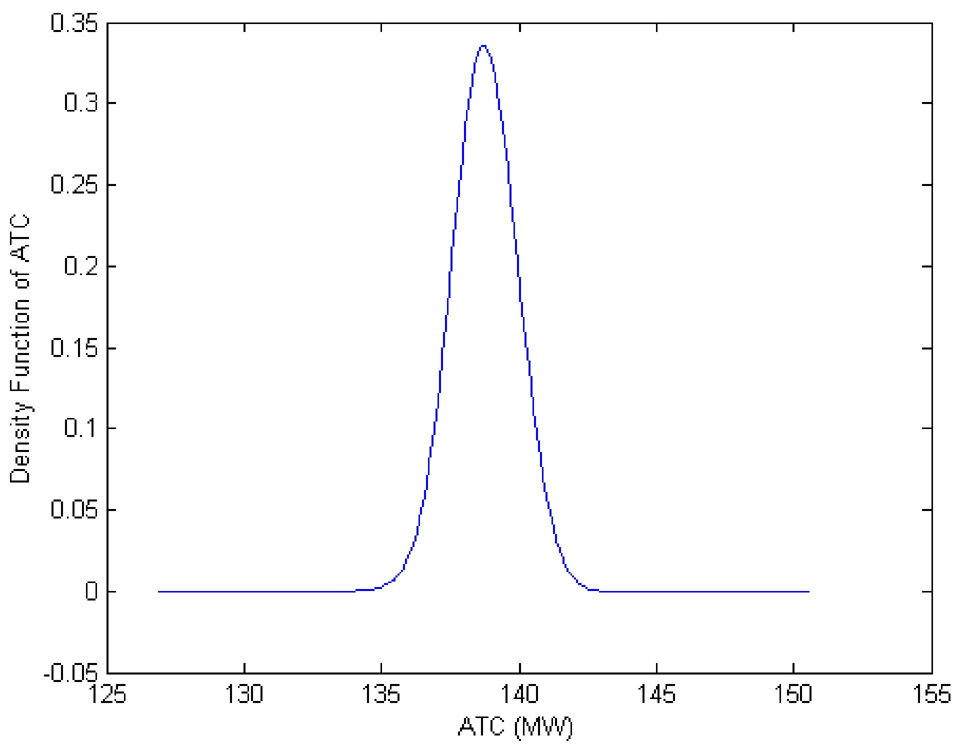

Figure 3.6 PDF of the ATC transfer from 1 to 2 in Example 4S

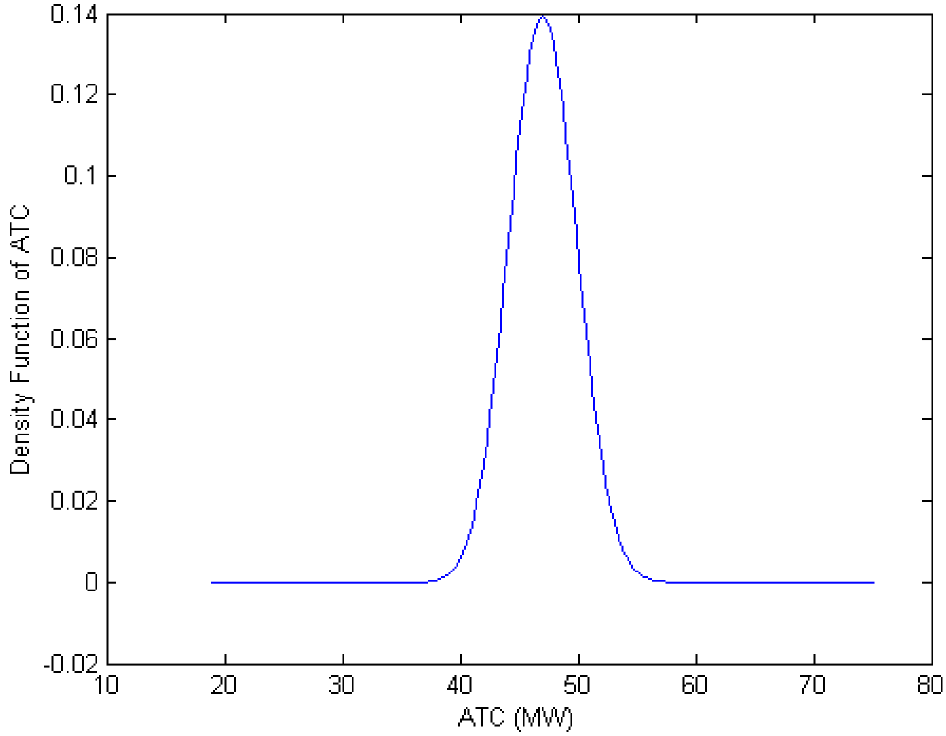

Figure 3.7 PDF of the ATC transfer from 1 to 3 in Example 4MC

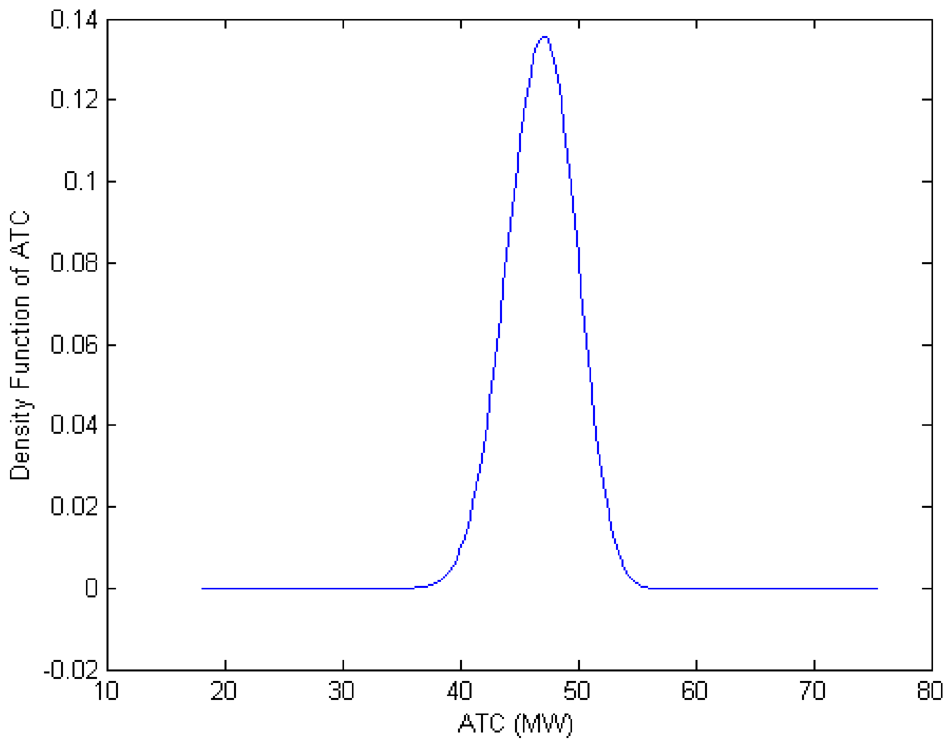

Figure 3.8 PDF of the ATC transfer from 1 to 3 in Example 4S

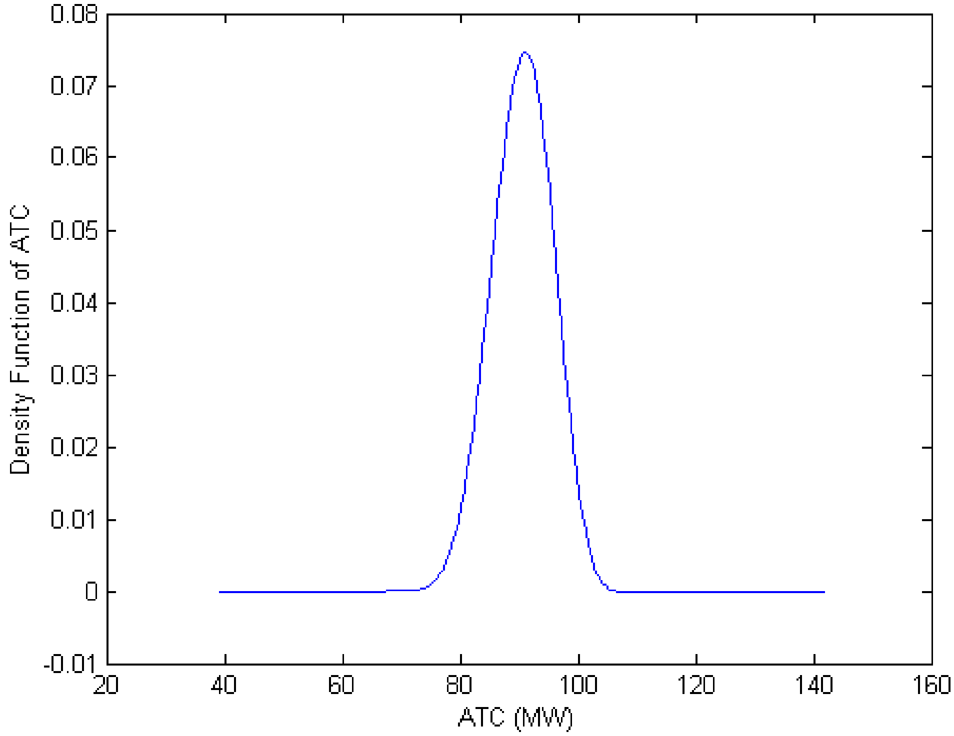

Figure 3.9 PDF of the ATC transfer from 1 to 4 in Example 4MC

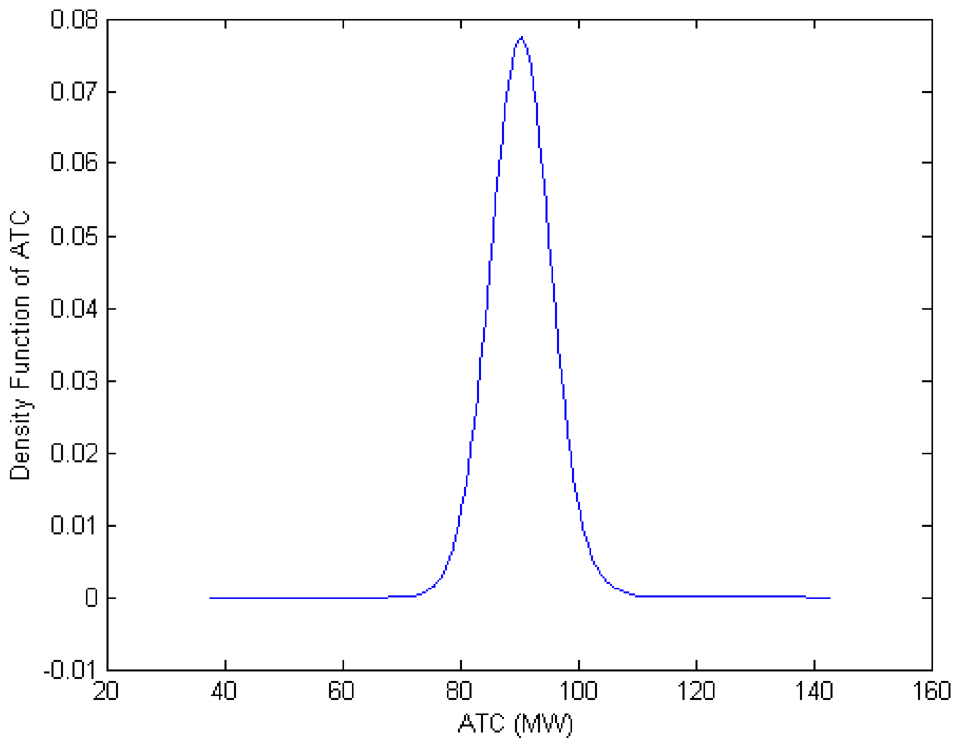

Figure 3.10 PDF of the ATC transfer from 1 to 4 in Example 4S

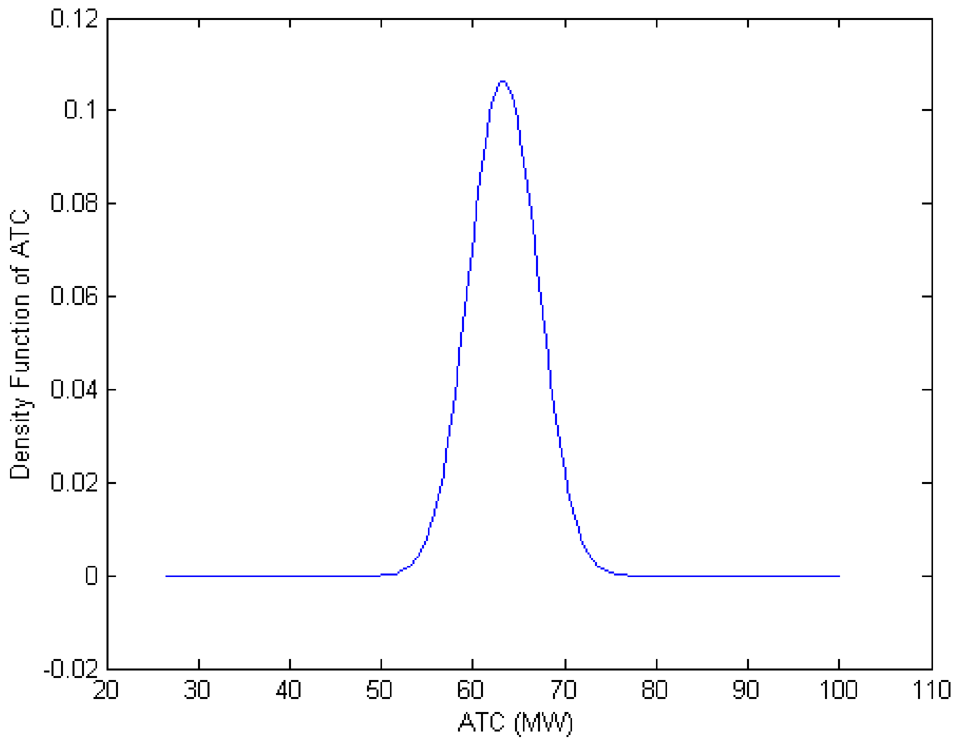

Figure 3.11 PDF of the ATC transfer from 2 to 3 in Example 4MC

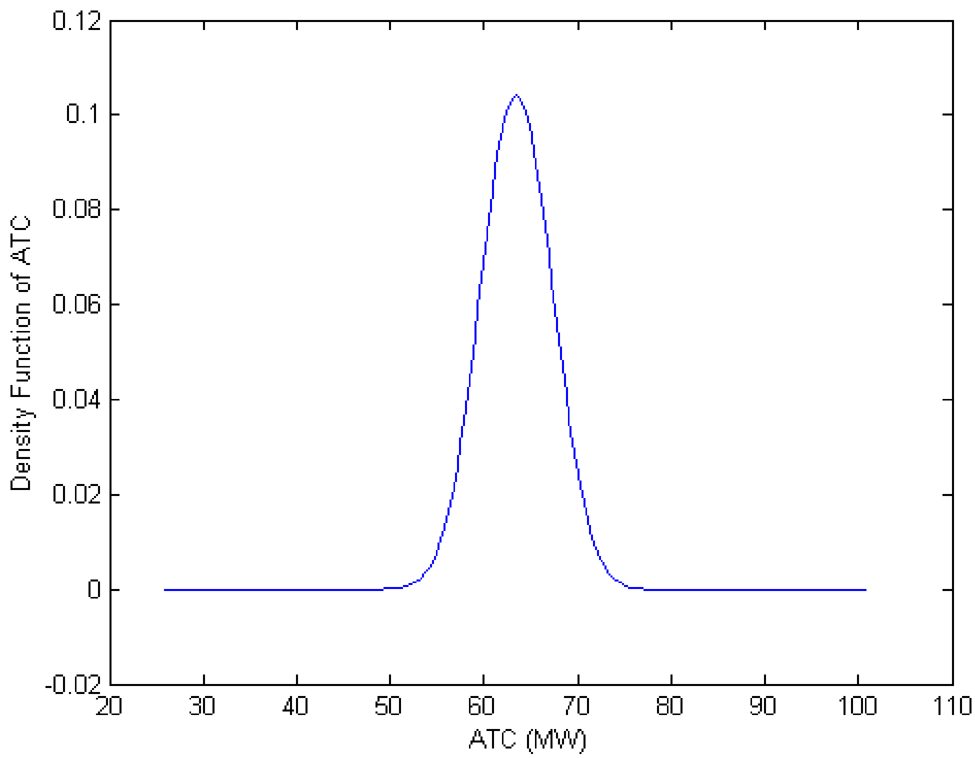

Figure 3.12 PDF of the ATC transfer from 2 to 3 in Example 4S

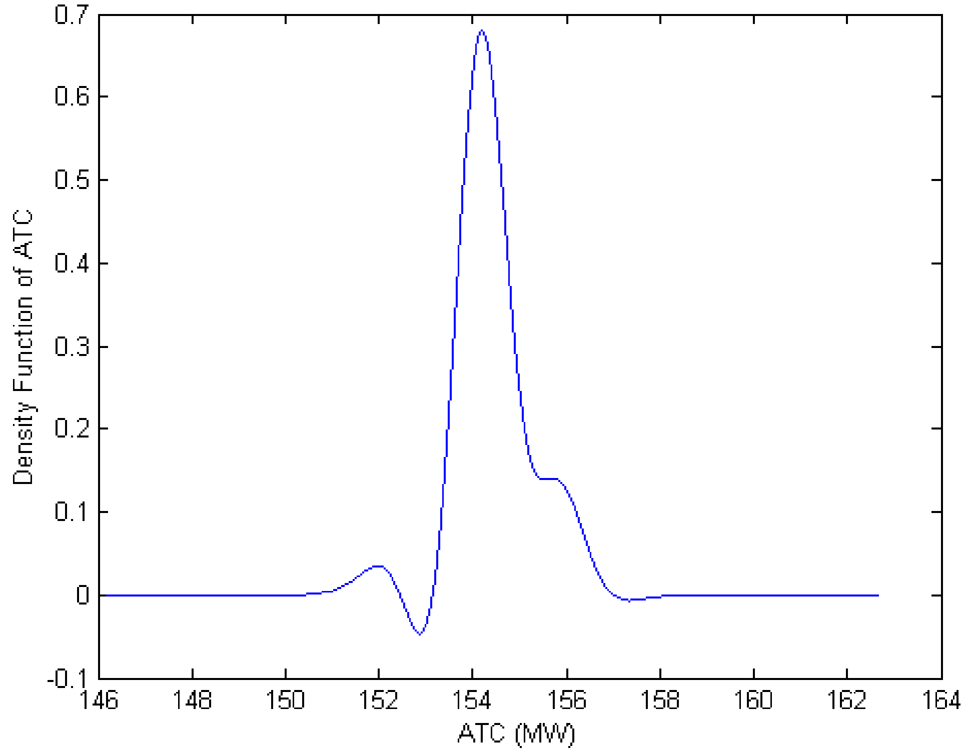

Figure 3.13 PDF of the ATC transfer from 2 to 4 in Example 4MC

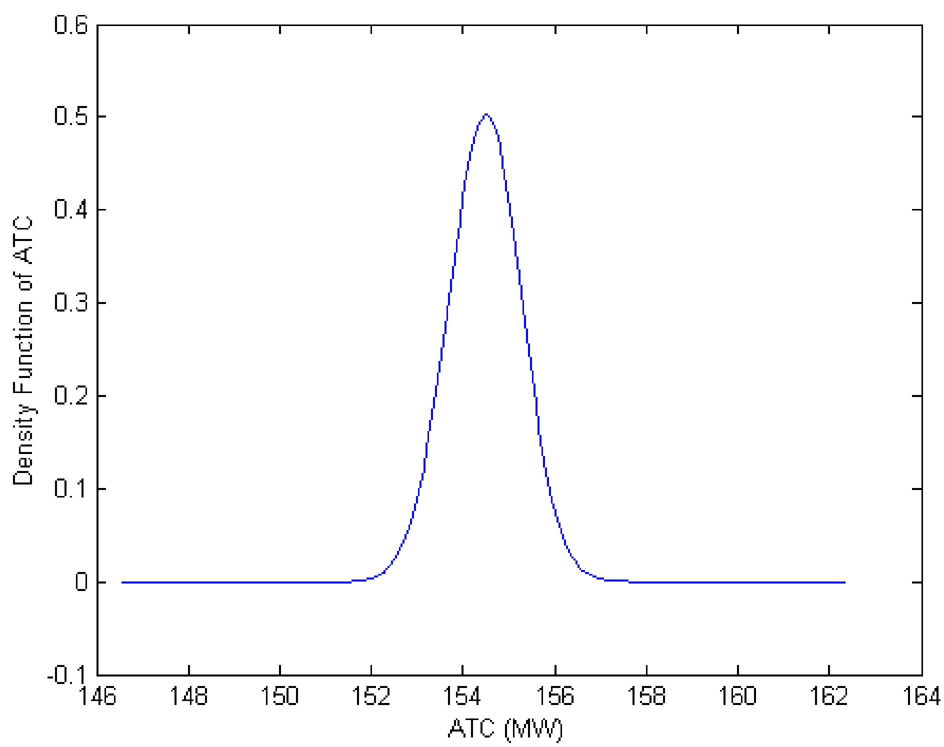

Figure 3.14 PDF of the ATC transfer from 2 to 4 in Example 4S

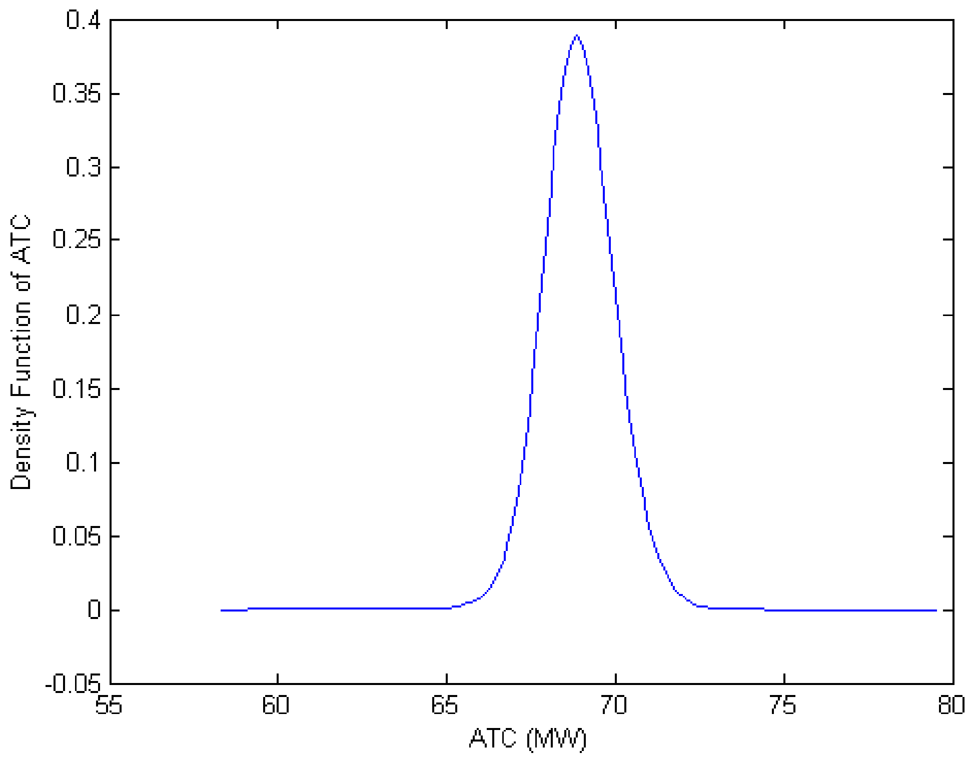

Figure 3.15 PDF of the ATC transfer from 3 to 4 in Example 4MC

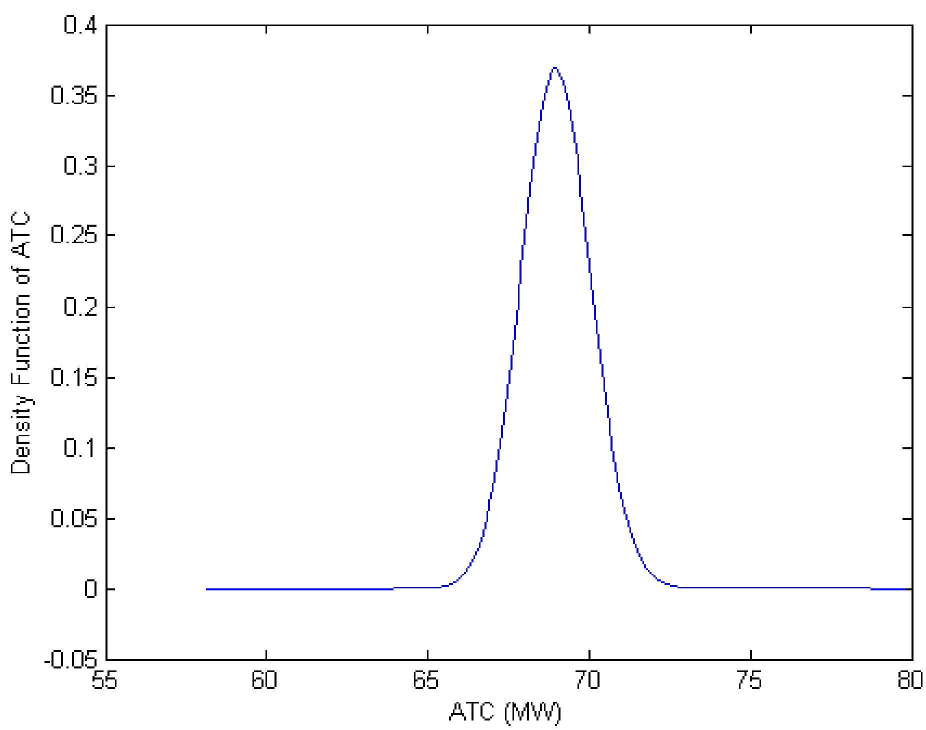

Figure 3.16 PDF of the ATC transfer from 3 to 4 in Example 4S

3.5 Expected price of transfer

For each of the transfers shown in Section 3.2, two different price functions from Equations (3.1) and (3.2) are used to illustrate the price of the transfer. The Gram Charlier Series Type A representation of the probability density for each transfer from the stochastic method, *f(x)*, is used in the determination the expected value of the price. The expected price is found by

$$E[\Pr_{ATC}] = \int_{-\infty}^{\infty} \Pr_{ATC}(x) f(x) dx . \tag{3.3}$$

