Optimal Bidding Strategies in Electricity Markets*

R. Rajaraman
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(*) New PSERC report co-authored with Prof. Fernando Alvarado
slated for release in early 2005

Opening Remarks

- Bidding by market participants is the cornerstone of any market

- Often assumed that the optimal bidding strategy in uniform price auctions is to bid generator's incremental costs
    - Only valid under restrictive assumptions
    - Optimal bidding strategy depends nontrivially on many factors

Five Significant Effects on Generator's Bidding Behavior

- Multiple markets, e.g.,
  - Energy and reserve markets
- Auction design rules, e.g.,
  - Pay-as-bid versus uniform price
  - Declining cost curves allowed?
  - Sequential versus simultaneous
- Discontinuity/non-convexity of generator costs
- Complex operational restrictions
- Effect of price uncertainty
  - Price volatility (not just its mean)
  - Price correlation over time and over markets

Generator Bidding Problem

- Given:
  - ➢ Exogenous *uncertain* future prices
  - ➢ Auction design rules
  - ➢ Generator operational restrictions
  - ➢ Generator operational costs

- How should a generator bid to maximize *expected* profits?

Talk Outline

- Examples
- Optimal bidding strategy theory
  - Rajaraman & Alvarado, “Optimal Bidding Strategy in Electricity Markets Under Uncertain Energy and Reserve Prices”, PSERC Report 03-05, April 2003
  - Note: A new version is due early 2005
- Conclusions

Example 1: Uniform-price vs. Pay-as-Bid

- Assume a single energy market
- Assume a single time period
- Assume a single-stair bid
- ExampleGen has incremental costs of $35/MWh (and no other costs or constraints) and forecasts with certainty that energy market will clear at $42/MWh
- [Uniform price auction] If every winning generator gets paid the **highest winning bid**, then ExampleGen's optimal bidding strategy is to bid $35/MWh
- [Pay-as-bid auction] If every winning generator gets paid **its own bid**, then ExampleGen's optimal bidding strategy is to bid $42/MWh

Example 1A

- Same assumptions as before excepting that:
  - ExampleGen forecasts that the highest winning bid is either $37/MWh, $42/MWh or $47/MWh with 1/3 probability each
- [Uniform price auction] ExampleGen's optimal bidding strategy is to bid $35/MWh
- [Pay-as-bid auction] ExampleGen's optimal bidding strategy is to bid $42/MWh
  - In effect, withholding output even when "market price" is $37/MWh

Example 1B

- Same assumptions as before excepting that:
  - ExampleGen forecasts that the highest winning bid is either $32/MWh, $42/MWh or $52/MWh with 33% probability each
- [Uniform price auction] ExampleGen’s optimal bidding strategy is to bid $35/MWh
- [Pay-as-bid auction] ExampleGen’s optimal bidding strategy is to bid $52/MWh
  - Recognize optionality of not producing when market price goes below incremental costs
  - But, in effect, withholding output when the “market price” is $42/MWh

Energy-Limited Generators

- Limited energy (MWh) over multiple periods

- If you have *perfect* energy price forecast, then
    - The highest valued period gets the most energy, followed by the second highest period, etc.

- If you have an *uncertain* energy price forecast
    - It depends on how you model the uncertainty!
    - If prices are high today, you *may* withhold under certain price scenarios ("optionality" feature)

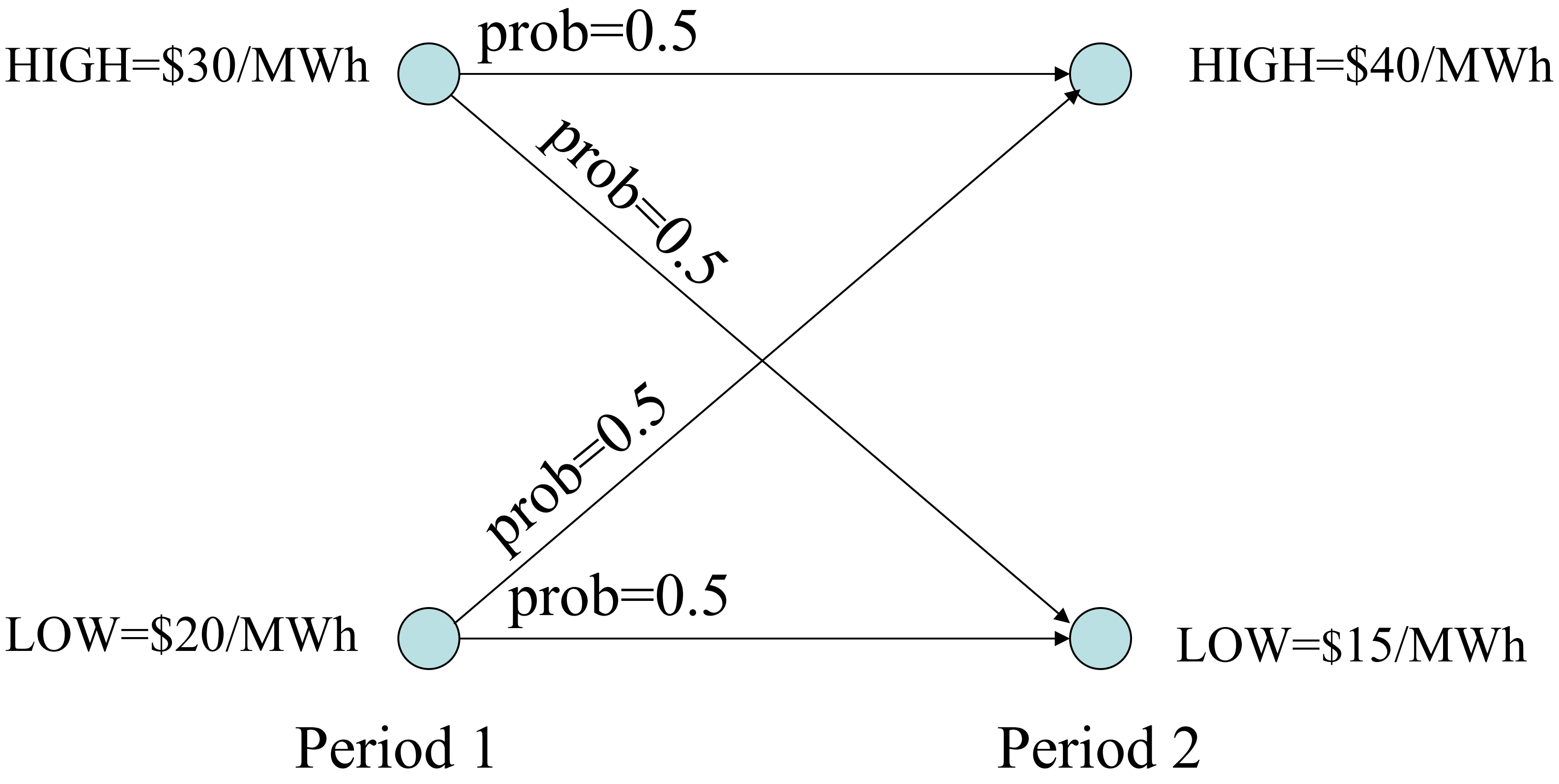
Example 2

- Two periods with a single energy market.
- The hydro reservoir has 100 MWh of energy.
- No inflow into the reservoir
- The cost of producing energy is zero.
- At the end of two periods, the generator must have no energy left.
- The minimum and maximum hourly power limits are 0 and 200 MW respectively.
- The generator is a price-taker.