In Table 3.5, the data for the expected price for all the cases are shown which are amounts of money that can be generated for each transfer. The two columns at the right in Table 3.5 correspond to the two ATC prices given in (3.1) and (3.2).

Table 3.5 Expected price of ATC in Example 4S for two price Equations (3.1), (3.2)

Transfer		$E[\Pr^4_{ATC-1}]$	$E[\Pr^4_{ATC-1}]$
Sending	Receiving	*($)*	*($)*
1	2	1.39E+04	3.31E+04
1	3	4.70E+03	6.92E+03
1	4	9.05E+03	1.73E+04
2	3	6.33E+03	1.03E+04
2	4	1.54E+04	3.93E+04
3	4	6.90E+03	1.17E+04

3.6 Example 14S

This section shows results of the stochastic ATC analysis in the 14 bus test bed. Since the test bed has 20 different power lines and devices, only 6 exemplary transfers are analyzed as shown in Table 3.6

Table 3.6 Transfers used for Example 14S

Transfer	
Sending Bus	Receiving Bus
3	6
2	9
3	13
1	13
2	14
1	3

For each of the power transfers in Table 3.6, first five statistical moments of the ATC and the expected price $E[\Pr_{ATC}^{14}]$ are determined and shown in Table 3.7. Figures 3.15-3.19 show the probability density functions for each transfer formed using the Gram Charlier Type A series.

Table 3.7 Statistical moments and expected prices for given transfers for Example 14S

Transfer		Statistical moments of the ATC*					$E[\Pr_{ATC}^{14}]$ (\$)
Sending bus	Receiving bus	1	2	3	4	5	
3	6	38.3338	7.45E+03	1.85E+05	1.62E+08	3.78E+09	3.83E+03
2	9	46.0474	1.28E+04	2.52E+05	5.14E+08	2.82E+10	4.60E+03
3	13	41.9471	6.80E+03	2.91E+05	1.23E+08	2.28E+09	4.19E+03
1	13	4.07E+01	7.59E+03	2.37E+05	1.62E+08	2.03E+09	4.07E+03
2	14	58.0429	1.91E+04	6.06E+05	1.13E+09	5.98E+10	5.80E+03
1	3	52.7573	9.45E+03	5.32E+05	2.18E+08	6.72E+09	5.28E+03