Example 2: Assumptions

- The auction is a single-price auction, i.e., all winners are paid the clearing price
- MW bids are restricted to be non-decreasing functions of price in $/MWh
- Period 1 auction is held; after it clears, period 2 bids are accepted and cleared

Example 2: Uncorrelated Price Forecast

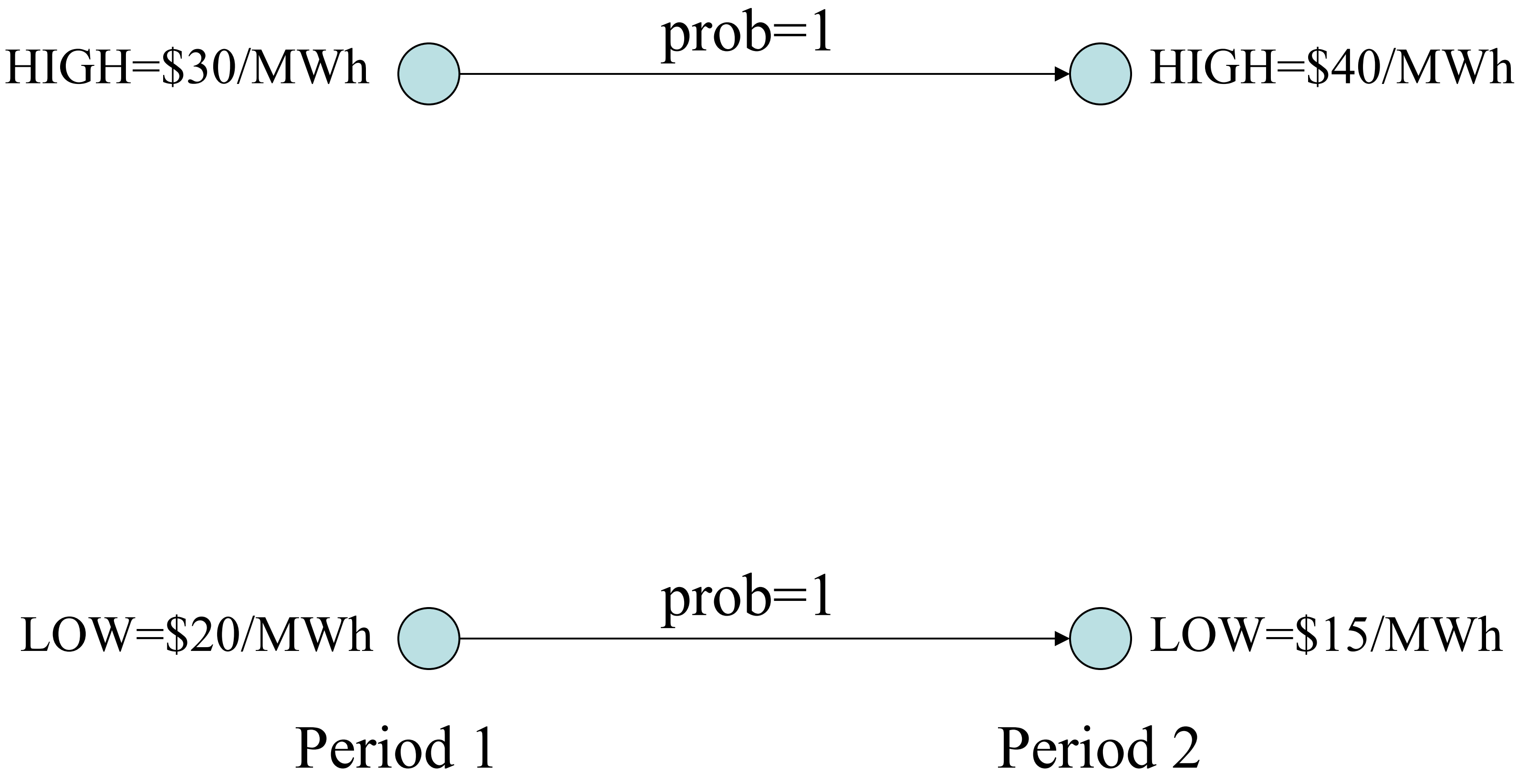


Optimal Bidding Strategy: Uncorrelated Prices

- In period 1, bid all 100 MW between \$20/MWh and \$30/MWh
- In period 2, bid the remaining energy (if any is left) at less than \$15/MWh

Expected profit for this strategy is \$2850

Example 2A:
Correlated Price Forecast



HIGH=$30/MWh —prob=1→ HIGH=$40/MWh

LOW=$20/MWh —prob=1→ LOW=$15/MWh

Period 1

Period 2

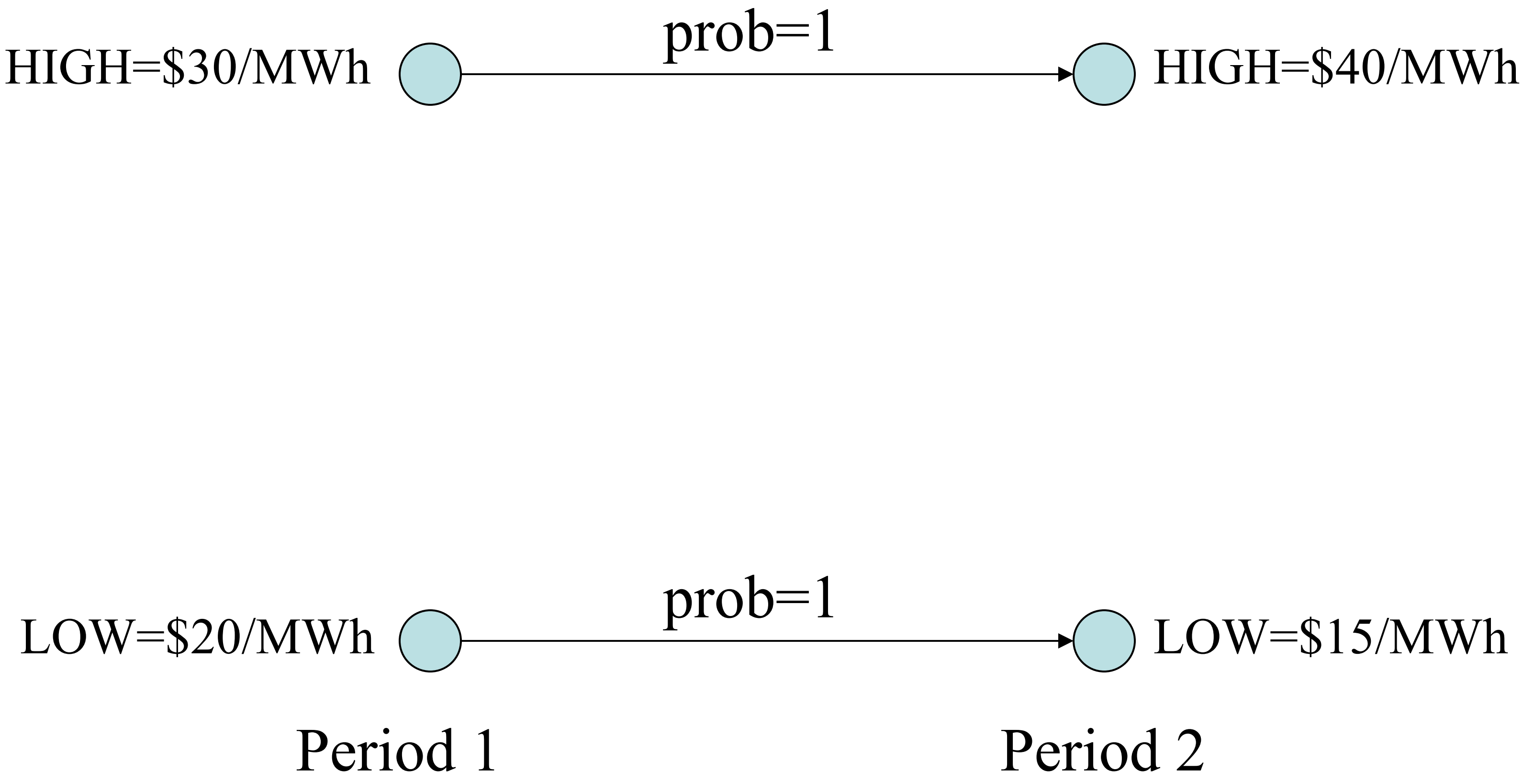
Optimal Bidding Strategy: Correlated Prices