* ATC given in MW

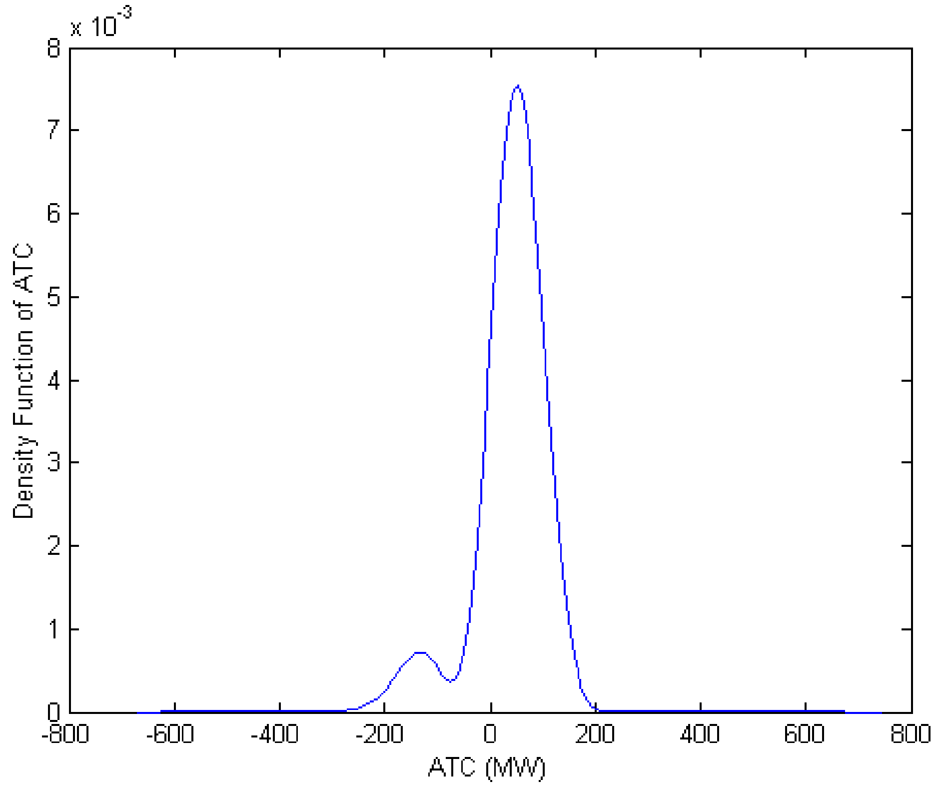

Figure 3.17 PDF of the ATC transfer from 3 to 6 in Example 14S

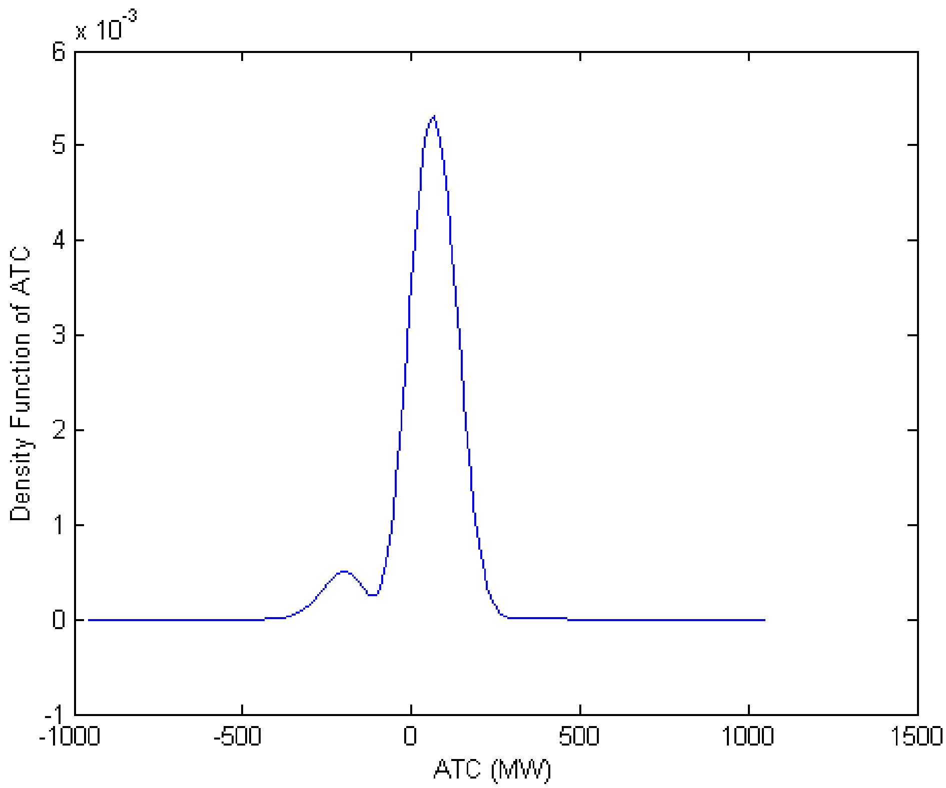

Figure 3.18 PDF of the ATC transfer from 2 to 9 in Example 14S

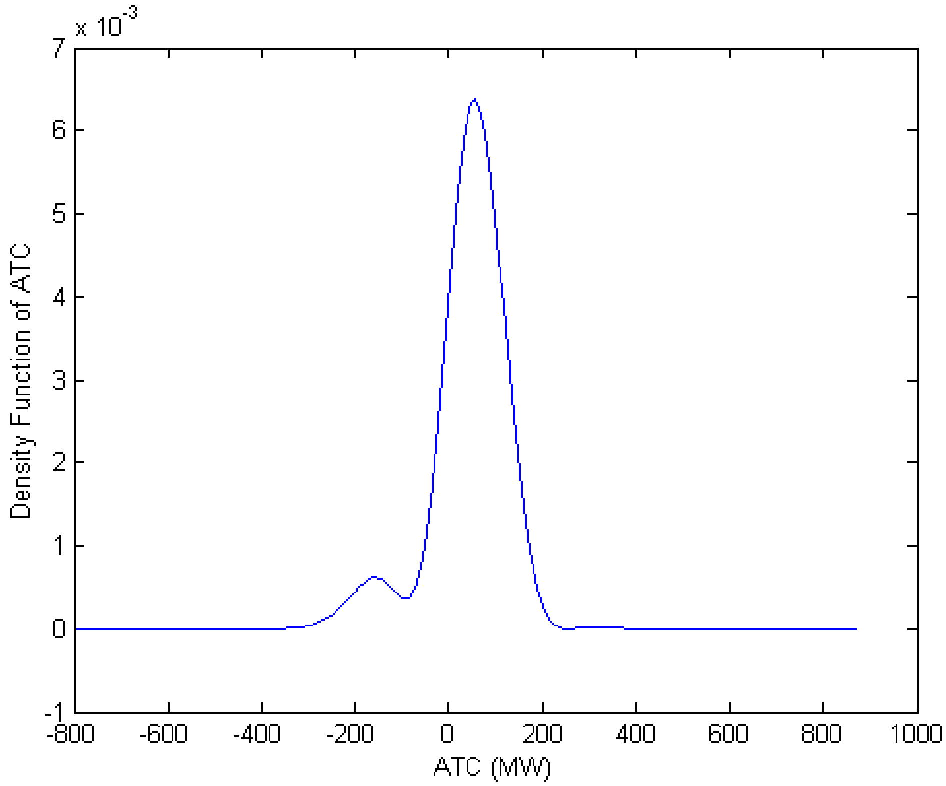

Figure 3.19 PDF of the ATC transfer from 3 to 13 in Example 14S

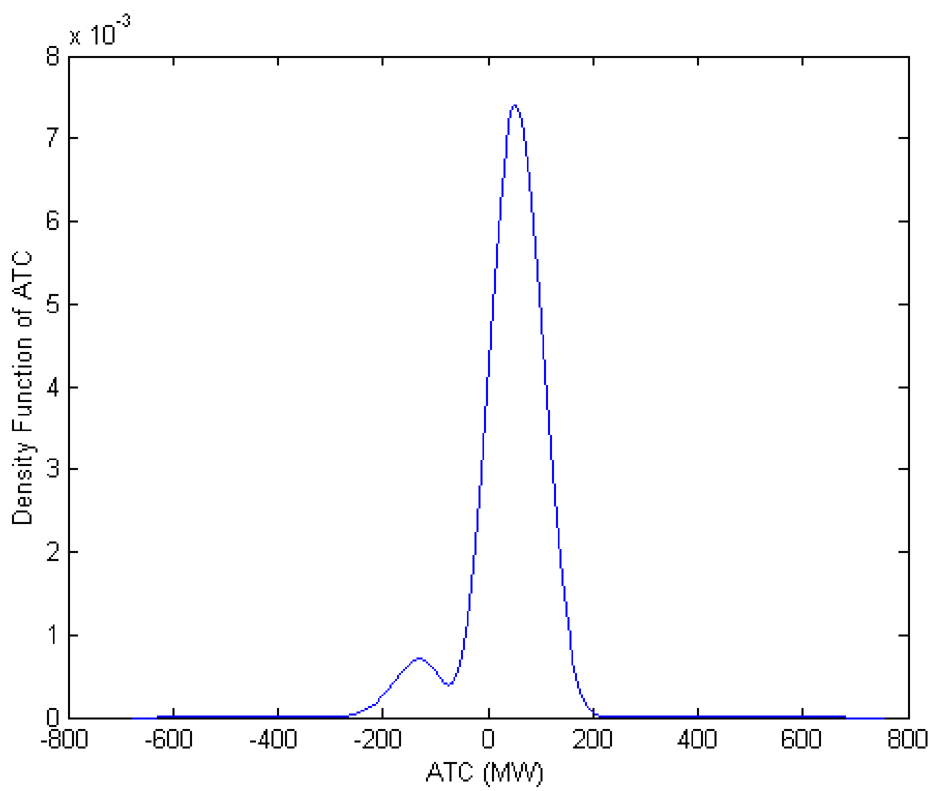

Figure 3.20 PDF of the ATC transfer from 1 to 13 in Example 14S

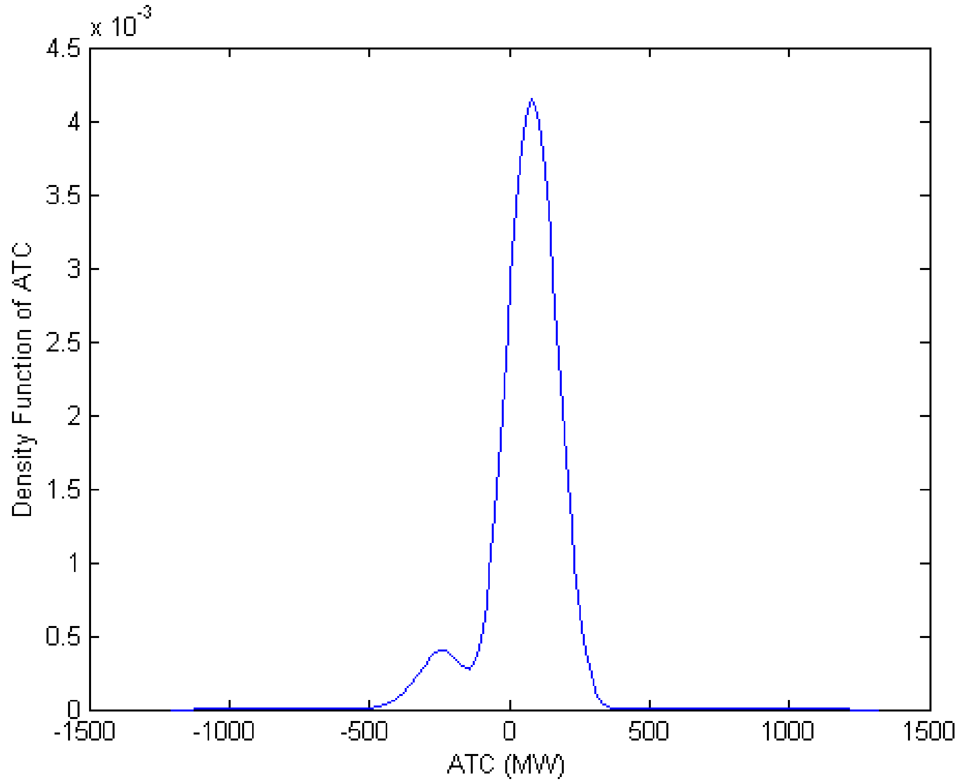

Figure 3.21 PDF of the ATC transfer from 2 to 14 in Example 14S

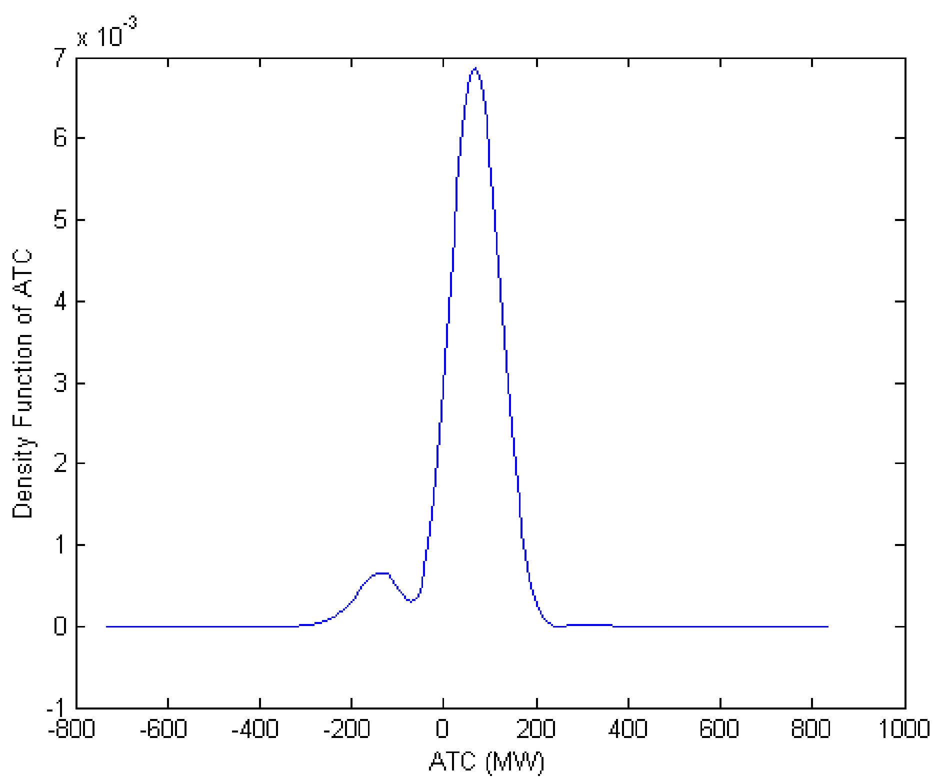

Figure 3.22 PDF of the ATC transfer from 1 to 3 in Example 14S

3.7 Concluding remark

The three examples shown in this chapter result in expected price of ATC. Tables 3.5 and 3.7 are these expectations. A comparison of the methods and general conclusion appear in Chapter 5.

4. The Effect of Transmission Line Outages on the Stochastic ATC Problem

4.1 Formulation of examples to illustrate line outage effects

In a power system, line maintenance can impact ATC. These scheduled outages can cause the ATC for a given transfer to change significantly. Forced outages have a more deleterious effect in that unplanned changes in the ATC can result in unexpected operating problems. For the examples in this chapter, the stochastic ATC is found for the line outage case only, but is then compared to the stochastic ATC is when no line outages exist in the system.

For the line outage cases in this chapter, the 14 bus test bed is used as described in Chapter 3. The first example, denominated as Example 14S2-4, evaluates the stochastic ATC of the 14 bus system with the line between bus 2 and bus 4 being a scheduled outage. The notation 14Sr-s refers to the line outage r-s. Three different ATC transfers from the stochastic ATC examples that are shown in Chapter 3, namely Example 14S, are compared to the same three transfers for Example 14S2-4 and are shown in Table 4.1.

Table 4.1 Lines used for the determination of the stochastic ATC in Example 14S2-4

Line	
From	To
3	6
2	9
3	13

The second example, denominated as Example 14S6-13, evaluates the stochastic ATC of the 14 bus system with the line between bus 6 and bus 13 being a scheduled outage. Three different ATC transfers that are shown in Chapter 3, Example 14S, are compared to the same three transfers for Example 14S6-13 and are shown in Table 4.2.

Table 4.2 Lines used for the determination of the stochastic ATC in Example 14S6-13

Line	
From	To
1	13
2	14
1	3

For both examples, the resulting probability density function plots, first five statistical raw moments and expected prices using the price function

$$P_{ATC}^{14}(x) = 100x \quad \$ / MW \tag{4.1}$$

are shown. The differences between the mean of the transfers and the expected price of the transfer are also shown. The differences indicate the impact that line outages have on the values of stochastic ATC.

4.2 Example 14S2-4

This example illustrates the stochastic ATC for a scheduled line outage for the line from bus 2 to bus 4. The algorithm used for Example 14S2-4 is the same as the illustration in Figure 3.4. The resulting statistical moments and expected price with the line from bus 2 to bus 4 out of service as well as the line in service are shown in Table 4.3. From Table 4.3 it can be seen that having a line out service will affect the ATC significantly for transfers that involve line 2-4. The differences between the mean of the ATC (i.e. the first moment) and the expected price of the ATC are shown in Table 4.4. Figures 4.1-3 show the probability density functions of the ATC with line 2-4 out of service.

Table 4.3 Statistical moments and expected price for all transmission lines online and line 2-4 out of service in Example 14S2-4

Transfer		Statistical moments of the ATC with line 2-4 in service*					$E[P_{ATC}^{14}]$ ($)
Sending	Receiving	$m_{ATC}^{(1)}$	$m_{ATC}^{(2)}$	$m_{ATC}^{(3)}$	$m_{ATC}^{(4)}$	$m_{ATC}^{(5)}$	
1	13	4.07E+01	7.59E+03	-2.37E+05	1.62E+08	-2.03E+09	4.07E+03
2	14	58.0429	1.91E+04	-6.06E+05	1.13E+09	-5.98E+10	5.80E+03
1	3	52.7573	9.45E+03	-5.32E+05	2.18E+08	-6.72E+09	5.28E+03
Transfer		Statistical moments of the ATC with line 2-4 out of service*					$E[P_{ATC2-4}^{14}]$ ($)
Sending	Receiving	$m_{ATC2-4}^{(1)}$	$m_{ATC2-4}^{(2)}$	$m_{ATC2-4}^{(3)}$	$m_{ATC2-4}^{(4)}$	$m_{ATC2-4}^{(5)}$	
1	13	9.22E+00	2.87E+04	-4.63E+05	3.34E+09	5.52E+10	9.22E+02
2	14	8.8002	1.60E+04	-2.36E+05	1.02E+09	-1.92E+09	8.80E+02
1	3	23.2916	1.74E+05	-9.52E+06	1.20E+11	5.14E+11	2.33E+03

*ATC in MW

Table 4.4 Differences of the mean ATC calculated with line 2-4 in service and out of service and the expected price of the ATC calculated with line 2-4 in service and out of service in Example 14S2-4

Transfer		$m_{ATC2-4}^{(1)} - m_{ATC}^{(1)}$ (MW)	$E[P_{ATC2-4}^{14}] - E[P_{ATC}^{14}]$ ($)
Sending	Receiving		
1	13	-31.477	-3147.702
2	14	-49.2427	-4919.9811
1	3	-29.4657	-2950.8

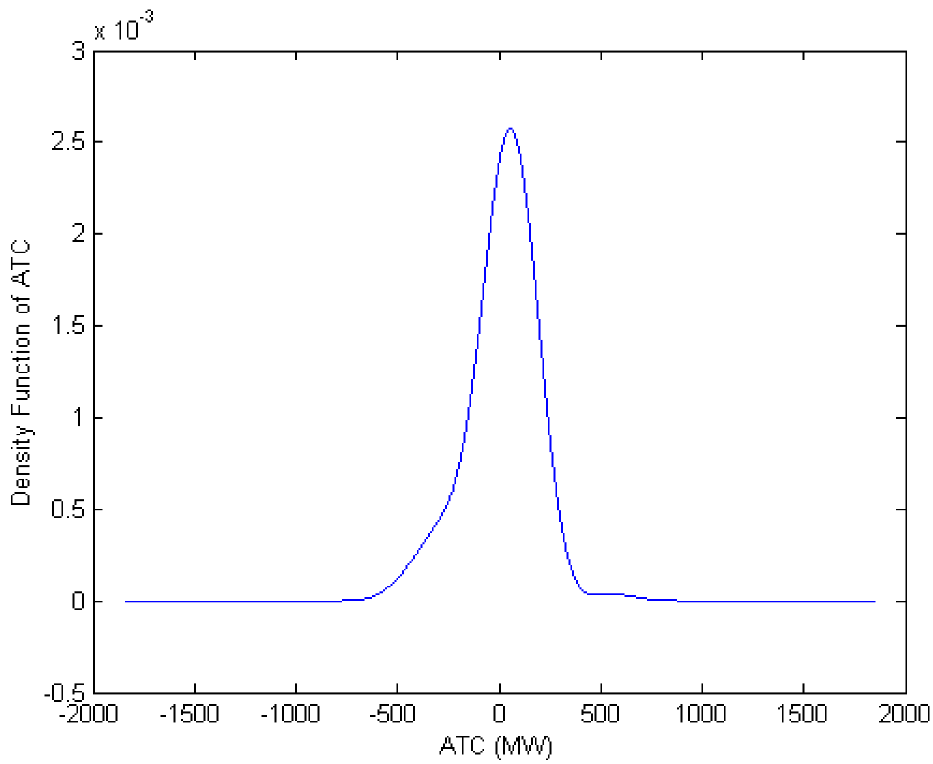

Figure 4.1 PDF of the ATC in line 1-13 in Example14S2-4

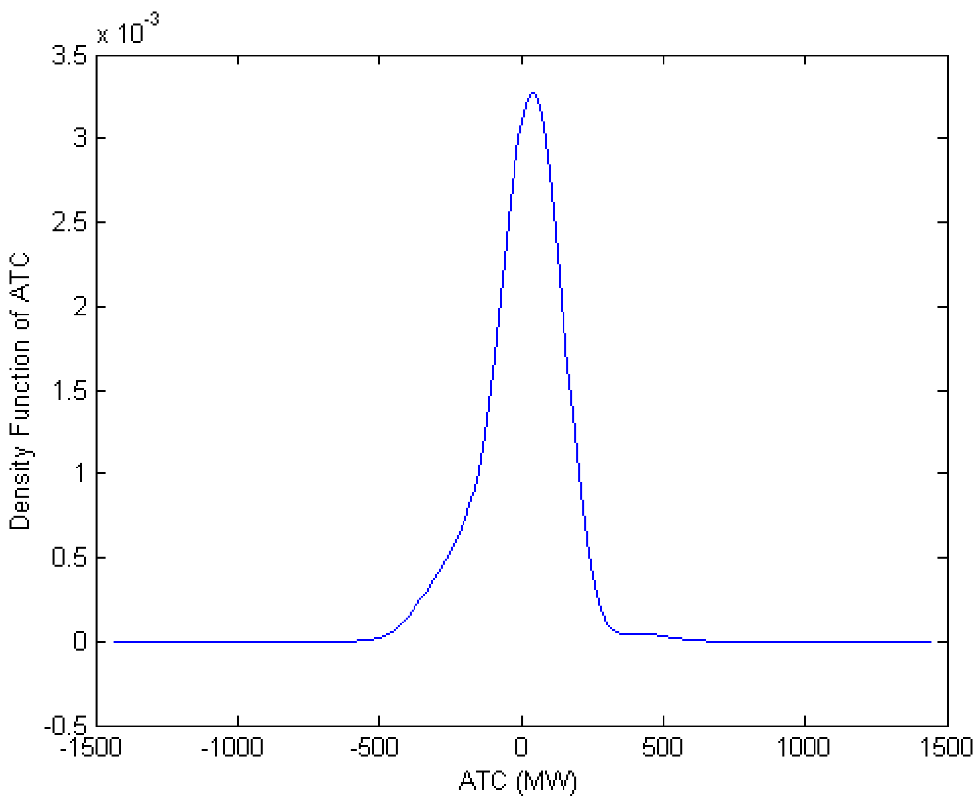

Figure 4.2 PDF of the ATC in line 2-14 in Example14S2-4

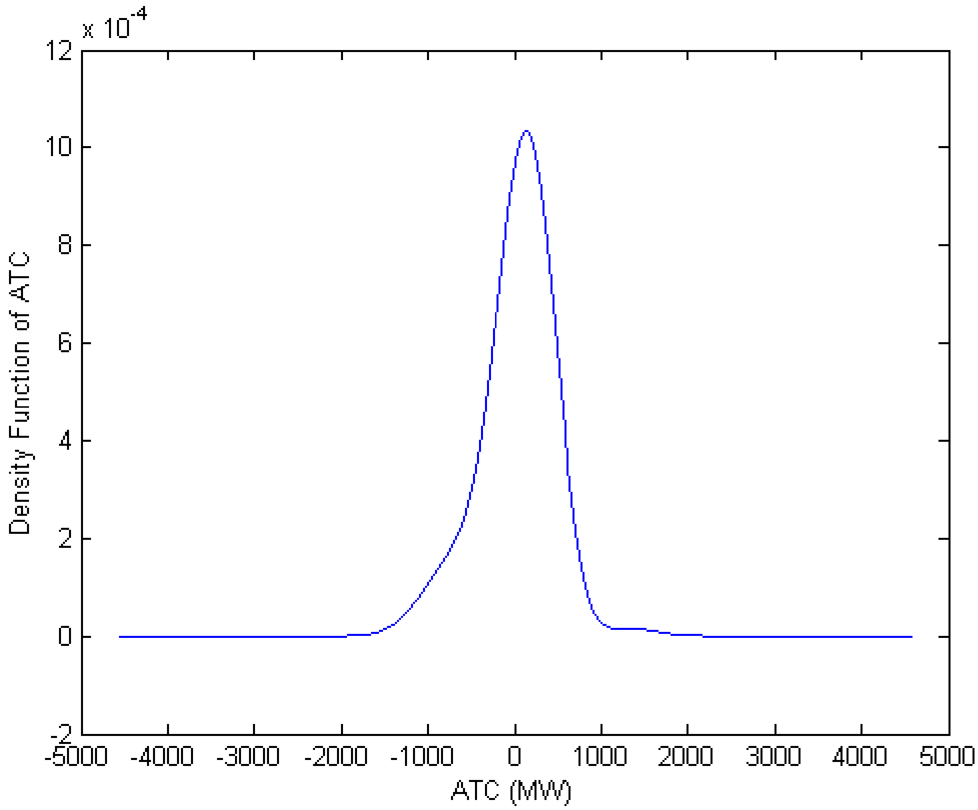

Figure 4.3 PDF of the ATC in line 1-3 in Example14S2-4

4.3 Example 14S6-13

This example illustrates the ATC for a scheduled line outage for the line from bus 6 to bus 14. The algorithm used for Example 14S6-13 is the same as the illustration in Figure 3.4. The resulting statistical moments and expected price with the line from bus 6 to bus 13 out of service as well as the line in service are shown in Table 4.5. From Table 4.6 it can be seen that having a line out service will affect the ATC for transfers that involve line 6-13. The differences between the mean of the ATC (i.e. the first moment) and the expected price of the ATC are shown in Table 4.6. Figures 5.4-6 show the probability density functions of the ATC with line 6-13 out of service.

Table 4.5 Statistical moments and expected price for all transmission lines online and line 6-13 out of service in Example 14S6-13

Transfer		Statistical moments of the ATC with line 6-13 in service*					$E[P_{ATC}^{14}]$
Sending	Receiving	$m_{ATC}^{(1)}$	$m_{ATC}^{(2)}$	$m_{ATC}^{(3)}$	$m_{ATC}^{(4)}$	$m_{ATC}^{(5)}$	($)
3	6	38.3338	7.45E+03	-1.85E+05	1.62E+08	-3.78E+09	3.83E+03
2	9	46.0474	1.28E+04	-2.52E+05	5.14E+08	-2.82E+10	4.60E+03
3	13	41.9471	6.80E+03	-2.91E+05	1.23E+08	-2.28E+09	4.19E+03
Transfer		**Statistical moments of the ATC with line 6-13 out of service***					$E[P_{ATC6-13}^{14}]$
Sending	Receiving	$m_{ATC6-13}^{(1)}$	$m_{ATC6-13}^{(2)}$	$m_{ATC6-13}^{(3)}$	$m_{ATC6-13}^{(4)}$	$m_{ATC6-13}^{(5)}$	($)
3	6	40.8527	7.85E+03	3.07E+05	1.75E+08	1.70E+09	4.09E+03
2	9	40.3125	1.22E+04	2.56E+05	4.82E+08	-1.90E+10	4.03E+03
3	13	26.211	2.00E+04	-1.11E+06	1.65E+09	-3.05E+11	2.62E+03

*ATC in MW

Table 4.6 Differences of the mean ATC calculated with line 6-13 in service and out of service and the expected price of the ATC calculated with line 6-13 in service and out of service in Example 14S6-13

Transfer			
Sending	Receiving	$m_{ATC6-13}^{(1)} - m_{ATC}^{(1)}$ (MW)	$E[P_{ATC6-13}^{14}] - E[P_{ATC}^{14}]$ ($)
3	6	2.5189	255.3
2	9	-5.7349	-568.8
3	13	-15.7361	-1568.9

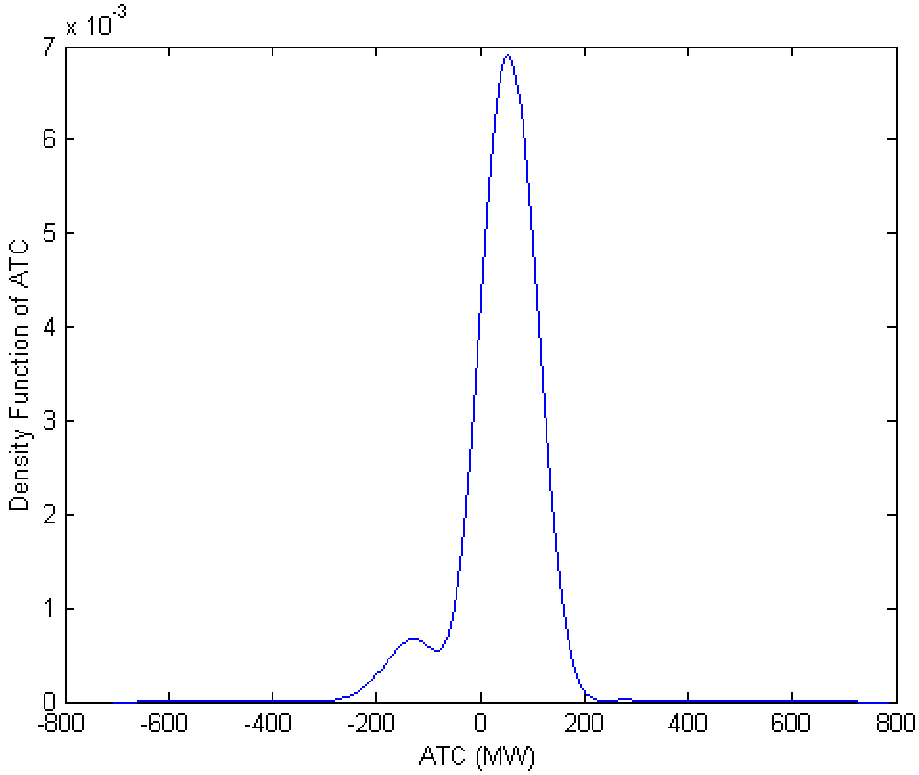

Figure 4.4 PDF of the ATC in line 3-6 in Example14S6-13

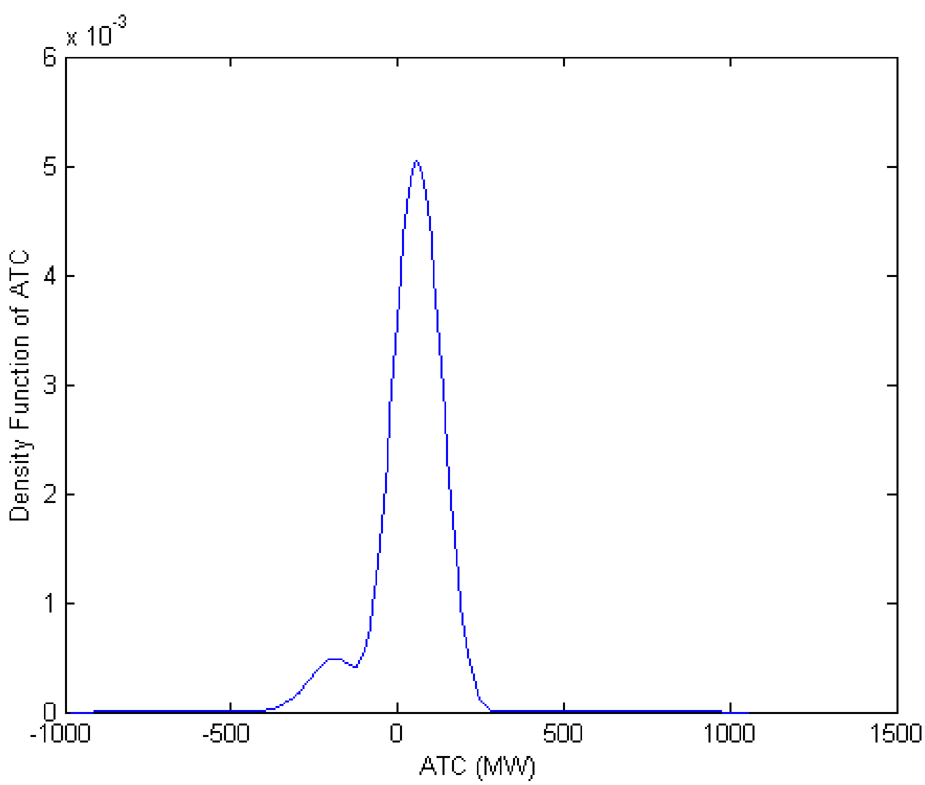

Figure 4.5 PDF of the ATC in line 2-9 in Example14S6-13

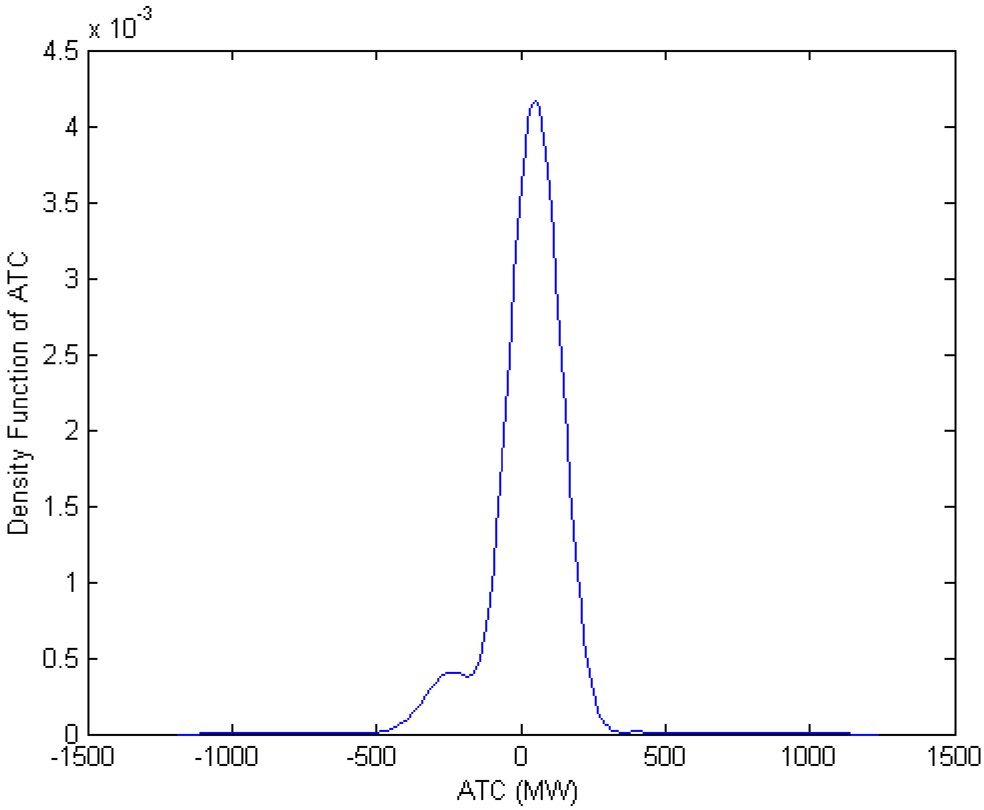

Figure 4.6 PDF of the ATC in line 3-13 in Example14S6-13

4.4 Concluding remark

The two examples shown in this chapter result in differences between the mean and expected price of the ATC. Tables 4.4 and 4.6 are these differences. Conclusions drawn from having a line outage in the sample 14 bus system appear in Chapter 5.