- In period 1, withhold all 100 MW
  - Equivalently, bid 100 MW at a price higher than $30/MWh
- In period 2, bid 100 MW at less than $15/MWh

Expected profit for this strategy is $2750

Example 2B:
Correlated Prices, No Bid Restriction

Decreasing MW versus price bids are allowed



HIGH=$30/MWh

prob=1

HIGH=$40/MWh

LOW=$20/MWh

prob=1

LOW=$15/MWh

Period 1

Period 2

Optimal Bidding Strategy: Correlated Prices, No Bid Restrictions

- In period 1, bid 100 MW if price is LOW but withhold everything if price is HIGH
  - That is, offer 100 MW at $20 but 0 MW at $30
- In period 2, bid the remaining MW at less than $15/MWh

Profit for this strategy is $3000

Energy-Limited Generators: Energy Plus Reserve Markets

- Limited energy over multiple periods
- If you have uncertain energy *and reserve* price forecasts, then optimal bidding strategy depends on:
  - How energy and reserve markets are cleared
  - How you model uncertainty
  - Are prices among periods correlated?
  - Are energy and reserves prices correlated?

Example 3: Multiple Markets, Sequential Auction

- There are two markets, energy and reserves
- The auctions are sequential:
    - Energy market is first cleared
    - Reserve market is cleared next
    - Each auction is uniform-price
    - MW vs. price bid must be non-decreasing

Example 3: Generator Parameters

- Generator's incremental cost is $30/MWh
- Generator can offer up to 100 MWh energy and up to 40 MW/h reserves,
    - ➢ But energy + reserves ≤ 100
- Energy price can be either $40 or $35 with equal probability
- Reserve price can be either $12 or $4 with equal probability

Optimal Bidding Strategy: Energy and Reserve Prices Correlated

- Energy and reserve prices are perfectly correlated:
  - $40 Energy price => $12 reserve price
  - $35 Energy price =>$4 reserve price
- Optimal strategy is:
  - Energy Market
    - Bid 60 MW of energy below $35
  - Reserve Market:
    - Bid 40 MW of reserves below $4

Expected profit from this strategy is $770

Optimal Bidding Strategy: Energy and Reserve Prices Uncorrelated

- Whether the energy price is \$40 or \$35, the reserve price is equally likely to be either \$12 or \$4
- Optimal strategy is:
  - Energy Market:
    - Bid a 60 MW "stair" below \$35
    - Bid a 40 MW "stair" between \$35 and \$40
  - Reserve Market:
    - Bid amount uncleared in the energy market below below \$4/MW/h

Expected profit from this strategy is \$810

Example 4: Effect of Inter-temporal Constraints

- Single energy market
- Two periods
- Energy price forecast:
  - Period 1 price is either \$33 or \$45 with equal probability
  - Period 2 price is correlated with period 1 price:
    - Period 1 price is \$45 => Period 2 price is either \$65 or \$5 with equal probability
    - Period 1 price is \$33 => Period 2 price is either \$65 or \$5 with probability 0.9 and 0.1 respectively

Example 4: Generator Parameters

- Generator has incremental costs of \$30/MWh and no other costs
- Generator minimum = 90 MW
- Generator maximum = 100 MW
- Generator is off-line currently
    - Once online, generator has to stay online for 2 consecutive periods

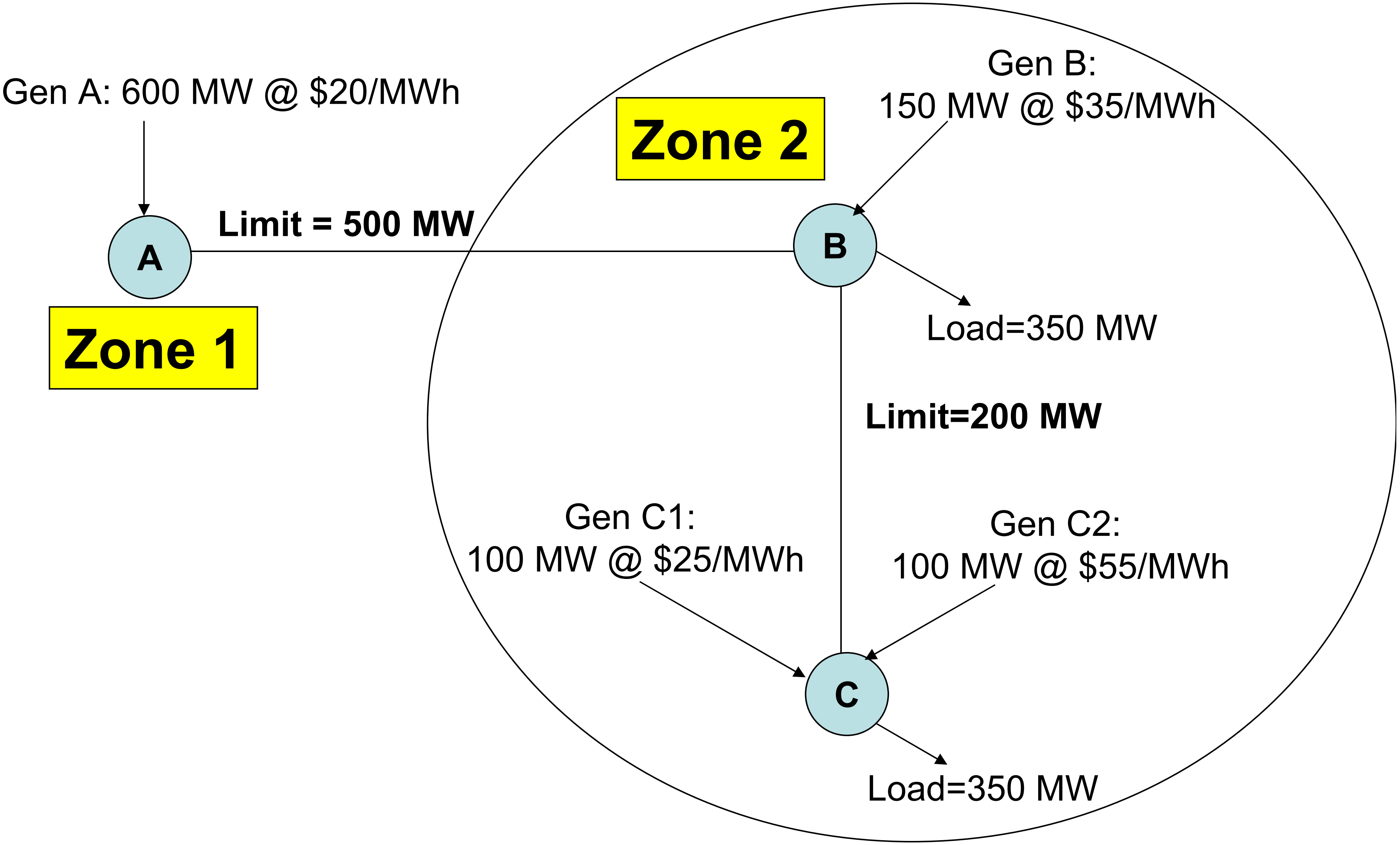
Optimal Bidding Strategy

- Period 1:
    - ➢ Bid 100 MW between $33 and $45 (say $44)
- Period 2 (only if selected in period 1)
    - ➢ Stair 1: Bid 90 MW below $5
    - ➢ Stair 2: Bid 10 MW at $30

Zonal Pricing System

- Many markets have adopted zonal pricing system for congestion management
- A select set of “commercially” significant transmission constraints (“inter-zonal”) are priced via market means
  - ➢ “Intra-zonal” congestion is socialized
- Typically, zonal congestion priced using “uniform price” auction
  - ➢ “Pay-as-bid” for resolving intra-zonal congestion

Example 5: Zonal Pricing



Gen A: 600 MW @ $20/MWh

Zone 2

Gen B:
150 MW @ $35/MWh

Limit = 500 MW

A

B

Zone 1

Load=350 MW

Limit=200 MW

Gen C1:
100 MW @ $25/MWh

Gen C2:
100 MW @ $55/MWh

C

Load=350 MW

Example 5: Zones

- Zone 1 has bus A
- Zone 2 has busses B and C
- Inter-zonal limit = 500 MW
- Intra-zonal limit (in Zone 2) = 200 MW

Example 5: Market Clearing Rules

- There are two rounds
- In round 1, intra-zonal congestion limit is ignored (i.e., set to infinity)
    - All winning bids are paid zonal clearing price
    - If there is no inter-zonal congestion, Zone 1 and Zone 2 prices are the same and are set by highest winning bid
    - If there is inter-zonal congestion, each zone’s prices are set by highest winning bid within zone

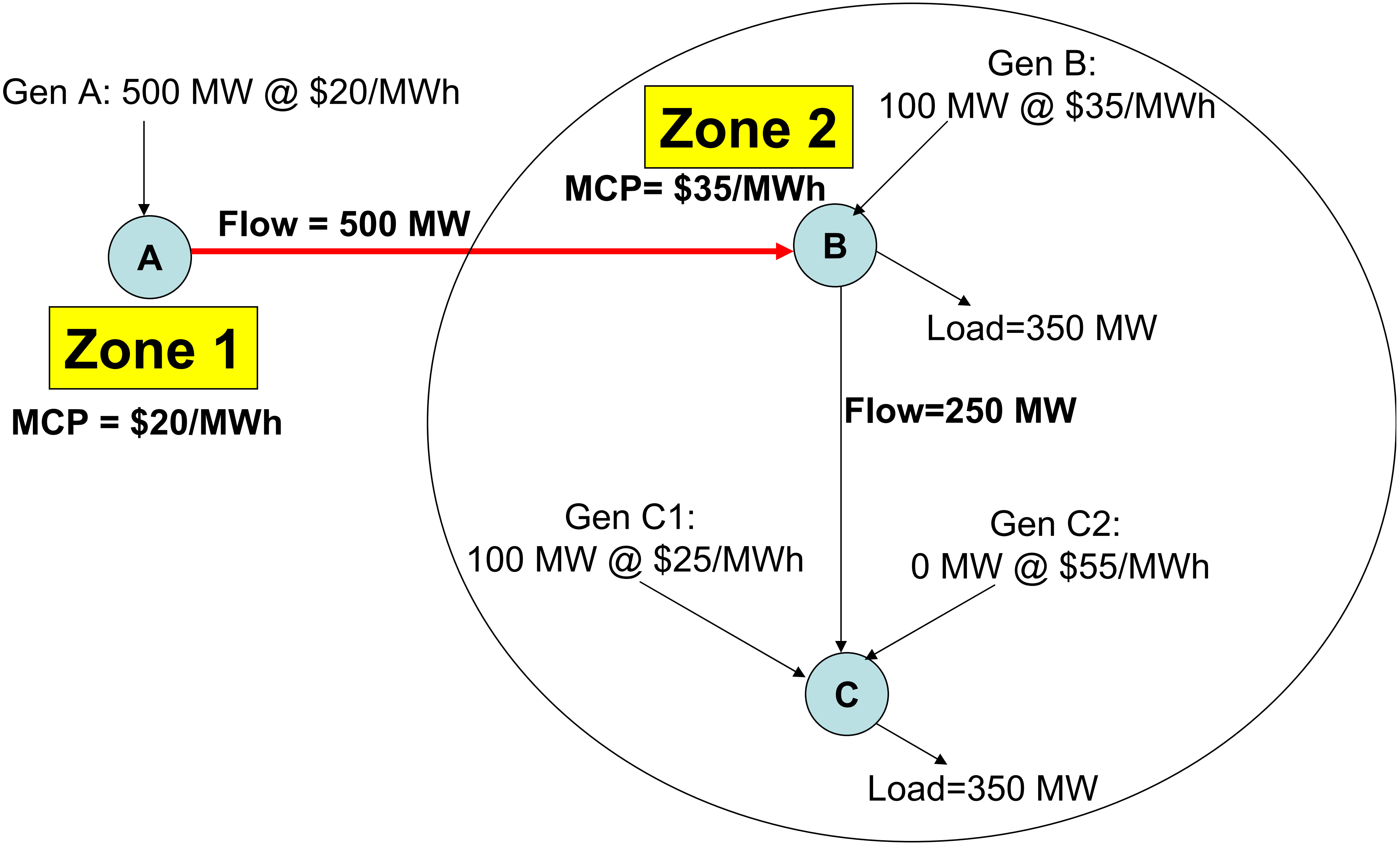
Market Clearing Rules: Round 2

- In round 2, if intra-zonal congestion limit is violated, then generators are re-dispatched
    - Generators that are “constrained up” relative to round 1 dispatch are paid their bid price
    - Generators that are “constrained down” relative to round 1 dispatch pay round 1 zonal price

Example 5: Optimal Bidding Strategy

- In the absence of market power, all generators will bid their incremental costs, excepting generator C1
- Generator C1 will bid just below $55/MWh
- To see why, assume first that everyone bids their incremental costs