Appendix B contains two extra line outage cases which have similar results in the differences between the mean and expected price of the ATC.

5. Conclusions Drawn From The Examples

5.1 Conclusions from Examples 4S, 4MC and 14S

Examples 4S and 4MC show an agreement between the Monte Carlo and stochastic analysis methods for evaluating the stochastic ATC for a small power system. The raw statistical moments of the ATC as calculated using the Monte Carlo method versus the stochastic analytical method have small differences (e.g., much less than 1%). Also, the expected price of the ATC shows small differences. The main reason for the difference between the two examples 4S and 4MC is the determination of the element in the power system which limits the power transfer. Since the Monte Carlo method is an iterative approach (i.e., multiple trials) with a fixed number of samples, the element that limits the ATC is determined for each case. This results in the possibility of multiple limiting elements. The stochastic analytical method identifies a unique limiting transmission element. Since only one limiting transmission element is determined in the stochastic analysis, slight deviations in the mean and the expected price of the ATC occur.

Example 14S replicates the main conclusions for the four bus system listed above. Tests indicate that the Monte Carlo method to obtain line flow statistics results in execution time that increases linearly with increasing N_{bus}. Tests also indicate that the stochastic analytical method to obtain line flow statistics results in execution time increases linearly with increasing N_{bus}. Fore the relatively simple tests done, the Monte Carlo tests were about 5 times more lengthy than the stochastic analytical tests. It is conjectured that stochastic low flow study methods could be used to find line loading statistics. Table 5.1 shows the computation times of stochastic load flow studies using the Monte Carlo and stochastic analytical method for the 4 and 14 bus system.

Table 5.1 Computation times for the Monte Carlo and stochastic analysis methods in Examples 4S, 4MC and 14S

System	Computation time using Monte Carlo method (s)	Computation time using stochastic analysis method (s)
4-bus	11.51	5.48
14-bus	63.39	8.91

5.2 Conclusions from the line outage Examples 14S2-4, 14S6-13

Examples 14S2-4 and 14S6-13 show that for line outages with lines 2-4 and 6-13 out of service, the stochastic analytical method of solving the ATC results in noticeable differences between the statistical moments of the ATC and expected price of the transfer. The line outage Examples 14S*l*-*m* indicate that a line outage results in the change of ATC price. The observations are that for some outages, ATC price increases, and for others, ATC price decreases. Example 14S2-4 illustrates that if a line is taken out of service, the expected price of the ATC decreases. Table 5.2 shows that for the line outage in Example 14S2-4, the limiting element that is connected between two buses differs between the "all lines in" and the line outage case. Analyzing Table 5.1, it is shown that for each case specified, the limiting element for each ATC transfer differs depending on whenever line 2-4 is in or out of service.

Table 5.2 Differences between the limiting elements in Example 14S2-4

Transfer		All lines in service Limiting Element	
Sending	Receiving	Bus No.	Bus No.
1	13	5	6
2	14	4	7
1	3	1	2
Transfer		Line 2-4 out of service Limiting Element	
Sending	Receiving	Bus No.	Bus No.
1	13	2	5
2	14	2	5
1	3	2	5

Table 5.3 Differences between the limiting elements in Example 14S6-13

Transfer		All lines in service Limiting Element	
Sending	Receiving	Bus No.	Bus No.
3	6	5	6
2	9	4	7
3	13	5	6
Transfer		Line 6-13 out of service Limiting Element	
Sending	Receiving	Bus No.	Bus No.
3	6	5	6
2	9	4	7
3	13	6	12

Since the topology of the system changes with line 2-4 out of service, the line flows will change. In Example 14S2-4, the line flows are higher for the limiting transmission elements (which are listed in Table 5.2). Therefore, the higher line flows can result in a smaller ATC value (e.g., if the higher loading occurs in a limiting transmission system element). For each of the transfers, the mean and expected price also shows a decrease of ATC. This can be expected for most transfers whenever a line outage occurs, but occasionally transfers can yield higher ATC values.

Example 14S6-13 illustrates that for one of the selected line outages (namely 6-13 outaged), the mean and expected price of stochastic ATC can rise. When line 6-13 is taken out of service, the transfer from bus 3 to bus 6 has a higher value for the expected price and mean. Table 5.3 shows that for the line outage in Example 14S6-13, the limiting element that is connected between two buses differs between the transfers of the non line outage case and the line outage case. For transfers between 2-9 and 3-13, the ATC decreases, but in transfer 3-6, the ATC increases.

The increase of ATC for transfer 3-6 can be attributed to a few different causes. Analyzing the topology of the system in Figure 5.3, the system can be divided into two sections separated by transformers connected between buses 5 and 6 and buses 4 and 9. Section 1 from Figure 5.3 carries all the system generation; therefore all the loads supplied in Section 2 are supplied from Section 1 through the two transformers. For the ATC between buses 3 and 6, the limiting element with or without line 6-13 in service is the transformer between buses 5 and 6. Since the limiting element for the transfer does not change, the increase of ATC 3-6 is attributed to a decrease in active power (in line 5-6), a decrease of the distribution factor $(DF_{5-6,3} - DF_{5-6,6})$, or both. Table 5.3 shows the active power and distribution factors for the transformer connected between buses 5 and 6 with line 6-13 in and out of service. In Table 5.4, the change in the distribution factor is small. Therefore it can be concluded that the rise of ATC for the transfer between bus 3 and 6 with a line outage between buses 6 and 13 is due to a decrease of active power in the transformer connected between buses 5 and 6.

Admittedly, the forgoing conclusions are example specific. Figure 5.2 is an attempt to depict generic results in the line outage case.

Table 5.4 Mean of the ATC and distribution factors for line 5-6 when line 6-13 is in and out of service in Example 14S6-13

Line 6-13	Mean of the active power between buses 5 and 6 (MW)	Distribution Factor
In service	45.041	0.66
Out of service	43.0497	0.67

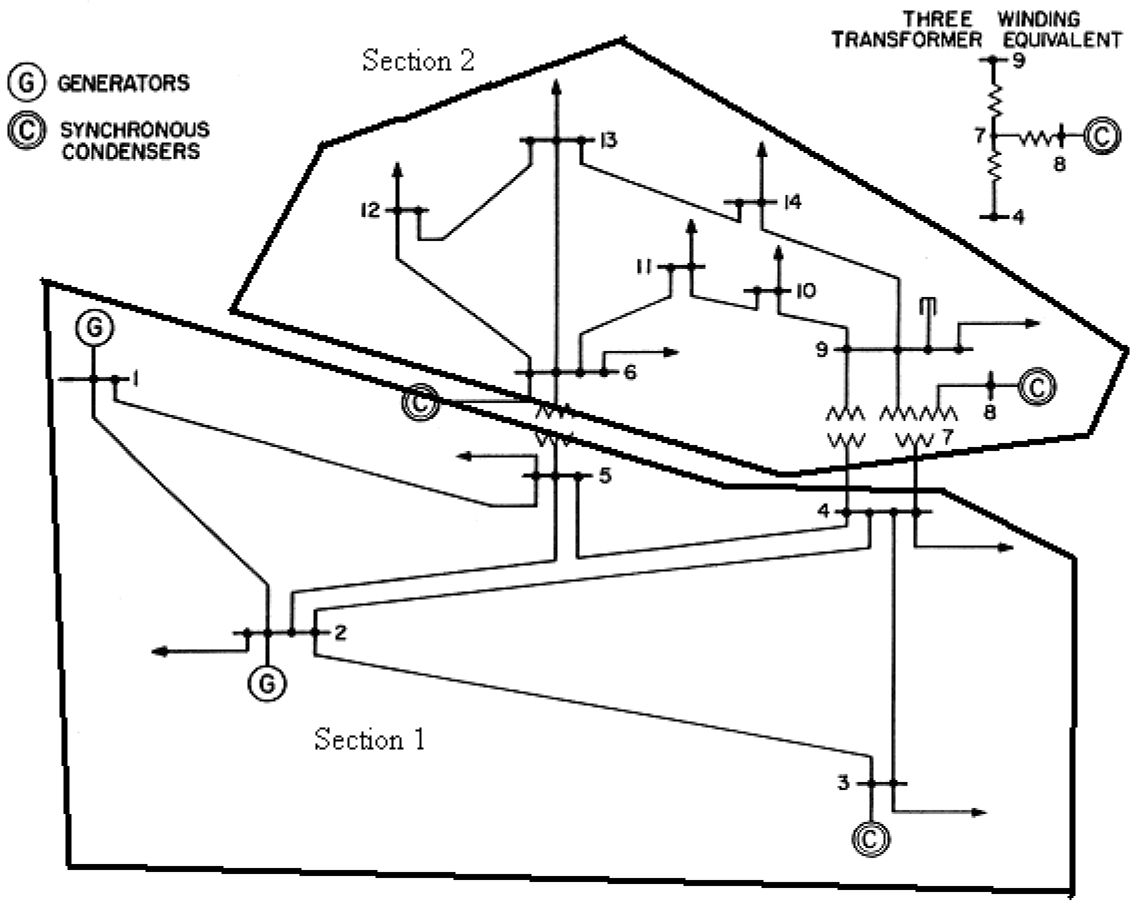

Figure 5.1 The 14 bus test bed

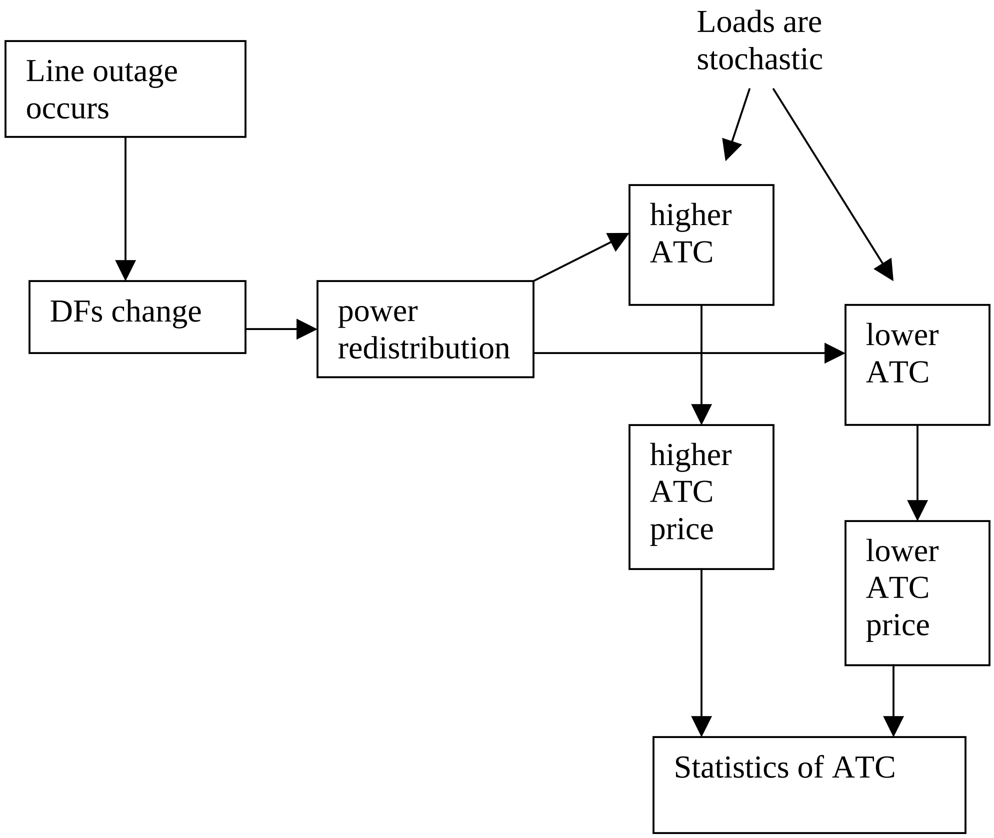

Figure 5.2 General results of a line outage case

6. Conclusions and Future Work

6.1 Original research

In this report, several innovative concepts are offered for ATC marketing utilizing stochastic methods. The main elements of innovative research are:

- Recognition of the ATC problem as stochastic
- Application of stochastic load flow to ATC
- Effects of line outages on the ATC
- Applications of probabilistic methods to marketing.

The secondary contributions are:

- New application of the Gram Charlier series
- The use of Monte Carlo methods to validate Gram Charlier series models
- An illustration of a calculation method to find the expected value of ATC.

6.2 Conclusions

Power marketers have the option to trade power within a network using the ATC. Since line power flows are an important part of the evaluation of ATC, a stochastic power flow problem can be used to find the probabilistic behavior of ATC. If bus load models can be statistically approximated using existing data, line flows can be found probabilistically resulting in multiple statistical moments for each line flow as well as a probability density function for each line.

Evaluating a stochastic ATC problem using a stochastic analysis method, as opposed to a Monte Carlo method, decreases the computational time to evaluate the stochastic ATC. The decrease in computational time can result in a more efficient method of evaluating the stochastic ATC as well as offering power marketers fast determination of the expected price of ATC for any given transfer.

Line outages within a network will generally change the ATC problem, which results in changes in the expected price of the ATC. Line outage cases of the stochastic ATC problem show that the mean and price of the ATC can change significantly. This also applies to transmission expansion of the system. Coupling future power market price with transmission expansion, the stochastic ATC problem has the potential to justify future transmission expansion projects.

6.3 Recommendations

Stochastic ATC can be useful in power marketing applications and it is conjectured that power marketing can benefit from improved statistical analysis of ATC. The evaluation shown in this report is only the basis for many different research topics that can be studied to improve stochastic ATC.

In both Chapters 3 and 4, reactive power and bus voltage magnitude have no limits modeled. In actual power systems, these limits can have an impact on line active power flows, possibly resulting in different probabilistic model of the ATC. It is recommended to model $Q-|V|$ limits in ATC calculation.

In stochastic load flow studies, bus data probabilistic, and load statistics are sometimes assumed to be Gaussian. Bus load statistical moments resulting from stochastic load flow studies are used in the stochastic ACT problem. It is recommended to examine non-Gaussian statistical models, including models that are tabular rather than formula based. There may be benefits attainable from more complex statistical models.

In most studies done using statistical analysis, is it useful to calculate confidence intervals. It is conjectured that power marketing may benefit not only from the expected ATC evaluation, but also from an evaluation of the confidence intervals of the calculation. Confidence interval calculations from the stochastic ATC problem can benefit from further research. For example, a buyer wants to contract 250 MW of the $ATC^{North-South}$. Let the range $240 \le ATC^{North-South} \le 260$ MW be attained with probability (0.9). Then the 90% confidence interval for the $ATC^{North-South}$ is (240,260) *MW*. The contract price could be negotiated depending on the range of the confidence interval. The recommendation is to calculate confidence intervals in this application.

There is a considerable focus in industry on a cost-to-benefit ratio of adding new transmission or upgrading transmission on limiting rights of way. Since the stochastic ATC problem is formulated using limiting transmission elements, the ATC can have an impact on the cost-to-benefit ratio. For example, the ATC transfer in Example 14S from bus 3 to bus 6 has an expected ATC value of 38.73 MW, with the limiting element 5-6. If the rating of the limiting element 5-6 is raised by 25% and the ATC is recalculated, the expected value of ATC rises to 64.99 MW. The recommendation is to incorporate the stochastic ATC problem into a cost-to-benefit ratio of adding new transmission or upgrading transmission on limiting rights of way.