Round 1: Everybody Bids Costs



Gen A: 500 MW @ $20/MWh

A

Zone 1

MCP = $20/MWh

Flow = 500 MW

Zone 2

MCP= $35/MWh

Gen B:
100 MW @ $35/MWh

B

Load=350 MW

Flow=250 MW

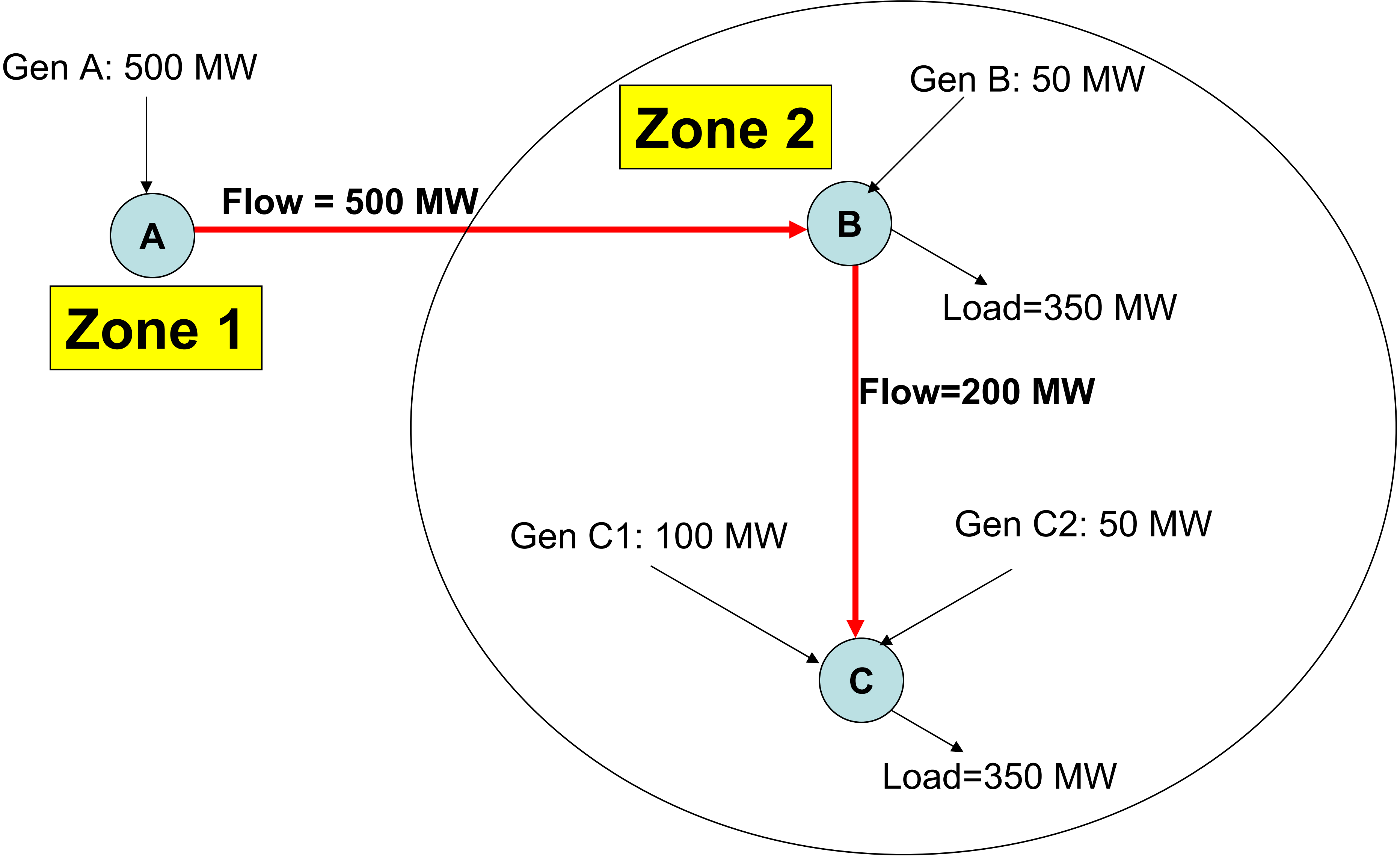
Gen C1:
100 MW @ $25/MWh

Gen C2:
0 MW @ $55/MWh

C

Load=350 MW

Round 2: Intra-Zonal Congestion Resolved



Example 5*: Observations

- Gen C2 gets paid its bid of $55/MWh and therefore sets the price at node C
- Therefore the intra-zonal pricing rules introduce "pay-as-bid" type features
- Optimal strategy for Gen C1 is to bid just below highest winning bid at node C, which is $55/MWh
    - ➢ *Aside: Note that this has the effect of increasing round 1 Zone 2 prices to $55/MWh*

*Example idea based on exchange between CAISO and Harvey & Hogan, see http://ksghome.harvard.edu/~whogan/zonal_Feb11.pdf

Optimal Bidding Strategy Theory*

*** Rajaraman and F. Alvarado, “Optimal Bidding Strategy in Electricity Markets Under Uncertain Energy and Reserve Prices”, PSERC Report 03-05, April 2003 *(note: A new version is due early 2005)***

Inputs

- Modeling generator characteristics
- Modeling exogenous prices
  - When generator has market power, need to model exogenous "residual demand curves"
- Modeling auction characteristics

Generator Cost Characteristics

- Generators costs include:
  - Incremental or marginal costs
  - Startup/shutdown costs
  - No-load costs
  - Ramping costs

Non-Convex/Discontinuous Costs

- Are there startup and shutdown costs to consider?
- Are there “valve points” or other regions of inefficient operation to worry about?
- Are the generator marginal costs declining?

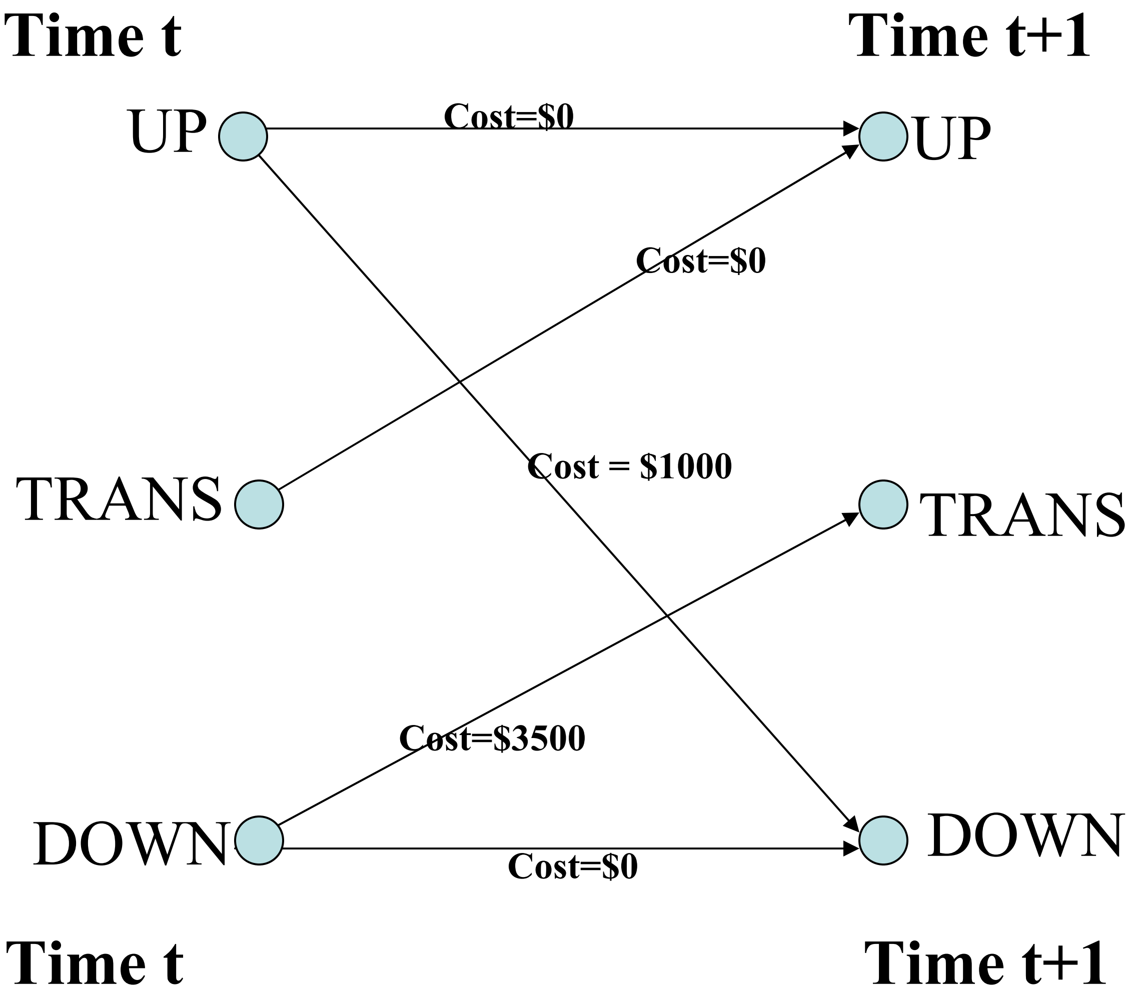
Generator Operational Constraints

- Generators have complex constraints:
  - MW limits on energy and reserves
  - Sum of energy and reserve MWs cannot exceed total capacity limits
  - Inter-temporal operating constraints

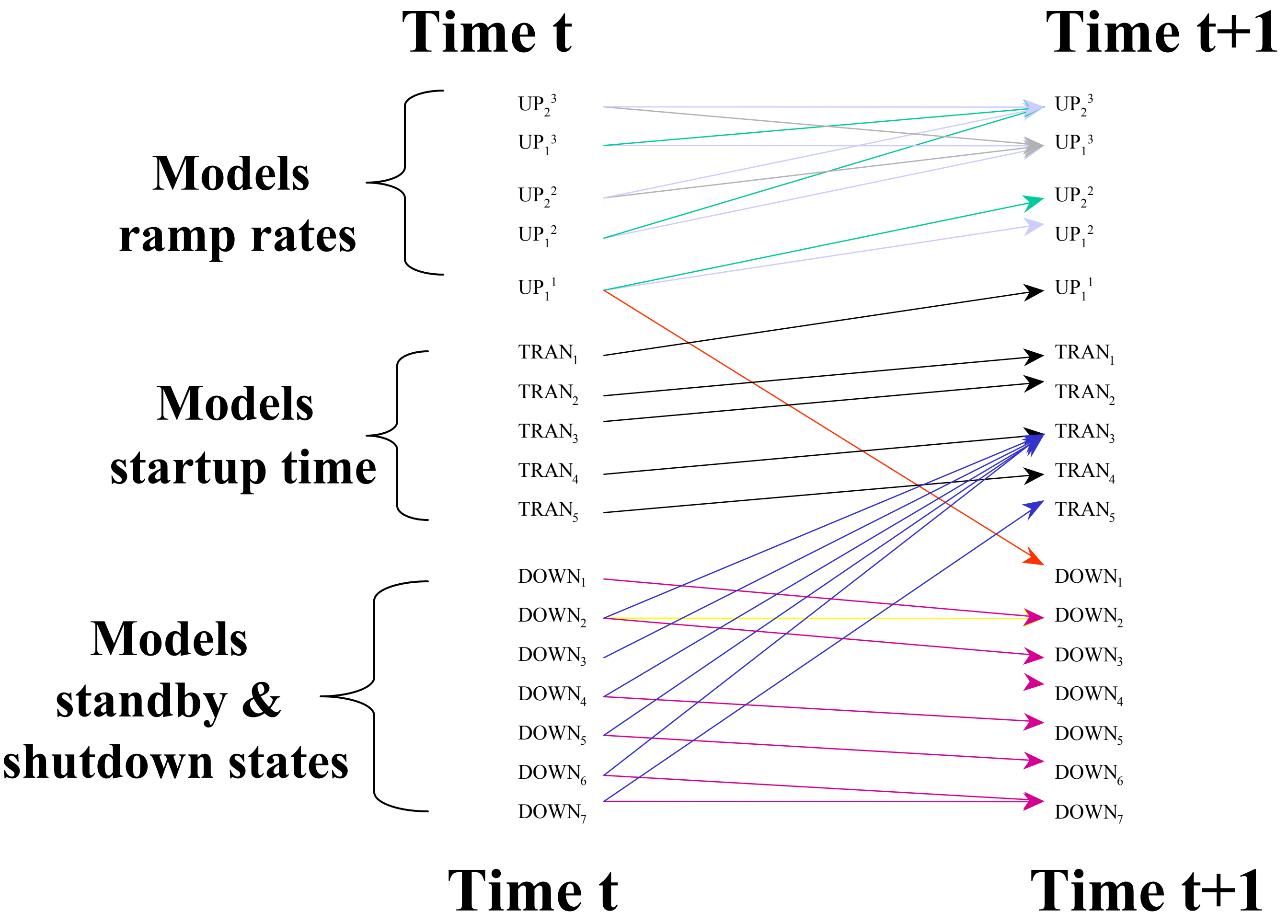
Inter-temporal constraints

- Does the generator have minimum up or down times?
- Are there startup delays?
- Does the generator have total energy or emissions constraints over multiple periods?
- What are the generator ramping rates?

States and State Transitions (1)



States and State Transitions (2)



Time t **Time t+1**

Models ramp rates

Models startup time

Models standby & shutdown states

UP_2^3
UP_1^3
UP_2^2
UP_1^2
UP_1^1
$TRAN_1$
$TRAN_2$
$TRAN_3$
$TRAN_4$
$TRAN_5$
$DOWN_1$
$DOWN_2$
$DOWN_3$
$DOWN_4$
$DOWN_5$
$DOWN_6$
$DOWN_7$

UP_2^3
UP_1^3
UP_2^2
UP_1^2
UP_1^1
$TRAN_1$
$TRAN_2$
$TRAN_3$
$TRAN_4$
$TRAN_5$
$DOWN_1$
$DOWN_2$
$DOWN_3$
$DOWN_4$
$DOWN_5$
$DOWN_6$
$DOWN_7$

Time t **Time t+1**

Handling Price Uncertainty

- Discrete (or continuous) price states are used to model uncertainty
  - E.g., High, Medium, Low
- Are prices correlated between time periods?
  - How is price in one period correlated with next period?
- Are prices correlated between markets?
  - How is energy price correlated with reserve price?

Auction Issues

- Auction type: Uniform price vs. pay as bid
  - ➢ Of course location (should) matter
- Single-part vs. multi-part bids
- Simultaneous vs. sequential auctions
- Specific rules of the auction

Electricity Markets

Market	Prices
Energy	$/MWh
Regulation	$/MW/h, $/MWh
Spinning Reserves	$/MW/h, $/MWh
Non-Spinning Reserve	$/MW/h, $/MWh
Backup Reserves	$/MW/h, $/MWh

Auction type

- Uniform Price Auction:
    - ➢ all winners are paid “the” market clearing price (*e.g.,* the lowest losing bid)
    - ➢ bidders have incentives to bid true costs
    - ➢ used in the major pools (NE, NY, PJM)
- Pay-as-Bid Auction:
    - ➢ all winners get paid what they bid
    - ➢ bidders want to bid above costs and close to expected market clearing prices
    - ➢ used in bilateral markets