Artificial Neural Networks (ANNs) have been applied to a large number of problems in engineering in general and in power systems. One of the interesting aspects of ANNs is the ability to speed up the calculation of complex expressions and possibly expressions of unknown form. The stochastic ATC can be found using a Monte Carlo approach, which utilizes multiple load flow studies. It is recommended to further analyze the impact that ANN has on the speed of the stochastic ATC problem.

Parallel processing can possibly help speed up the calculation of the stochastic ATC problem. For large systems, running multiple Monte Carlo load flows on one computer can take a long time. Multiple machines running a smaller number of load flow studies can possibly lower the computation time of finding the stochastic ATC. It is recommended to integrate parallel processing for this application.

The work done in this report is purely theoretical and has not been applied to a real system. It is recommended to test the concepts proposed in this report on real systems to fully evaluate the usefulness of those concepts.

After the August 2003 blackout in the Northeast United States, there has been an interest in line outage studies and their effects on bulk power exchange. It is recommended to extend the studies shown in Chapter 4 (i.e., single line outages) to multiple outages. The calculation of expected ATC in the face of multiple transmission outages may be of considerable interest to system operators.

7. References

[1] J. Vorsic, V. Muzek, G. Skerbinek, "Stochastic Load Flow Analysis," Proceedings of the Electrotechnical Conference, Vol. 2, May 22-24, 1991, pp. 1445 – 1448.

[2] B. Borkowska, "Probabilistic Load Flow," IEEE Trans. on Power Apparatus and Systems, PAS -93, No. 3, 1974, pp. 752-759.

[3] J. Dopazo, O. Klitin, A. Sasson, "Stochastic Load Flows," IEEE Trans. on Power Apparatus and Systems, PAS-94, No. 2, 1975, pp. 299 -309.

[4] P. Sauer, "A Generalized Stochastic Power Flow Algorithm," Ph. D. Thesis, Purdue University, W. LaFayette, IN, 1977.

[5] S. Patra, R. B. Misera, "Probabilistic Load Flow Solution Using Method of Moments," IEEE 2nd International on Advances in Power Systems Control, Operation and Management, Dec. 1993, pp. 922-934.

[6] L. Sanabria, T. Dhillion, "Stochastic Power Flow Using Cumulants and Von Mises Functions," International Journal of Electric Power and Energy Systems, Vol. 8, 1986, pp. 47-60.

[7] Federal Energy Regulatory Commission, "Promoting Utility Competition Through Open Access, Non-Discriminatory Transmission Service By Public Utilities: Recovery of Stranded Costs By Public Utilities And Transmitting Utilities," Order No. 888, Final Rule, FERC, April 24, 1996.

[8] Federal Energy Regulatory Commission, "Open Access Same-Time Information System And Standards of Conduct," Order No. 889, Final Rule, FERC, April 24, 1996.

[9] North American Electric Reliability Council, "Available Transfer Capability Definitions and Determination," June 1996.

[10] North American Electric Reliability Council, "Transmission Transfer Capability," May 1995.

[11] M. Gravener, C. Nwankpa, T. Yeoh, "ATC Computational Issues," Proceedings of the 32nd Hawaii International Conference on System Sciences, Jan. 5-8, 1999. pp. 1-6.

[12] G. Ejebe, J. Waight, M. Santos-Nieto, W. Tinney, "Fast Calculation of Linear Available Transfer Capability," IEEE Transactions on Power Systems, Vol. 15, No. 3, Aug. 2000, pp. 1112-1116.

[13] S. Grijalva, P. Sauer, "Reactive Power Considerations in Linear ATC Computation," Proceedings of the 32nd Hawaii International Conference on System Sciences, Jan. 5-8, 1999, pp. 1-11.

[14] G. Ejebe, J. Tong, J. Waight, J. Frame, X. Wang, W. Tinney, "Available Transfer Capability Calculations," IEEE Transactions on Power Systems, Vol. 13, No. 4, November 1998, pp. 1521-1527.

[15] P. Zhang, S. T. Lee, "Probabilistic Load Flow Computation Using the Method of Combined Cumulants and Gram-Charlier Expansion," IEEE Transactions on Power Systems, Vol. 19, No. 1, February 2004, pp. 676-682.

[16] G. T. Heydt, Computer Analysis Methods for Power Systems, Macmillan, New York, 1996.

[17] P. Jorgensen, J. S. Christensen, J. O. Tande, "Probabilistic Load Flow Calculation Using Monte Carlo Techniques for Distribution Network with Wind Turbines," Proceedings on 8th International Conference on Harmonics And Quality of Power, Vol. 2 , 14-16 Oct. 1998, pp.1146–1151.

[18] A.B. Owen, " Monte Carlo Extension of Quasi-Monte Carlo," Winter Simulation Conference Proceedings, Vol. 1 , 13-16 Dec. 1998, pp. 571–577.

[19] C. Bensk, E. Cabau, "New Benchmark for Unreplicated Experimental-Design Analysis," Proceedings on Reliability and Maintainability Symposium, pp. 242–245.

[20] I. Beich, F. Sullivan, "The Importance of Importance Sampling," Computing in Science & Engineering, Vol. 1, March-April 1999, pp. 71–73.

[21] J.T Saraiva, V. Miranda, M.A.C.C Matos," Generation and Load Uncertainties Incorporated in Load Flow Studies," Proceeding on the 6th Mediterranean Electrotechnical Conference, 22-24 May 1991, pp. 1339-1342.

[22] A. Dimitrovski, K. Tomsovic," Boundary Load Flow Solutions," IEEE Transactions on Power Systems, Vol. 19 , Feb. 2004, pp. 348–355.

[23] J. Choi, S. Moon, H. Kim, B. Lee, R. Billinton," Development of the ELDC and Reliability Evaluation of Composite Power System Using Monte Carlo Method," Power Engineering Society Summer Meeting, 2000, Vol. 4 , 16-20 July 2000, pp. 2063–2068.

[24] R. Billinton, L. Gan," A Monte Carlo Simulation Model for Adequacy Assessment of Multi-Area Generating Systems," Third International Conference on Probabilistic Methods Applied to Electric Power Systems, 3-5 Jul 1991, pp. 317–322.

[25] G.M. Huang, P. Yan, "Composite System Adequacy Evaluation Using Sequential

Monte Carlo Simulation for Deregulated Power Systems," 2002 IEEE Power Engineering Society Summer Meeting, Vol. 2 , 21-25 July 2002, pp. 856–861.

[26] H. Cramer, Mathematical Methods of Statistics, Princeton University Press, Princeton, New Jersey, 1954.

[27] M. G. Kendall and A. Stuart, The Advanced Theory of Statistics, Vol. 1, Second Edition, Hafner Publishing Company, New York, N. Y., 1963.

[28] N. R. Goodman, "Statistical Analysis Based on a Certain Multivariate Complex Gaussian Distribution (An Introduction)," Annals of Mathematical Statistics, Vol. 34, 1963, p. 152.

[29] W. Feller, An Introduction to Probability Theory and Its Applications, Vol. II, Second Edition, John Wiley and Sons, Inc. New York, NY, 1971.

[30] A. Papoulis, Probability, Random Variables, and Stochastic Processes, McGraw Hill, New York, NY, 1965.

[31] K. S. Miller, Multidimensional Gaussian Distributions, Wiley and Sons, New York, NY, 1964.

[32] R. V. Hogg, A. T. Cragin, Introduction to Mathematical Statistics, Third Edition, Macmillan Publishing Company, Inc., New York, New York, 1970.

[33] C. U. L., Charlier, "Application de la theorie des probabilities a l'astronomie," Vol. II, Part IV of the Traite edited by Borel, Paris, France, 1931.

[34] F. Y. Edgeworth, "The Law of Error," Transactions Cambridge Philosophical Society, Vol. 20, 1905-1908, p. 45.

[35] A. L. Bowley, "F. Y. Edgeworth's Contributions to Mathematical Statistics," Reprint by Augustus M. Kelly Publishers, Clifton, New Jersey, 1972.

[36] P. J. Bickel, "Edgeworth Expansions in Nonparametric Statistics," The Annals of Statistics, Vol. 2, No. 1, 1974, pp. 1-20.

[37] R. Christie, "Power System Test Archive," 1999.
http://www.ee.washington.edu/research/pstca/pf14/pg_tca14bus.htm

A. The Gram Charlier Series Subroutine

```
%%%%%%%%%%%%%%%%%%%%%%%%%
% The Gram Charlier Series Subroutine   %
%        Jonathan W. Stahlhut                  %
%%%%%%%%%%%%%%%%%%%%%%%%%
function [f,y] = GCS(Moments_ATC)

x = -10:.1:10; %initial range of x for the standard Gram Charlier Series(GCS)
n = size(Moments_ATC,2); %how many moments of the ATC are used to approximate
the GCS, for all examples will equal 5

% Subroutine to find the first n Hermite Polynomials
b = 1;
for a = 1:n
   if b == 1
      t = (a+1)/2;
      b = 0;
   else
      t = (a+2)/2;
      b = 1;
   end
   H(a,:) = X.^a;
   kk = 1;
   for k = 2:t
      kk = kk*(2*k-3);
      H(a,:) = H(a,:) + (-1)^(k-1)*(X.^(a-2*(k-1)))*(factorial(a)/(factorial(2*(k-
1))*(factorial(a-2*(k-1)))))*kk;
   end
end

% the first five GCS constants cj
c = [ 0
Moments_ATC(2)/Moments(ATC(2))-1
Moments_ATC(3)/Moments_ATC(2)^(3/2)
Moments_ATC(4)/Moments_ATC(2)^(4/2)-6*Moments_ATC(2)/Moments_ATC(2)+3
Moments_ATC(5)/Moments_ATC(2)^(5/2)-
10*Moments_ATC(3)/Moments_ATC(2)^(3/2)
];

%Gaussian Function
G = (1/(sqrt(2*pi)))*exp((-X.^2)/2);
```

```
%iterative procedue to find the standard GCS with the Hermite Polynomials and the GCS
constants
GCS_CH = 1;
for g = 1:5
GCS_CH = GCS_CH + (1/factorial(g))*c(g)*H(g,:);
end

% the standard Gram Charlier Series
GCS = G.*(GCS_CH);

% the GCS using the real values for the ATC
f = GCS./sqrt(Moments_ATC(2));
y = sqrt(Moments_ATC(2)).*X + Moments_ATC(1);
```

B. Additional Line Outage Examples

B.1 Formulation of examples to illustrate line outage effects

For the line outage cases in this appendix, the 14 bus test bed is used as described in Chapter 3. The first example, denominated as Example 14S10-11, evaluates the stochastic ATC of the 14 bus system with the line between bus 10 and bus 11 being a scheduled outage. The three different ATC transfers with line 10-11 in and out of service compared are shown in Table B.1.

Table B.1 Lines used for the determination of the stochastic ATC in Example 14S10-11

Line	
From	To
6	9
2	9
3	13

The second example, denominated as Example 14S1-5, evaluates the stochastic ATC of the 14 bus system with the line between bus 1 and bus 5 being a scheduled outage. The three different ATC transfers with line 1-5 in and out of service compared are shown in Table B.2.

Table B.2 Lines used for the determination of the stochastic ATC in Example 14S1-5

Line	
From	To
1	5
2	5
2	13

For both examples, the resulting probability density function plots, first five statistical raw moments and expected prices using the price function

$$P_{ATC}^{14}(x) = 100x \quad \$/MW \tag{B.1}$$

are shown. The differences between the expected value of the transfers and the expected price using (B.1) of the transfer are also shown. The differences indicate the impact that line outages have on the values of stochastic ATC.

B.2 Example 14S10-11

This example illustrates the stochastic ATC for a scheduled line outage for the line from bus 10 to bus 11. The algorithm used for Example 14S10-11 is the same as the illustration in Figure 3.4. The resulting statistical moments and expected price with the line from bus 10 to bus 11 out of service as well as the line in service are shown in Table B.3. From Table B.3 it can be seen that having a line out service will affect the ATC significantly for transfers that involve line 10-11. The differences between the expected value of the ATC (i.e. the first moment) and the expected prices, $E[P_{ATC}^{14}]$ and $E[P_{ATC10-11}^{14}]$, of the ATC are shown in Table B.4. Figures B.1-6 show the probability density functions of the ATC with line 10-11 in and out of service.

Table B.3 Statistical moments and expected price for all transmission lines online and line 10-11 out of service in Example 14S10-11

Transfer		Statistical moments of the ATC with line 10-11 in service*					$E[P_{ATC}^{14}]$ ($)
Sending	Receiving	$m_{ATC}^{(1)}$	$m_{ATC}^{(2)}$	$m_{ATC}^{(3)}$	$m_{ATC}^{(4)}$	$m_{ATC}^{(5)}$	
3	6	38.3338	7.45E+03	-1.85E+05	1.62E+08	-3.78E+09	3.83E+03
2	9	46.0474	1.28E+04	-2.52E+05	5.14E+08	-2.82E+10	4.60E+03
3	13	41.9471	6.80E+03	-2.91E+05	1.23E+08	-2.28E+09	4.19E+03
Transfer		Statistical moments of the ATC with line 10-11 out of service*					$E[P_{ATC10-11}^{14}]$ ($)
Sending	Receiving	$m_{ATC10-11}^{(1)}$	$m_{ATC10-11}^{(2)}$	$m_{ATC10-11}^{(3)}$	$m_{ATC10-11}^{(4)}$	$m_{ATC10-11}^{(5)}$	
3	6	40.8527	7.85E+03	3.07E+05	1.75E+08	1.70E+09	4.09E+03
2	9	40.3125	1.22E+04	2.56E+05	4.82E+08	-1.90E+10	4.03E+03
3	13	26.211	2.00E+04	-1.11E+06	1.65E+09	-3.05E+11	2.62E+03

*ATC in MW

Table B.4 Differences of the mean ATC calculated with line 10-11 in service and out of service and the expected price of the ATC calculated with line 10-11 in service and out of service in Example 14S10-11

Transfer		$m_{ATC10-11}^{(1)} - m_{ATC}^{(1)}$ (MW)	$E[P_{ATC10-11}^{14}] - E[P_{ATC}^{14}]$ ($)
Sending	Receiving		
3	6	2.5189	255.3
2	9	-5.7349	-568.8
3	13	-15.7361	-1568.9

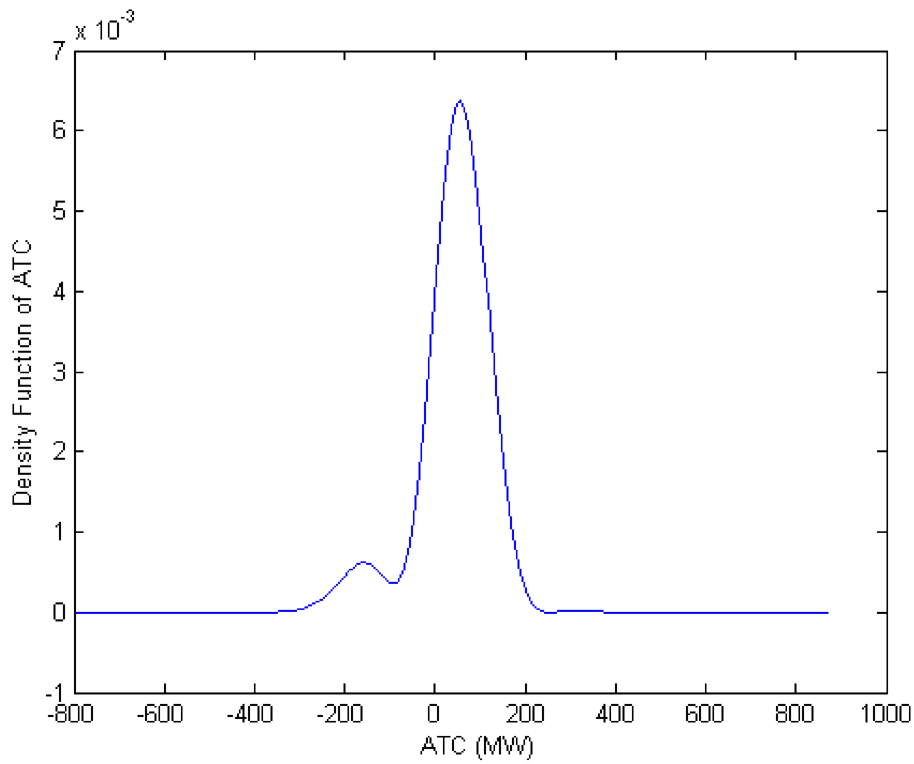

Figure B.1 PDF of the ATC between buses 3-13 in Example14S10-11 with line 10-11 in service

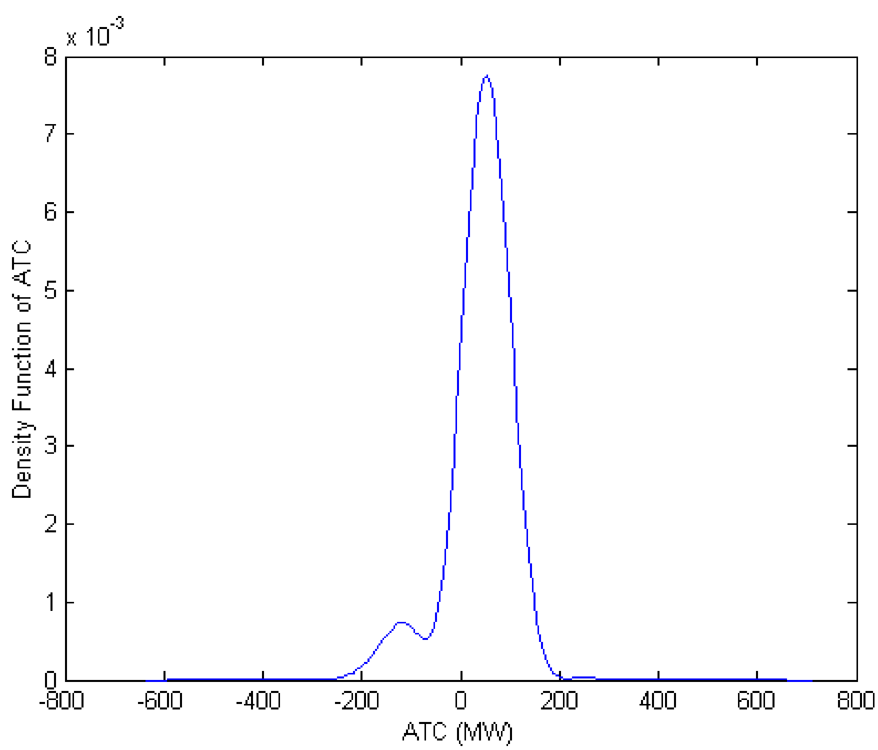

Figure B.2 PDF of the ATC in line 3-13 in Example14S10-11 with line 10-11 out of service

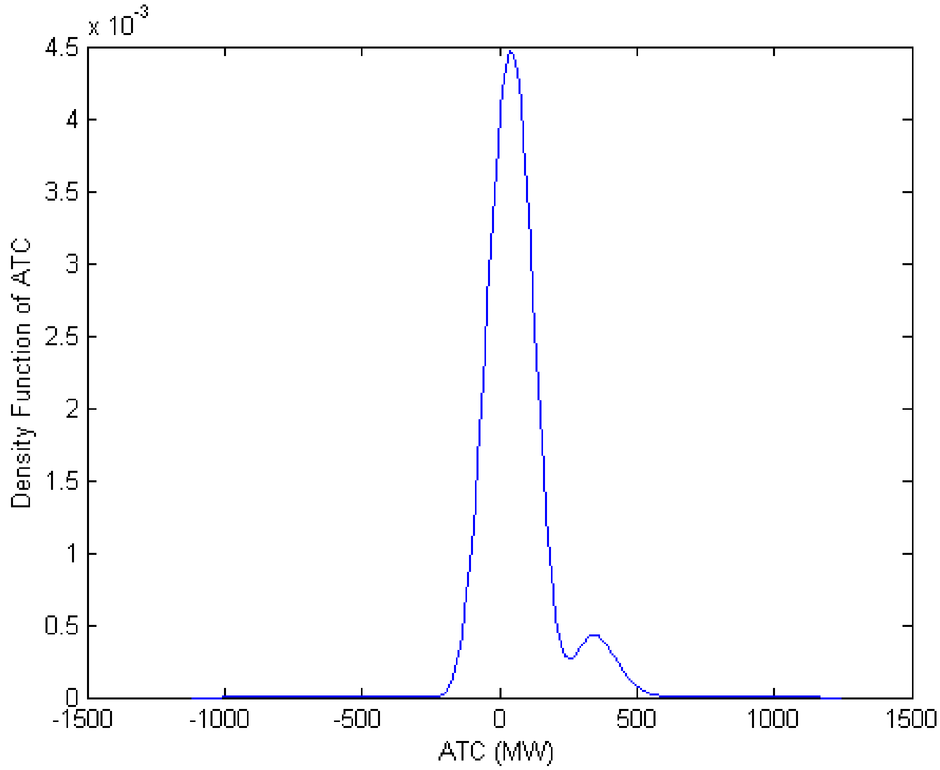

Figure B.3 PDF of the ATC in line 6-9 in Example14S10-11 with line 10-11 in service

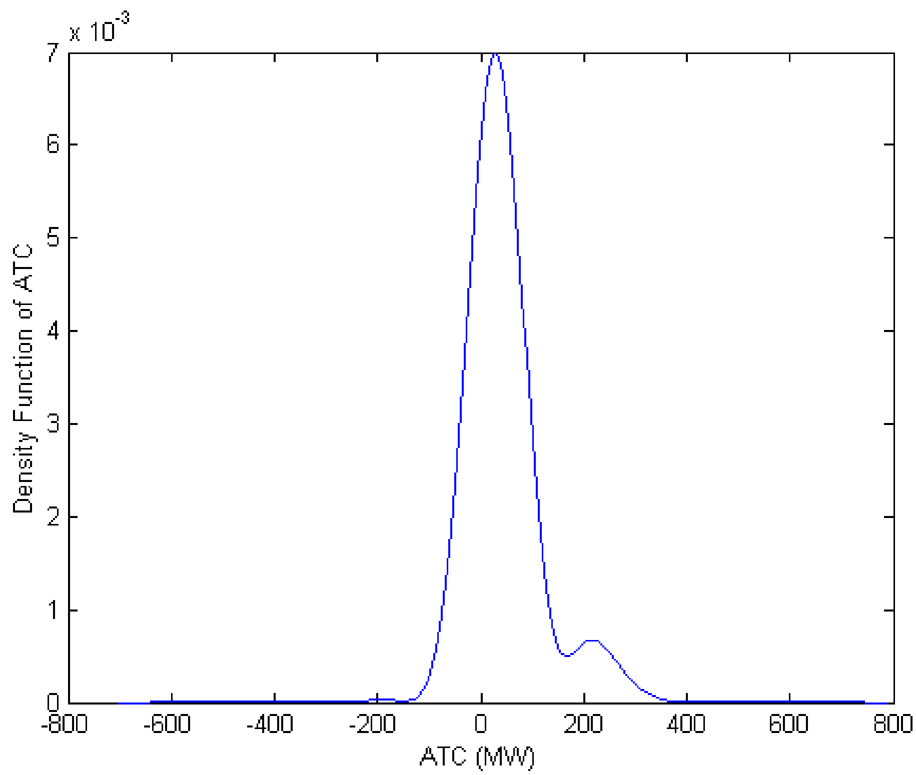

Figure B.4 PDF of the ATC in line 6-9 in Example14S10-11 with line 10-11 out of service

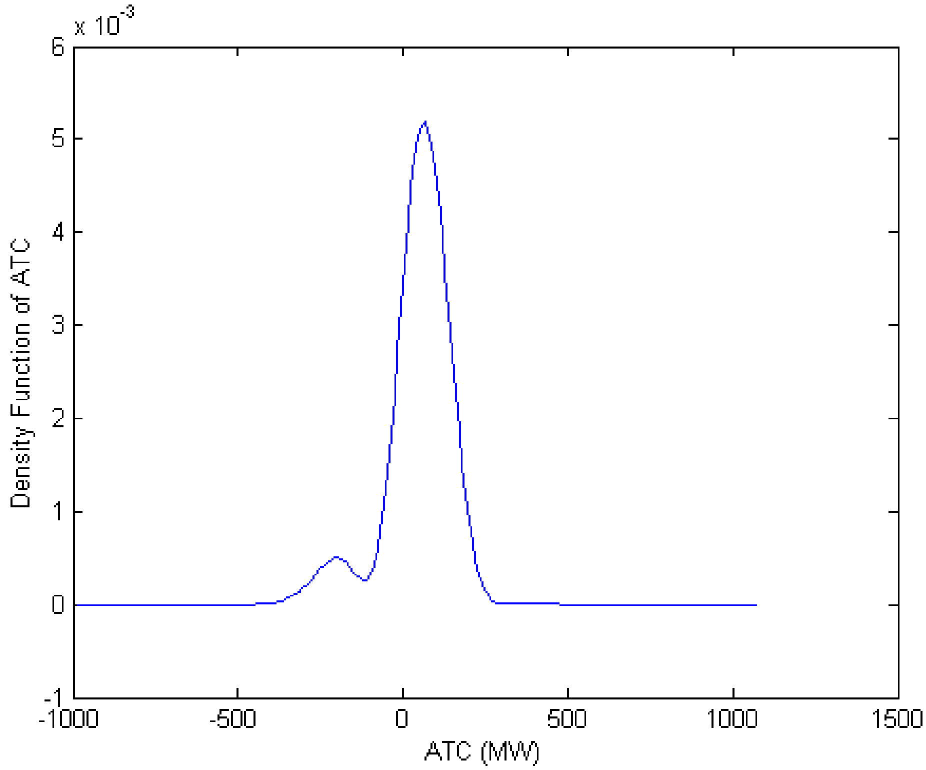

Figure B.5 PDF of the ATC in line 2-9 in Example14S10-11 with line 10-11 in service

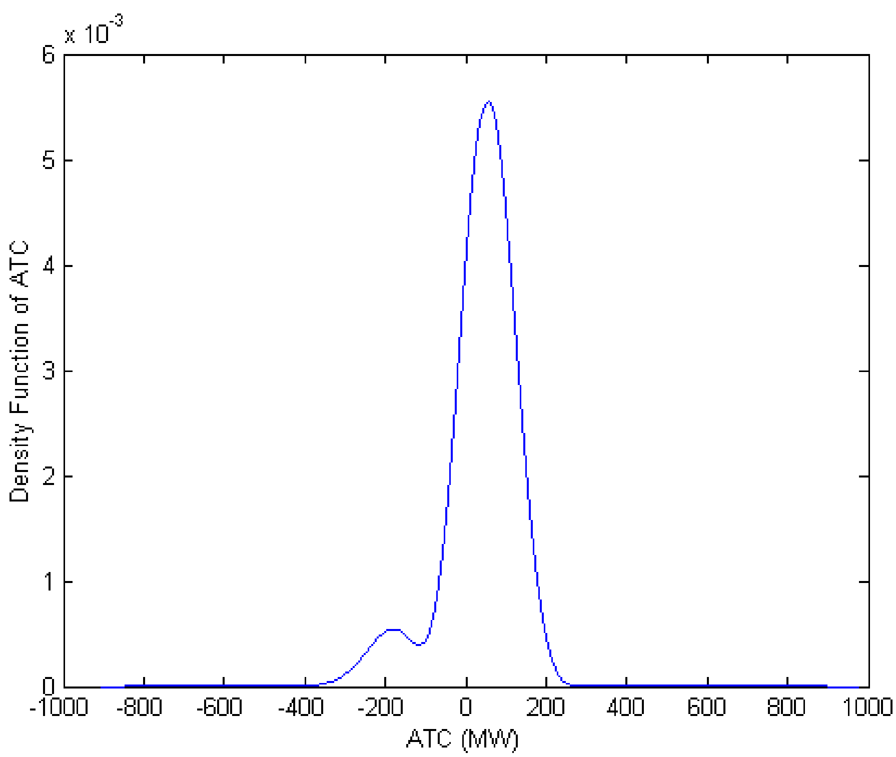

Figure B.6 PDF of the ATC in line 2-9 in Example14S10-11 with line 10-11 out of service

B.3 Example 14S1-5

This example illustrates the ATC for a scheduled line outage for the line from bus 1 to bus 5. The algorithm used for Example 14S1-5 is the same as the illustration in Figure 3.4. The resulting statistical moments and expected price with the line from bus 1 to bus 5 out of service as well as the line in service are shown in Table B.5. From Table B.6 it can be seen that having a line out service will affect the ATC for transfers that involve line 1-5. . The differences between the expected value of the ATC (i.e. the first moment) and the expected prices, $E[P^{14}_{ATC}]$ and $E[P^{14}_{ATC1-5}]$, of the ATC are shown in Table B.6. Figures B.7-12 show the probability density functions of the ATC with line 1-5 in and out of service.

Table B.5 Statistical moments and expected price for all transmission lines online and line 1-5 out of service in Example 14S1-5

Transfer		Statistical moments of the ATC with line 1-5 in service*					$E[P^{14}_{ATC}]$ ($)
Sending	Receiving	$m^{(1)}_{ATC}$	$m^{(2)}_{ATC}$	$m^{(3)}_{ATC}$	$m^{(4)}_{ATC}$	$m^{(5)}_{ATC}$	
1	5	63.5553	1.17E+04	9.63E+05	3.18E+08	2.54E+10	6.36E+03
2	5	94.9556	4.00E+04	3.14E+06	4.42E+09	-5.03E+10	9.50E+03
2	13	41.3538	8.52E+03	2.47E+05	2.11E+08	-4.20E+09	4.14E+03
Transfer		Statistical moments of the ATC with line 1-5 out of service*					$E[P^{14}_{ATC1-5}]$ ($)
Sending	Receiving	$m^{(1)}_{ATC1-5}$	$m^{(2)}_{ATC1-5}$	$m^{(3)}_{ATC1-5}$	$m^{(4)}_{ATC1-5}$	$m^{(5)}_{ATC1-5}$	
1	5	15.96	4.70E+04	-2.12E+07	1.71E+10	-1.25E+13	1.60E+03
2	5	15.9725	4.23E+04	-1.82E+07	1.40E+10	-9.74E+12	1.60E+03
2	13	14.2658	3.52E+04	-1.41E+07	9.92E+09	-6.36E+12	1.43E+03

*ATC in MW

Table B.6 Differences of the mean ATC calculated with line 1-5 in service and out of service and the expected price of the ATC calculated with line 1-5 in service and out of service in Example 14S1-5

Transfer		$m^{(1)}_{ATC1-5}$ - $m^{(1)}_{ATC}$ (MW)	$E[P^{14}_{ATC1-5}]$ - $E[P^{14}_{ATC}]$ ($)
Sending	Receiving		
1	5	-47.5953	-4759.5
2	5	-78.9831	-7898.3
2	13	-27.088	-2708.8