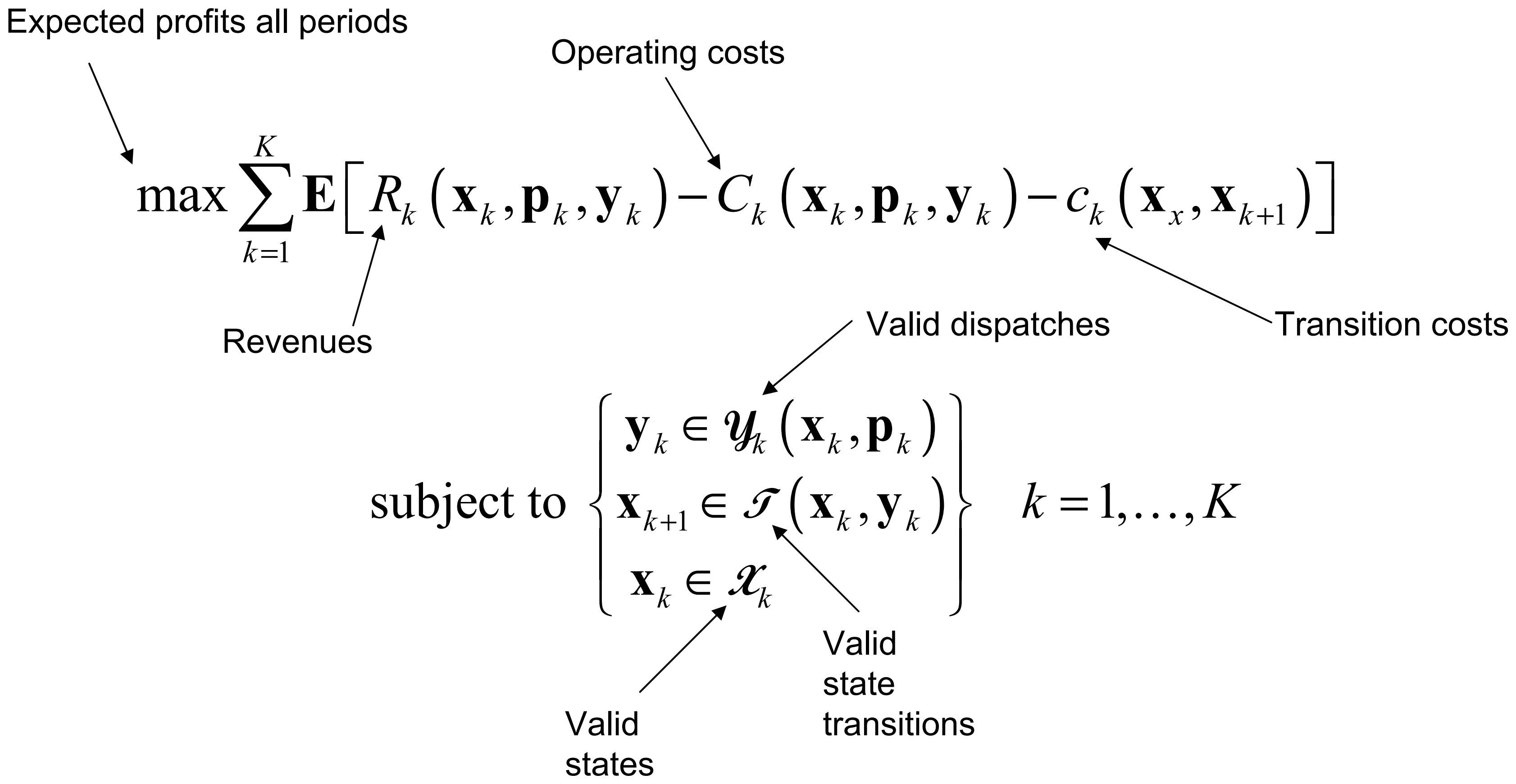
Simultaneous Auction

- ISO clears all markets simultaneously:
    - ➢ determines uniform prices for each service
    - ➢ determines winning schedules
    - ➢ guarantees profit optimality for all bidders
- Price-taking bidders bid their cost curves
    - ➢ At ISO-determined market prices, no bidder can increase profits by changing schedules in the different markets
- Example: NYISO

Sequential Auction

- Markets clear separately:
  - ➢ First energy, regulation, spin reserves, etc.
  - ➢ Any capacity that is unused after one market clears is bid into the subsequent market
- Winners are paid the uniform market clearing price in each market
  - ➢ Example: CAISO's old market design

The Model:
Maximize *Expected Ex-Ante Profits*



Expected profits all periods

Operating costs

$$\max \sum_{k=1}^{K} \mathbf{E}\left[R_k\left(\mathbf{x}_k, \mathbf{p}_k, \mathbf{y}_k\right) - C_k\left(\mathbf{x}_k, \mathbf{p}_k, \mathbf{y}_k\right) - c_k\left(\mathbf{x}_x, \mathbf{x}_{k+1}\right) \right]$$

Revenues

Valid dispatches

Transition costs

$$\text{subject to} \left\{ \begin{array}{c} \mathbf{y}_k \in \mathcal{Y}_k\left(\mathbf{x}_k, \mathbf{p}_k\right) \\ \mathbf{x}_{k+1} \in \mathcal{T}\left(\mathbf{x}_k, \mathbf{y}_k\right) \\ \mathbf{x}_k \in \mathcal{X}_k \end{array} \right\} \quad k = 1, \ldots, K$$

Valid
state
transitions

Valid
states

This model leads to a *nested* Dynamic Programming Problem with *uncertainty*

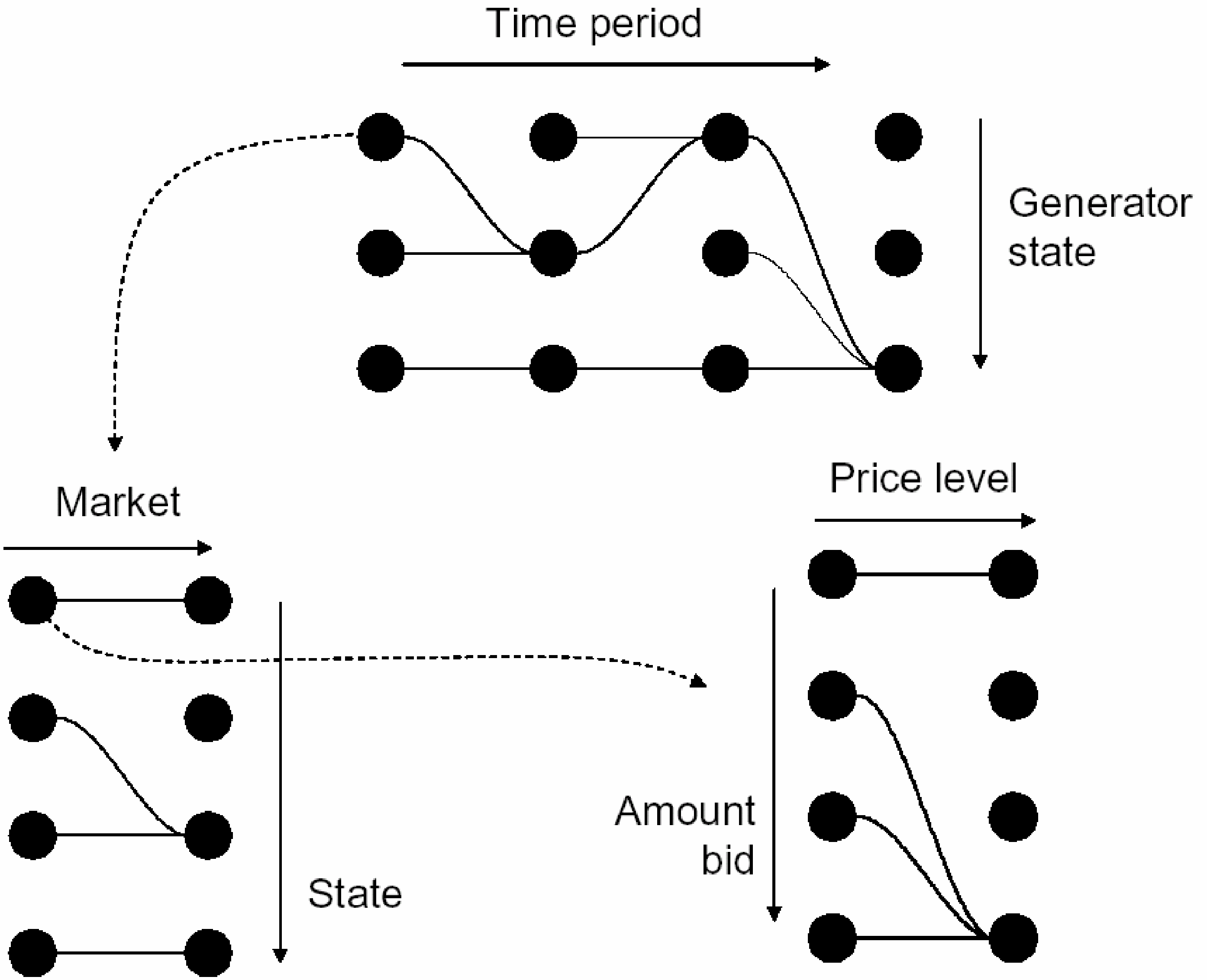
Optimal Bidding Strategy

- The optimal bidding strategy is a **function of MW versus price**, not just an expected value of the dispatch
    - ➢ This takes full account of generator optionality
- This **function** depends on both the *generator state* and on *exogenous* prices
- The **function** must satisfy all market (bidding and other) rules
    - ➢ 3-part versus 1-part bids
    - ➢ Simultaneous versus sequential auctions