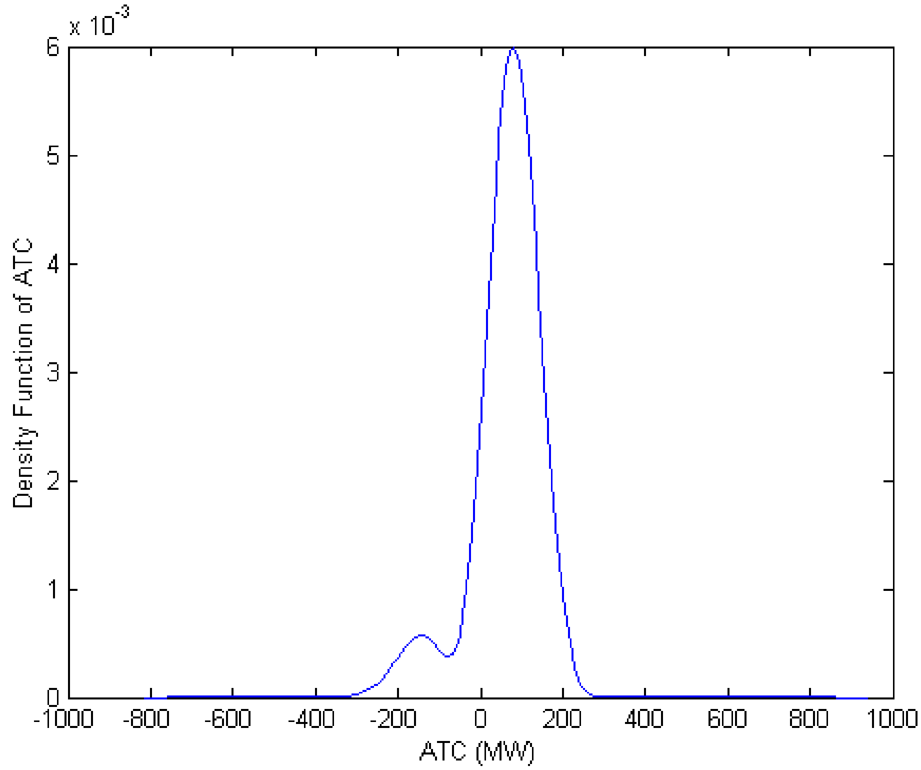

Figure B.7 PDF of the ATC in line 1-5 in Example14S1-15 with line 1-5 in service

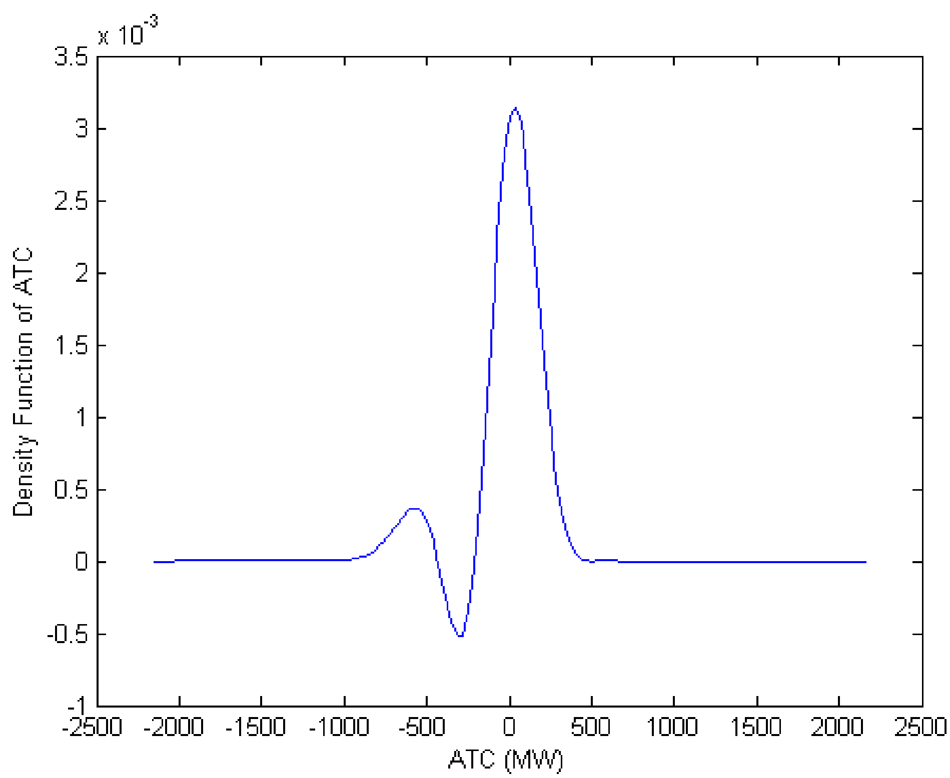

Figure B.8 PDF of the ATC in line 1-5 in Example14S1-15 with line 1-5 out of service

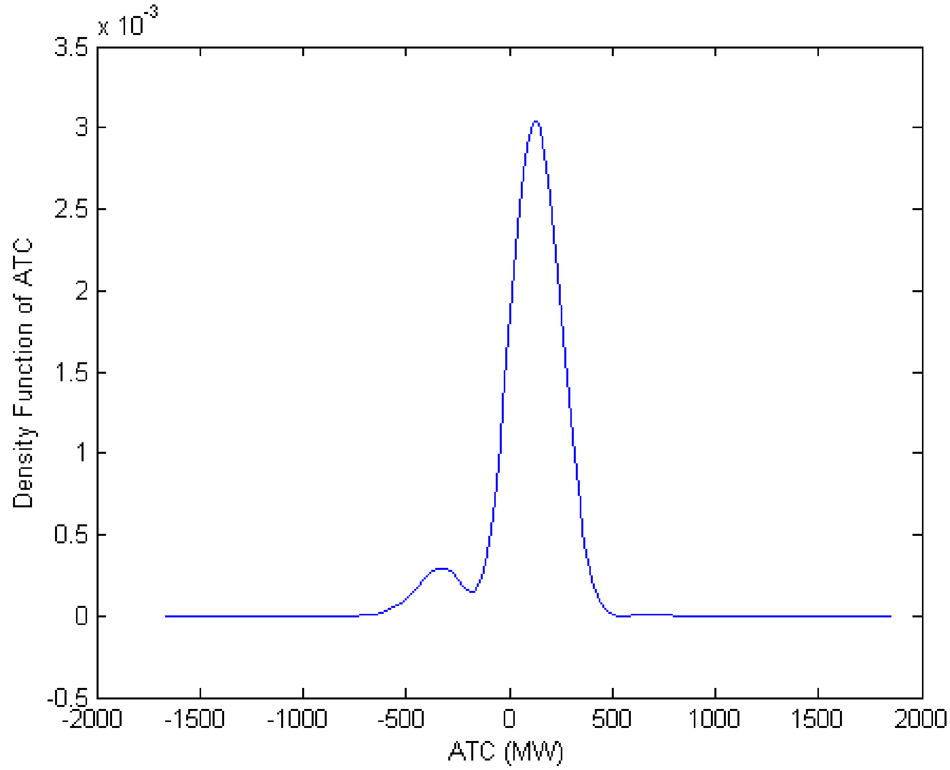

Figure B.9 PDF of the ATC in line 2-5 in Example14S1-15 with line 1-5 in service

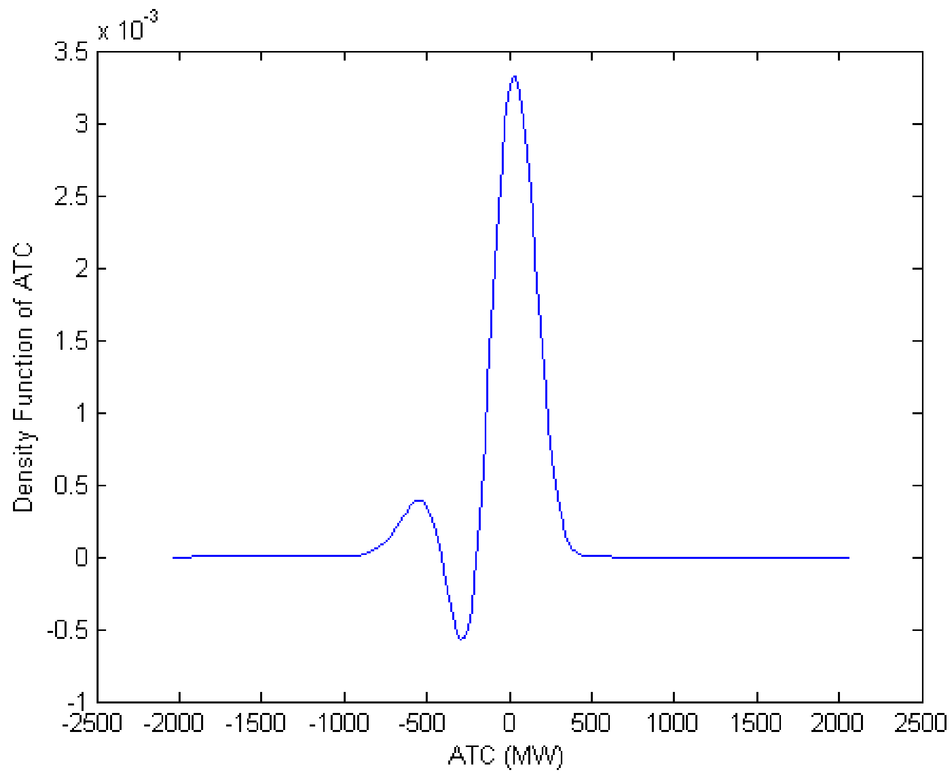

Figure B.10 PDF of the ATC in line 2-5 in Example14S1-15 with line 1-5 out of service

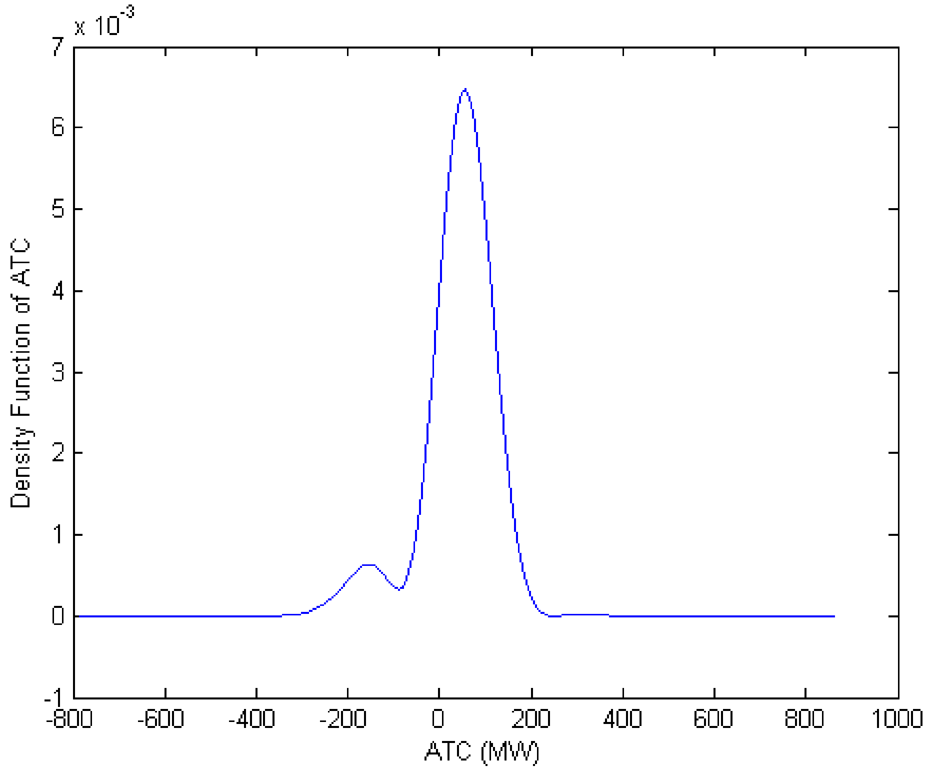

Figure B.11 PDF of the ATC in line 2-13 in Example14S1-15 with line 1-5 in service

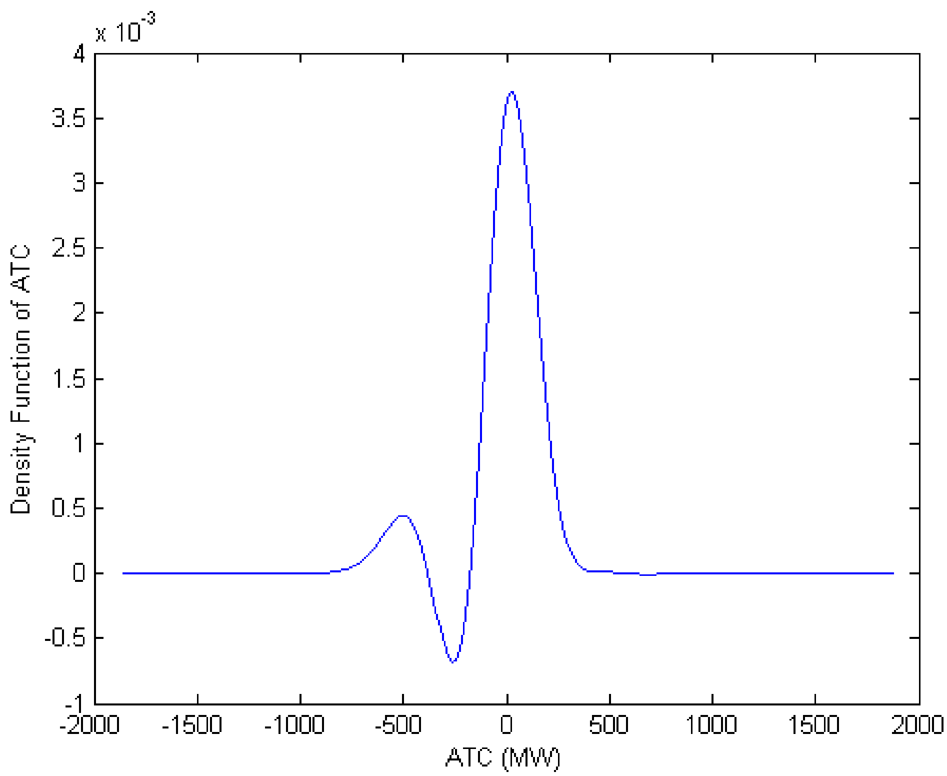

Figure B.12 PDF of the ATC in line 2-13 in Example14S1-15 with line 1-5 out of service

C. List of the Examples

This appendix shows a list and explanation for all examples in this report. Tables C.1 and C.2 show the list of examples and the conditions for the report. In Table C.1, for each example, an "x" in the corresponding box shows which analysis and test beds are used. In Table C.2, for each example, an "x' in the corresponding box are the ATC transfers shown in that example. For each example, the corresponding first five statistical moments, expected price and PDF plots are shown.

Table C.1 The analysis, test bed, and line outage case possibility for all examples in the report

Example	Stochastic analysis	Monte Carlo analysis	4 bus test bed	14 bus test bed	Line outage
4MC		x	x		
4S	x		x		
14S	x			x	
14S2-4	x			x	x
14S6-13	x			x	x
14S10-11	x			x	x
14S1-5	x			x	x

Table C.2 Transfers that are shown for each example in the report

ATC Transfers Shown in for each example											
Example	All possible transfers	1-13	2-14	1-3	3-6	2-9	3-13	6-9	1-5	2-5	2-13
4MC	x										
4S	x										
14S		x	x	x	x	x	x				
14S2-4			x		x	x					
14S6-13		x		x			x				
14S10-11						x	x	x			
14S1-5									x	x	x

Example 4MC illustrates the Monte Carlo analysis method of finding the stochastic ATC. The conditions for Example 4MC are

- all system elements in service
- all possible ATC transfers shown.

Example 4MC is used in comparison with Example 4S to validate the stochastic analysis method to find the stochastic ATC.

Example 4S illustrates the stochastic analysis method of finding the stochastic ATC. The conditions for Example 4S are

- all system elements in service
- all possible ATC transfers shown.

Example 4S is used in comparison with Example 4MC to validate the stochastic analysis method to find the stochastic ATC.

Example 14S illustrates the stochastic analysis method of finding the stochastic ATC for a 14 bus system. The conditions for Example 14S are

- all system elements in service
- the ATC transfers illustrated in Table C.2

Example 14S2-4 illustrated the stochastic analysis method of finding the stochastic ATC for the 14 bus system when the transmission line from bus 2 to bus 4 is out of service. The conditions for Example 14S2-4 are

- all system elements in service except for the element connected between buses 2 and 4
- the ATC transfers illustrated in Table C.2

The purpose for this example is to illustrate how the ATC is affected whenever a system element is taken out of service

Example 14S6-13 illustrated the stochastic analysis method of finding the stochastic ATC for the 14 bus system when the transmission line from bus 6 to bus 13 is out of service. The conditions for Example 14S6-13 are

- all system elements in service except for the element connected between buses 6 and 13
- the ATC transfers illustrated in Table C.2

The purpose for this example is to illustrate how the ATC is affected whenever a system element is taken out of service

Example 14S10-11 illustrated the stochastic analysis method of finding the stochastic ATC for the 14 bus system when the transmission line from bus 10 to bus 11 is out of service. The conditions for Example 14S10-11 are

- all system elements in service except for the element connected between buses 10 and 11
- the ATC transfers illustrated in Table C.2

The purpose for this example is to illustrate how the ATC is affected whenever a system element is taken out of service

Example 14S1-5 illustrated the stochastic analysis method of finding the stochastic ATC for the 14 bus system when the transmission line from bus 1 to bus 5 is out of service. The conditions for Example 14S1-5 are

- all system elements in service except for the element connected between buses 1 and 5

- the ATC transfers illustrated in Table C.2

The purpose for this example is to illustrate how the ATC is affected whenever a system element is taken out of service