Nested Backward DP

- We want to optimize over time periods
  - This involves a backward DP with time periods as stages
- We want to optimize among markets (energy and reserves)
  - This is a backward DP with auction rounds as stages
- We want to our bids to be "biddable"
  - This involves a backward DP where larger bids require larger prices (no downward transitions allowed)

Three-Level Nested DP



$$
\begin{aligned}
J_2^*(x_2, p_2, y_2, x_3) &= p_2 y_2 \\
J_2^*(x_2, p_2, y_2) &= p_2 y_2 \\
x_3^*(x_2, p_2, y_2) &= 0 \\
y_2^*(x_2, p_2) &= x_2 \\
x_3^*(x_2, p_2) &= 0 = \mathrm{EMPTY} \\
J_1^*(x_1, p_1, y_1, x_2) &= \begin{cases} 30y_1 + 40x_2 & \text{if } p_1 = \$30/\mathrm{MWh} \\ 20y_1 + 15x_2 & \text{if } p_1 = \$20/\mathrm{MWh} \end{cases}
\end{aligned}
$$

Energy-limited hydro

- Two periods/rounds
- 100 MWh in reservoir
- No new water
- Must use all water
- Single price auction
- Sequential clearing
- Unrestricted bids
- 50 MW "chunks"
- Correlated prices

- Period 1: bid 100 MW if price is LOW **and 0 MW if price is HIGH**
- Period 2: bid remainder at LOW

Expected profits if "full": $3000

$$
\begin{aligned}
J_1^*(x_1 = \mathrm{FULL}, p_1 = \mathrm{HIGH}, 0) &= 30 \cdot 0 + 40 \cdot 100 = 4000 \\
J_1^*(x_1 = \mathrm{FULL}, p_1 = \mathrm{LOW}, 0) &= 20 \cdot 0 + 15 \cdot 100 = 1500 \\
J_1^*(x_1 = \mathrm{FULL}, p_1 = \mathrm{HIGH}, 50) &= 30 \cdot 50 + 40 \cdot 50 = 3500 \\
J_1^*(x_1 = \mathrm{FULL}, p_1 = \mathrm{LOW}, 50) &= 20 \cdot 50 + 15 \cdot 50 = 1750 \\
J_1^*(x_1 = \mathrm{FULL}, p_1 = \mathrm{HIGH}, 100) &= 30 \cdot 100 + 40 \cdot 0 = 3000 \\
J_1^*(x_1 = \mathrm{FULL}, p_1 = \mathrm{LOW}, 100) &= 20 \cdot 100 + 15 \cdot 0 = 2000 \\
x_2^*(x_1 = \mathrm{FULL}, p_1, 0) &= \mathrm{FULL} \\
x_2^*(x_1 = \mathrm{FULL}, p_1, 50) &= \mathrm{HALF} \\
x_2^*(x_1 = \mathrm{FULL}, p_1, 100) &= \mathrm{EMPTY}
\end{aligned}
$$

	Time Period	
p_k	1	2
HIGH	30	40
LOW	20	15

Optimal profits

	Time Period	
(x_k, p_k)	1	2
(EMPTY,LOW)	0	0
(EMPTY,HIGH)	0	0
(HALF,LOW)	1000	750
(HALF,HIGH)	2000	2000
(FULL,LOW)	**2000**	1500
(FULL,HIGH)	**4000**	4000

Optimal dispatch

	Time Period	
(x_k, p_k)	1	2
(EMPTY,LOW)	0	0
(EMPTY,HIGH)	0	0
(HALF,LOW)	50	50
(HALF,HIGH)	0	50
(FULL,LOW)	100	100
(FULL,HIGH)	0	100

Fill these two tables working "backwards"

Optimal profits for period 1, state is FULL

	p_1	
y_1 in MW	LOW (\$20/MWh)	HIGH (\$30/MWh)
0	1500	**4000**
50	1750	3500
100	**2000**	3000

Computational details

- Solve a nested *backward* dynamic programming problem
    - This is best done by creating a number of intermediate "tables"
    - See paper for details of examples

- A model called "GenOptimizer" has been developed by Laurits R. Christensen Associates (www.LRCA.com)

GenOptimizer

- The model implements some of our ideas

- The model can:
  - find profit maximizing commitment and dispatch,
  - assist in finding optimal bidding strategies
  - be used to investigate allegations of market power,
  - and prove highly educational and informative

GenOptimizer Inputs

- **Energy and reserve price forecasts**
- **Price volatilities**
- **Fuel costs**
- **Generator heat rate**
- **Minimum and maximum energy dispatch constraints**
- **Maximum reserve dispatch constraints**
- **Likelihood that offered reserve services will be called**
- **Start up time of a cold generator vs. a hot generator**
- **Minimum down time of a generator**

GenOptimizer Inputs (2)

- **Time it takes for a hot generator to become cold**
- **Ramping rate of the generator**
- **Cost to start a cold generator vs. a hot generator**
- **Cost to shut down the generator from a low dispatch vs. a high dispatch**
- **Banking costs**
- **No-load costs**
- **Ramping costs**
- **Planned generator outages and must-run conditions**

GenOptimizer Execution

- Backward Dynamic Programming determines the optimal strategy in every time period, generator dispatch state, and price level
    - Takes account of price uncertainty
    - Takes account of all operational constraints
- Monte Carlo algorithm used to evaluate the performance of the optimal commitment strategy in the face of price volatility.
- Finds the optimal energy and reserve dispatches for given price levels

GenOptimizer Results (1)

- Expected revenue, costs, and profit by hour for energy and reserve services
- Standard deviation of expected profit
- Minimum and maximum profit achieved over the set of Monte Carlo runs
- Distribution of profit over the set of Monte Carlo runs

GenOptimizer Results (2)

- Aggregated analysis of the commitment strategy taken by the generator over the set of Monte Carlo runs.

- Analysis of optimal dispatch strategy

Conclusions (1)

- In ideal uniform pricing auctions, optimal bidding strategy is to bid “costs”, but
- Many seemingly uniform pricing auctions have “pay-as-bid” features, e.g.,
  - Zonal pricing
  - Sequential auctions
- 3-part bid feature of Northeast ISOs comes closest to “uniform pricing”, but:
  - Cannot handle opportunity costs beyond 24-hr time frame (e.g., hydro, emission costs)
  - Cost structure may not fit into bidding rules

Conclusions (2)

- In practice, optimal bidding strategy must consider the following:
  - Auction design rules
  - Inter-temporal constraints and energy limitations
  - Multiple markets
  - Price volatility and price correlation

Conclusions (3)

- Optimal bidding strategy maximizes *ex-ante expected* profits
  - ➢ Because prices are *uncertain*, optimal strategy can *seem* sub-optimal in hindsight
- Using these methods,
  - ➢ Generators can better optimize profits and audit past performance
  - ➢ Regulators can investigate market power allegations

Conclusions (4)

- Optimal bidding strategy theory
  - ➢ Rajaraman & Alvarado, "Optimal Bidding Strategy in Electricity Markets Under Uncertain Energy and Reserve Prices", PSERC Report 03-05, April 2003
  - ➢ Note: A new version is due early 2005
  - ➢ LRCA's GenOptimizer
- Questions?
  - ➢ Email me at: rrajaraman@LRCA.com
  - ➢ LRCA website: www.LRCA.com